

Update Algorithm on One-Step-Lag Out-of-Sequence Measurement with Correlated Noise Based on Particle Filter

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Key Words: OOSM; nonlinear; correlated noise; forward prediction; particle filter

Abstract. In the target tracking system, sensor measurements may arrive at the fusion center out of sequence because of the different communication delays, which results in the Out-of-Sequence Measurement(OOSM) problem. In order to solve one-step-lag OOSM problem with corrected process noise and measurement noise in nonlinear system, a new algorithm has been proposed. By combining the framework of the forward prediction filtering, wipe off the correlation, and use Particle filtering to estimate the state. Simulations verify the effectiveness of the proposed algorithm.

Introduction

In a centralized multi-sensor target tracking system, various sensors have different sampling rates, pretreatment time and communication delay, measurement synchronizations are difficult to ensure, which can cause measurement from the same target arrive to the fusion center out of sync, the asynchronous problem occurred [1,2,3]. Further, there will be a phenomenon that the measurement observed earlier arrives later, which can be called OOSM(Out-of-Sequence Measurement) phenomenon. And the traditional filtering algorithm cannot directly handle these OOSM, need to study the corresponding filter algorithm.

At present, the most suitable for the real-time processing of OOSM is direct update method, using OOSM and target state estimator in full stored to update the current state estimation, working out a new state estimation and estimation error covariance matrix [5]. On account of the small storage volume and calculated amount of direct update method, and no lag output, scholars have proposed the A1, B1, AA1^[5] linear filtering algorithm to deal with OOSM.

For weakly nonlinear Gaussian system, [4] discusses EKF-A1 algorithm, linearize the nonlinear system, and apply the existed A1 algorithm to it. The above algorithm is relatively simple, but there may be a larger filtering error. UT transform was proposed to solve jacobian matrix or hessian matrix of nonlinear measurement equations^[4], but this method cannot apply to the situation that the system equations is nonlinear, and the approximation error is bigger. For strongly nonlinear system, particle filter was presented to solve the OOSM problem in [6][9], but process and measurement noise was not taken a sufficient consideration. When observations in common environment, may be interfered by the same noise, the process noise and measurement noise is often corrected. Some algorithms were raised to address the corrected problem in [7][8], but they worked only when the system was linear.

Aiming at this, under the forward prediction framework, nonlinear system equations are identical transformation to be uncorrected, and then update the state with particle filter. Algorithm can effectively deal with one-step-lag OOSM problem.

Formulation of the problem

Consider a target dynamic model evolving from t_{k-1} to t_k

$$x_k = f_{k,k-1}(x_{k-1}) + \Gamma_{k,k-1}w_{k,k-1} \quad (1)$$

$$z_k = h_k(x_k) + v_k \quad (2)$$

where, $f_{k,k-1}$ is the state transition matrix to time t_k from time t_{k-1} , $h_k(\bullet)$ is the measurement

matrix, Γ_{k-1} is noise input matrix, $w_{k,k-1}$ is the process noise for this interval, v_k is the measurement noise, z_k is the measurement of sensor at time t_k , the process and measurement noise is white gaussian noise.

Assumption 1 $w_{k,k-1}$ and v_k is corrected, its statistical characteristics are as follows

$$\begin{aligned} E(w_{k,k-1}) &= q, \text{Cov}(w_{k,k-1}, w_{j,j-1}^T) = Q_k \delta_{kj} \\ E(v_k) &= r, \text{Cov}(v_k, v_j^T) = R_k \delta_{kj} \\ \text{Cov}(w_{k,k-1}, v_j^T) &= S_k \delta_{kj} \end{aligned}$$

where, Q_k is the negative definite symmetric matrix, R_k is symmetric positively defective matrix, δ_{kj} is kronecker- δ function.

Assumption 2 The initial state $x(0)$ is not corrected to $w_{k,k-1}$ or v_k , its statistical characteristics are as follows

$$\begin{aligned} \hat{x}_0 &= E(x_0) \\ P_0 &= \text{Cov}(x_0) = E[(x(0) - \hat{x}_0)(x(0) - \hat{x}_0)^T] \end{aligned}$$

by (1)

$$x_k = f_{k,d}(x_d) + \Gamma_{k,d} w_{k,d} \quad (3)$$

At time t_k one has

$$\hat{x}_{k|k} \triangleq E[x_k | Z^k], P_{k|k} \triangleq \text{Cov}[x_k | Z^k] \quad (4)$$

where Z^k is the cumulative set of measurement at time t_k .

Subsequently, the earlier measurement from time t_d , denoted from now on with discrete time notation as d ,

$$z_d = h_d(x_d) + v_d \quad (5)$$

arrives after the state estimate (4) has been calculated. As shown in fig.1.

The problem is as follows: update the posteriori distribution $p(x_k | z_{1:k})$ with OOSM z_d , to obtain $p(x_k | z_{1:k}, z_d)$, so that the latest state estimate and the covariance matrix can be calculated.

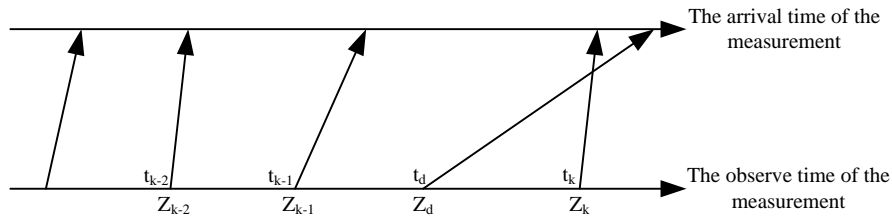


Fig.1 One-step-lag OOSM situation

Algorithm design

Aiming at last given nonlinear discrete system, considering the same time process noise and measurement noise corrected situation, put forward to solve the problems of the one-step-lag OOSM filtering algorithm.

Solve the corrected process noise and measurement noise

The measurement equation can be rewritten as

$$z_k - h_k(x_k) - v_k = 0 \quad (6)$$

then (1) can be expressed as

$$\begin{aligned} x_k &= f_{k,k-1}(x_{k-1}) + \Gamma_{k-1}w_{k,k-1} + J_{k-1}[z_{k-1} - h_{k-1}(x_{k-1}) - v_{k-1}] \\ &= \Psi_{k,k-1}(x_{k-1}) + \Gamma_{k,k-1}\bar{w}_{k,k-1} \end{aligned} \quad (7)$$

(7) and (1) are equivalent, where

$$\Psi_{k,k-1}(x_{k-1}) = f_{k,k-1}(x_{k-1}) + J_{k-1}[z_{k-1} - h_{k-1}(x_{k-1})] \quad (8)$$

$$\bar{w}_{k,k-1} = w_{k,k-1} - \Gamma_{k,k-1}^{-1}J_{k-1}v_{k-1} \quad (9)$$

J_{k-1} is undetermined coefficient.

Making the noise uncorrected, that is

$$\begin{aligned} \text{Cov}(\bar{w}_{k,k-1}, v_k^T) &= \text{Cov}\left[(w_{k,k-1} - \Gamma_{k,k-1}^{-1}J_{k-1}v_{k-1}), v_k^T\right] \\ &= \text{Cov}(w_{k,k-1}, v_k^T) - \Gamma_{k,k-1}^{-1}J_{k-1}\text{Cov}(v_{k-1}, v_k^T) \\ &= S_k - \Gamma_{k,k-1}^{-1}J_{k-1}R_k = 0 \end{aligned} \quad (10)$$

where $J_{k-1} = \Gamma_{k,k-1}S_kR_k^{-1}$. So (1)(2) can be express as

$$x_k = \Psi_{k,k-1}(x_{k-1}) + \Gamma_{k,k-1}\bar{w}_{k,k-1} \quad (11)$$

$$z_k = h_k(x_k) + v_k \quad (12)$$

where $\bar{w}_{k,k-1}$ and v_k is uncorrected white gaussian noise, its statistical characteristics are as follows

$$E(\bar{w}_{k,k-1}) = E(w_{k,k-1} - \Gamma_{k,k-1}^{-1}J_{k-1}v_{k-1}) = q_k - S_kR_k^{-1}r_k \quad (13)$$

$$\text{Cov}(\bar{w}_{k,k-1}, \bar{w}_{j,j-1}^T) = (Q_k - S_kR_k^{-1}S_k^T)\delta_{kj} \quad (14)$$

$$\text{Cov}(\bar{w}_{k,k-1}, v_j^T) = 0 \quad (15)$$

Calculate the state $\hat{x}_{d|d}$ and the corresponding covariance $P_{d|d}$

Here one has $\hat{x}_{k-1|k-1}$ and $P_{k-1|k-1}$. After the fusion center receives the OOSM z_d , based on the posterior probability $p(\hat{x}_{k-1|k-1})$ at time t_{k-1} , generates particle swarm $\{x_{k-1|k-1}^i\}_{i=1}^N$, the weights of particle are $\left\{w_{k-1}^i = \frac{1}{n}\right\}_{i=1}^N$.

Single step estimation for particles is

$$\left\{x_{d|k-1}^i = \Psi_{d,k-1}(x_{k-1|k-1}^i) + \Gamma_{d,k-1}\bar{w}_{d,k-1}\right\}_{i=1}^N \quad (16)$$

Update the weights of particle with measurement z_d

$$\left\{w_d^i = w_{k-1}^i p(z_d | x_d^i) = w_{k-1}^i p_{vd}(z_d - h(\hat{x}_{d|k-1}^i))\right\}_{i=1}^N \quad (17)$$

The normalized weights

$$\left\{\tilde{w}_d^i = w_d^i / \sum_{i=1}^N w_d^i\right\}_{i=1}^N \quad (18)$$

As a result, state estimation and its covariance matrix can be obtained

$$\hat{x}_{d|d} \approx \sum_{i=1}^N \tilde{w}_d^i x_{d|d}^i \quad (19)$$

$$P_{d|d} = \sum_{i=1}^N \tilde{w}_d^i (x_{d|d}^i - \hat{x}_{d|d})(x_{d|d}^i - \hat{x}_{d|d})^T \quad (20)$$

Calculate the state $\hat{x}_{k|k,d}$ and the corresponding covariance $P_{k|k,d}$

Regard $\hat{x}_{k|k}$ as the measurement at time t_k , repeat the above step to get the last state $\hat{x}_{k|k,d}$.

Thus, get update algorithm on one-step-lag OOSM with corrected noise based on particle filter. Compared with the existing algorithms, the new algorithm has better performance, this is mainly because:

(1) make the noise corrected system uncorrected, reduced the filtering error when processing OOSM problem.

(2) the new algorithm uses forward filter, so it need not to store a large number of particles and their weight, reducing the storage.

In addition, the optimizing distribution of sampling filter can be used in the algorithm, such as extended kalman particle filter, unscented kalman particle filter, to promote the quality and effectiveness of the particle is improved.

Simulation results

The following dynamic system was considered as

$$x_k = F_{k,k-1}x_{k-1} + \Gamma_{k,k-1}\omega_{k,k-1} \quad (21)$$

where, $F_{k,k-1} = \begin{bmatrix} 1 & \frac{\sin \omega T}{\omega} & 0 & -\frac{1 - \cos \omega T}{\omega} \\ 0 & \cos \omega T & 0 & -\sin \omega T \\ 0 & \frac{1 - \cos \omega T}{\omega} & 1 & \frac{\sin \omega T}{\omega} \\ 0 & \sin \omega T & 0 & \cos \omega T \end{bmatrix}$, $\Gamma_{k,k-1} = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}$,

$x_k = [X_k \ \dot{X}_k \ Y_k \ \dot{Y}_k]^T$, estimated the position state $[x_k \ y_k]^T$. $\omega_{k,k-1}$ is white discretized continuous time process noise.

The measurement equation is

$$z_k = h(x_k) + v_k \quad (22)$$

where, v_k is white discretized continuous time measurement noise, and

$$q_k = 0.2, Q_k = 0.04, r_k = 0.3, R_k = 0.09, S_k = 0.1.$$

In the process of actual measurement, getting the distance γ and deflection angle θ of the target, measurement equation can be rewritten as

$$\begin{bmatrix} \gamma \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{X^2 + Y^2} \\ \arctan(Y/X) \end{bmatrix} + v_k \quad (23)$$

Sampling period is $T = 1$, initial value is $x_0 = [20 \ -2 \ 10 \ -1]^T$, $P_0 = I$, $\omega = \pi/9$, sampling number of particles is $N = 500$.

Assume that the sensor obtains 5 measurement, including OOSM, finally the measurement sequence is: z_1, z_2, z_4, z_3, z_5 . Use different algorithms dealing with this set of measurement, to verify the performance of each algorithm.

- (1) Experiment 1: deal with the measurement sequence z_1, z_2, z_4, z_3, z_5 with algorithm in [6];
- (2) Experiment 2: deal with the measurement sequence z_1, z_2, z_4, z_3, z_5 with new algorithm;
- (3) Experiment 3: Deal with the measurement sequence z_1, z_2, z_4, z_5 with discarding the measurement algorithm.

Table 1. Filtering results compared

Experiment	X Axis RMSE	Y Axis RMSE	Trace
Experiment 1	0.8369	3.2064	0.2920
Experiment 2	0.4681	1.9923	0.2589
Experiment 3	0.8644	3.1009	0.4372

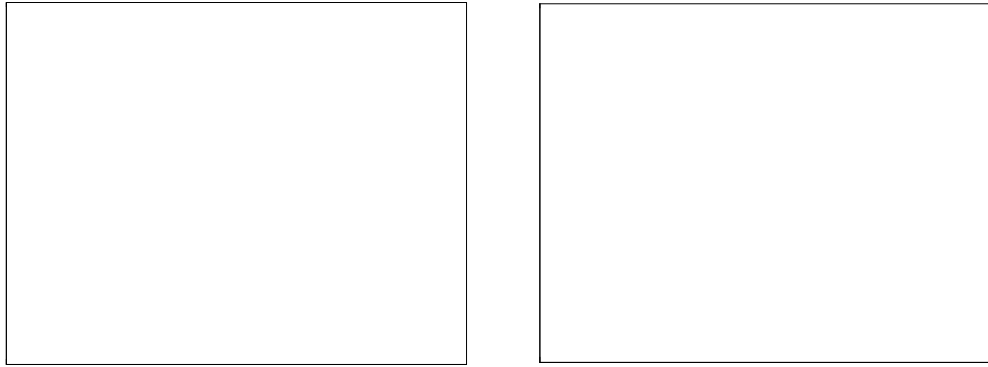


Fig.2 X axis and Y axis position estimation

Table 1 shows the root mean square error (RMSE) of the location and the covariance matrix Trace. Fig.2 shows the comparison between the estimation and true value of the X axis and Y axis of the three experiments. As show in the above results, the performance of the new algorithm is much better than the discarding algorithm.

Summary

In order to solve the one-step-lag OOSM problem with corrected noise in nonlinear system, a new algorithm is proposed here. The new algorithm is based on particle filter and its performance is independent with the discrete time model of the process noise. By using identical transformation, the process noise and measurement noise of the system is not corrected. And by using the particle filter, the state estimation becomes more precise. The computer simulation is also verified the feasibility and validity of the algorithm.

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