The analysis of the heat transfer in water

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Abstract. The High-speed development of Self-regulation temperature control technique gives a lot convenience to most people. Essentially people feel the temperature of water by personal perception. This article discusses the circumstances that people use personal feeling to make a judgment and manual control the faucet.

A hot bath is becoming a part and parcel of daily life. People relax themselves and clean their body by having a hot bath. However some questions just hide behind these normal things. Although the bathtub is simple, the process of the thermal conduction in the bathtub is considerably complex.

Introduction

Firstly, we establish a static cooling temperature field mode. The model includes the thermal conduction and natural convection between air and water. Secondly, set up a dynamic model of adding hot water and divide it into two elementary models. Use the heat convection minimum entropy generation to prove that put the faucet at the bottom of the bathtub is good way for saving energy.

Assumptions

1. The destiny and heat capacity of the water will not change with the change of temperature.
2. Thermal conductivity of the water will not change with the change of temperature.
3. Water can recover stabilization immediately after faucet stop adding hot water.
4. (The hot water added by faucet is turbulent jets, it is difficult to determine the time to recover stabilization .Besides, and this time is much shorter than cooling process)
5. The wall of the bathtub and the floor are adiabatic.

Three dimensional heat conduction of water

After the man and the water reached the dynamic equilibrium state, the temperature distribution of the water in the bathtub is different, and the temperature of any moment at any point in the bathtub is expressed by $T(x,y,z,t)$ the unsteady three-dimensional heat conduction equation of water is as follows:

$$\frac{\partial T}{\partial t}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \qquad (1)$$

The boundary conditions are as follows:

$$\begin{cases} t=0, T=T_f \\ z=0,\dfrac{\partial T}{\partial z}=0; z=H_{surface}(H_{surface}<H), -\lambda\dfrac{\partial T}{\partial z}=h(T-T_{air}) \\ x=0,\dfrac{\partial T}{\partial x}=0; x=m,\dfrac{\partial T}{\partial x}=0 \\ y=0,\dfrac{\partial T}{\partial y}=0; y=n,\dfrac{\partial T}{\partial y}=0 \end{cases} \qquad (2)$$

In the formula: $H_{surface}$ represents the water surface height at the initial time, B indicates the vertical height of the bathtub. m 、 n represents the length and width of the bathtub. T_{air} indicates the temperature of the air layer between the surface of water and bathtub edge level. λ Represents the thermal conductivity of water, h represents the local surface heat transfer coefficient of convective heat transfer on water surface, the unit is W/ (m *K), The size of surface heat transfer coefficient is related to many factors in the process of convective heat transfer. It depends not only on the physical properties of the fluid as well as the shape, size and arrangement of the heat transfer surface, but also closely related to the flow velocity.

Considering the convection heat transfer between water and air in the rectangular air layer between the horizontal surface and the edge of the bathtub. Because of incompressible and regular physical property, In the case of Newton fluid neglecting viscous dissipation, the three dimensional convection heat transfer differential equations are as follows:

$$\begin{cases} h_{x,y} = -\dfrac{\lambda}{T_{air} - T}\dfrac{\partial T}{\partial z}\Big| z = H_{surface} & \text{(I)} \\ \dfrac{\partial T}{\partial t} + u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} + w\dfrac{\partial T}{\partial z} = a\left(\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}\right) & \text{(II)} \\ \vec{f} - \dfrac{1}{p}\nabla p + v\nabla^2\vec{v} = \dfrac{d\vec{v}}{dt} & \text{(III)} \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0 & \text{(IV)} \end{cases} \qquad (3)$$

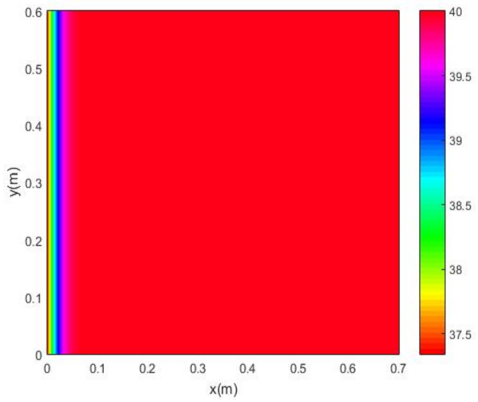
Above, in the formula (I) $h_{x,y} = h$; (II) said energy differential equation, a is the thermal diffusivity; (III) the corresponding partial differential equations, which are used to solve the incompressible fluid flow velocity; (IV) show the continuity equation, u, v, w respectively, indicates that the air layer of a point of air in the x, y, z flow velocity component in three directions.

The boundary conditions are as follows:

$$\begin{cases} t = o, T(x, y, h, t) = T_s \\ z = H_{surface}, w = 0, -\lambda\dfrac{\partial T}{\partial z} = h(T - T_{air}) \\ x = 0 \ or \ m, u = 0, \dfrac{\partial T}{\partial x} = 0 \\ y = 0 \ or \ n, v = 0, \dfrac{\partial T}{\partial y} = 0 \end{cases} \qquad (4)$$

Numerical solution results

Assumed initial time ambient temperature is $T_S = 20^\circ C$, the length bathtub is $m = 1.5m$, the height of bathtub is $H = 0.6m$, Air natural convection heat transfer coefficient is $h = 5w/(m^2 * k)$,Coefficient of thermal conductivity of water is $0.64w/(m*k)$, initial temperature of water is $T_0 = 40^\circ C$,in x, y, z three directions to take the number of nodes are 100, using MATLAB software to solve. The results as shown below:



$t = 600s$ *and* $t = 1800s$

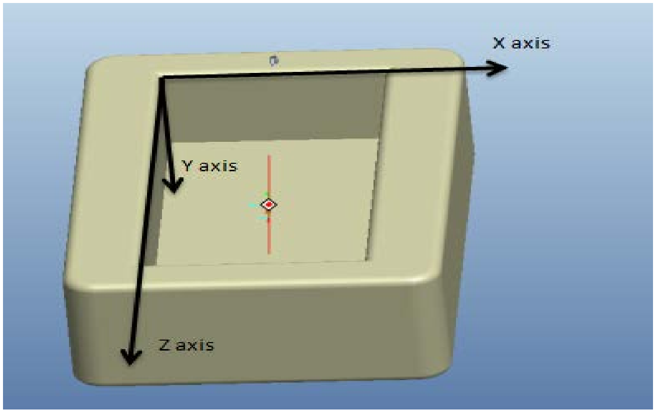


Figure 1 The section temperature distribution curves in z direction

Summary

According to the formula, we got the result of the section temperature distribution curves in z direction. We can know the temperature of the water in the bathtub.And the solution of adding water to keep a stable temperature.

Reference

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