Better Bath, Better Life

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Abstract. In the paper, we develop a model of temperature of tub water with energy conservation law. Meanwhile, we make a best way that can both keep temperature fixed and save water consumptions. First, we individually consider the heat exchange in three situations, which separately correspond model 1, model 2: water in the bathtub at a standstill and an equation of inflow and discharge. Energy conservation law and thermal balance equation are utilized to analysis heat exchange. Second, we calculate heat changes due to heat convection, conduction and radiation. Finally, we conduct sensitivity analysis on our model. It demonstrates that the model has a great dependency on the volume of the tub.

Introduction

Early bathtubs in England tended to be made of cast iron, or even tin and copper with a face of paint applied that tended to peel with time [1]. In the U.S., the commoditized bathtub first appeared is in the 1840s [2]. Comparing to the cast-iron bath, ceramic bathtub is much easier to clean up and have a greater possibility of a wide range of shape. Moreover, it is better to display modernity, simple texture .The bathtub accommodated the most basic of human functions, but Americans used modern technology, plumping, fixtures, and bathroom spaces as identifications of their social status [3].

Our team investigates a strategy to keep comfortable temperature constant and save water.

Assumptions

- About the heat exchange between the person and water in the bathtub, we only consider the effect of convection.
- Indoor air flows slowly, so we think thermal conduction is the main method of heat exchange between the bathtub and outside air.

Parameters: In the section, we use some symbols for constructing the model as follows:

Table 1. Model Parameters

Parameters	Meaning
C	Specific heat of water
m_0	Mass of the water when people get into the tub
T	Real-time temperature of the water in the bathtub
T_1	Temperature of the hot water from the hot faucet
T_2	Indoor air temperature
T_3	Temperature of the out surface of the tub wall
T_r	Temperature of the people
S_0	Area of the bottom of the tub
S_1	Contact area between water and the surface of the tub wall
S_2	Contact area between water and air
S_r	Superficial area of the people
λ	Thermal conductivity of acrylic
d	Thickness of the tub wall
v	Average relative speed between the water and human

Other symbols instructions will be given in the text.

The Model 1: Water in the Bathtub at a Standstill

When the person get into the tub, the temperature and height of the water in the bathtub is comfortable. m_0 is the mass of water at this stage. Later, none of water is injected into bathtub and the water content is certain. We consider a period $\left(t, t+dt\right)$ and the temperature is $\left(T, T+dT\right)$.

Step 1. Q_1 **Variation of heat energy of the water in the bathtub:** By knowing the density of water, one can determine the mass flow rate based on the volumetric flow rate and then solve the energy [5]. We have

$$Q_1 = Cm_0(T + dT - T) = Cm_0 dT \qquad (1)$$

Step 2. Q_2 **Energy loss due to the heat exchange between the tub wall and air:**

$$Q_2 = \left(H_1 S_1 (T_3 - T_2) + \frac{S_1 \lambda (T - T_2)}{d} + \varepsilon \sigma_1 S_1 (T_3^4 - T_3^3) \right) dt \qquad (2)$$

Step 3. Q_3 **Energy loss due to the heat exchange between the water surface and outside air:**

$$Q_3 = \left(H_2 S_2 (T - T_2) + W + \varepsilon \sigma_2 S_2 (T_3^4 - T_3^3) \right) dt \qquad (3)$$

Step 4. Q_4 **Energy loss due to the heat exchange between the person and the water in the tub:**

$$Q_4 = H_r S_r (T - T_r) dt + \frac{S_1 \lambda_r (T - T_r)}{d_r} dt + \varepsilon_r \sigma_r S_r (T_r^4 - T_r^3) dt \quad (4)$$

Step 5. Q_5 **Total energy loss:** Total energy loss is a sum of the heat exchange between tub wall and air, water surface and air, the person and water in the bathtub. We have

$$Q_5 = Q_2 + Q_3 + Q_4 \quad (5)$$

Step 6. Equilibrium relation: As we all know, the law of conservation of energy states that the total energy of an isolated system remains constant—it is said to be conserved over time. Energy can neither be created nor destroyed; rather, it transfo**rms from one form to another** . On the basis of it, we can get a balanced equation as follows:

$$\begin{cases} Cm\dfrac{dT}{dt} = -\dfrac{S_1 \lambda (T - T_3)}{d} - H_2 S_2 (T - T_3) - H_r (T - T_r) S_r \\ T(t=0) = T_0 \end{cases} \qquad (8)$$

In the process, the heat energy variation of the water in the tub is the total decrement of the heat energy of the water.

The Model 2: an Equation of Inflow and Discharge

Step 1. Q_6 Heat given off by the hot water from the tab: This part is as same as Model 2: Extra hot water with extra energy is taken to the bathtub. We have

$$Q_6 = \rho q C (T_1 - T) dt$$

Step 2. Equilibrium relation: The equilibrium equation is as the following:

$$Q_1 = -Q_5 + Q_6$$

$$Cm_0 dT = \left(q\rho (T_1 - T) - \frac{S_1 \lambda (T - T_3)}{d} - H_2 S_2 (T - T_3) - H_r (T - T_r) S_r \right) dt$$

The final model is

$$\begin{cases} Cm_0 \dfrac{dT}{dt} = q\rho(T_1 - T) - \dfrac{S_1\lambda(T-T_3)}{d} - H_2S_2(T-T_3) - H_r(T-T_r)S_r \\ T(t=0) = T_0 \end{cases} \quad (12)$$

Numerical Computation and Results Analysis

We determine values for some parameters.Specific heat of water C is $4200 J/(kg \cdot K)$; Density of water ρ is $1000\ kg/m^3$. The person get into the bathtub when the surface elevation is 500mm, we can calculate $m_0 = 555.945kg$. The initial contact area $S_1 = 3.646725m^2$ The thermal conductivity of acrylic is $0.18\ w/(m \cdot K)$; The indoor temperature is $298.13K$; Body temperature is $310.13K$; Average relative speed between the water and human is about $0.5m/s$; Superficial area of the people S_r is $0.921915\ m^2$; Thickness of the tub wall is $0.0035\ m$; H_r and H_2 are empirical values. We have $H_r = \sqrt[3]{270v^3 + 23}$ and $H_2 = 0.1987J/(m^2 \cdot s \cdot K)$.

The Model 1: In Figure 1, the green line shows the temperature decreases approximately as a linear function. We can get the relationship between the temperature and time:

$$T = 15.754 / \exp(0.0005993t) + 298.38$$

The time of temperature from upper limits to lower temperature is about 1680s

The Model 2: As shown in Figure 1, The equation is

$$T = 320.15 \text{ - } 8.0169/\exp(0.00049731t)$$

The time from lower limits to upper limits is $577.06s$

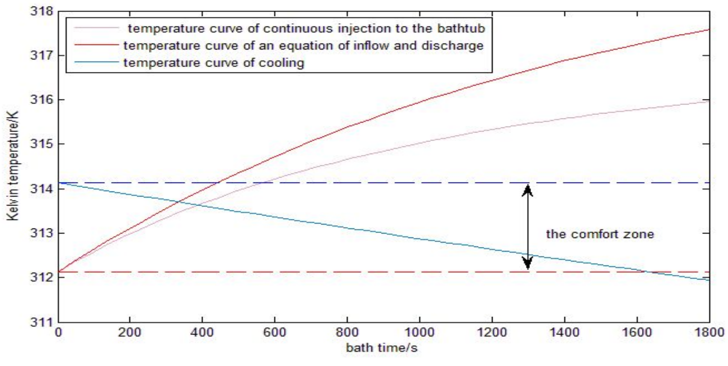


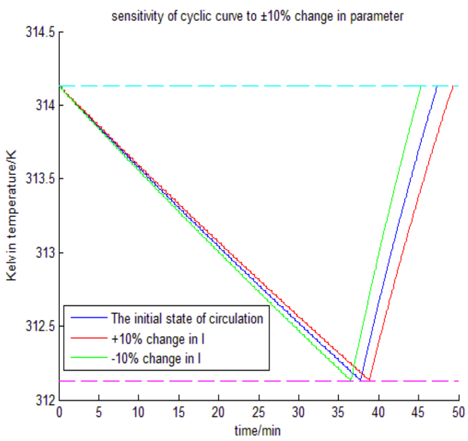
Figure 1. Curves of temperature variation

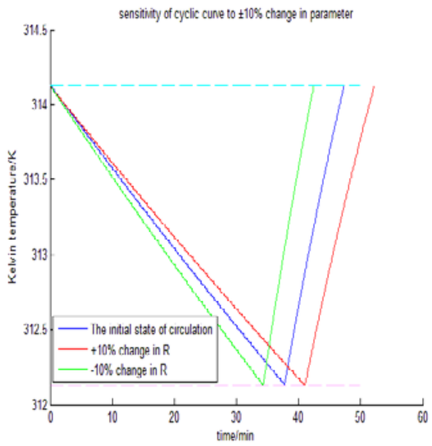
Sensitivity

Ordinary values: t_1 is the total time of losing heat free without hot water injection. t_2 is the total time when the hot water is into bathtub. $t_2/(t_1 + t_2)$ can measure the total amount of water in some way.

Table 2. Sensitivity of heating water time and free cooling time to ±10% change in parameter values

Parameter	Percentage change in time for +10% in parameter			-10% change in parameter		
	t_1	t_2	$t_2/(t_1+t_2)$	t_1	t_2	$t_2/(t_1+t_2)$
h	+1.1%	+13.91%	+9.85%	-1.34%	-13.22%	-9.835%
v	-0.034%	+0.01%	+0.04%	+0.03%	-0.008%	-0.03%
V_r	-1.02%	-1.02%	0	+1.02%	+1.02%	0
T_r	+4.52%	-1.095%	-4.329%	-4.146%	+1.12%	+4.33%
R	+8.78%	+16.37%	+5.48%	-9.089%	-15.245%	-5.47%
l	-2.88%	+8.96%	+4.66%	-3.155%	-8.749%	-4.657%
H_2	-0.146%	+0.04%	+0.15%	+0.15%	-0.037%	-0.146%





Conclusion

Without water inflow, the model 1 shows a linear function that the temperature decreases gradually. In the model 2, it has an equation of inflow and discharge and shows that the water in the tub is warming faster. we conduct sensitivity analysis on our model. When the shape/volume/temperature of the person is changed separately from -10% to 10%, the results change is no more than 4.52%. When the motions made by people is changed, it also just has a slight wave. We validate that our result is correct and robust. While we change the volume of bathtub, the model has a 16.37% change. It demonstrates that the model has a great dependency on the volume of the tub.From the sensitivity analysis, we know the model has a great stability and is reliable.

Reference

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[3] Jennings J. Le Corbusier's "Naked":"Absolute Honesty" and (Exhibitionist) Display in Bathroom Settings[J]. Interiors, 1917, 2(3): 307-332.

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