One World, One Dream

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Abstract: This paper established the Measurement Model of Sustainable Development to determine the level of sustainable development of one country by processing the data with some fuzzy sets methodologies. Then We took Madagascar for an example, using the Principal Component Analysis and the data inside World Bank to get the main integrated indicators of the assessment of Sustainable Development and calculate the overall score. The result shows that environment and society improves slightly, but the population and the economy deteriorate. The improved Model of $GM(1,1)$ Model in Grey Theory was used to predict the level of development of Madagascar. This method selects the Index values in each subsystem every year as the initial sequence and obtains the Time Response Sequence by MATLAB to test if the given policy is valid.

Introduction

We have only one planet, with the rapid development in economic and in the population explosion today, our environment is getting poorer, resources are fewer and fewer, so the sustainable development has become an inevitable trend. One of the largest challenges in the era is to balance the relationship between the increasing population and the limited resources in the earth.

Model

First,we need to design a model to describe the sustainability of a country. This model should provide a measure to distinguish between countries and policy's sustainability. We will divide the research for a country's sustainability into four aspects, that is, to evaluate the coordinated development about population, resources and environment, economic and social. It is a quantitative analysis to discover the coordinated development about population, resources and environment, economy and society, is by the qualitative analysis stage into the implementation stage, using the quantitative mathematical methods and means to describe and evaluate the coordinated development status and trend of population, resources and environment, economic and social, to provide decisions and practical basis for guiding the coordinated development of the society, in order to better promote the sustainable development of the whole country. Specific contents should include: construction of the assessment indicator system, making sure the comprehensive development level, building the coordination degree and continued degree measure model those three parts.

Construction of the assessment indicator system

In order to be able to make a comprehensive, objective evaluation to these aspects, according to the connotation of coordinated development, together with the complexity of those systems, based on the scientific principle, realistic principle, system, locality principle and dynamic principle we divide the country's sustainable development evaluation into the evaluation of the following four subsystems.

- The population subsystem
- Resources and environment subsystem
- Economic subsystem
- Social subsystem

Determination of comprehensive development level

We use dimensionless method by adopting $Z-sore$ standard transform method:

1.First of all, build a sample matrix of index system, assuming X is a matrix that the subsystem of population, resources and environment and coordinated development of economy and society corresponding to the n indexes of year m.

x_{ij} is the $i-th$ evaluation object and the $j-th$ evaluation value, so $X=\left(x_{ij}\right)_{m\times n}$.The formula of $Z-sore$ **standard transform method** is shown as the following:

$$b_i=\frac{x_i-\overline{x}}{s_i}, s_i=\sqrt{\frac{1}{p-1}\sum_{i=1}^{p}\left(x_i-\overline{x}\right)^2} \quad 1$$

indent In the formula, b_i shows standardized value of the $i-th$ evaluation index. x_i is standard deviation of the $i-th$ evaluation index. $\overline{x}$ is original value of the $i-th$ is average value of this group of index data; and $i=1,2,\cdots n$.

Therefore, we can get the standardized matrix:

$$B=\left(b_{ij}\right)_{m\times n} \quad 2$$

2.calculate the correlation coefficient matrix of standardized matrix

$$R\left(r_{ij}\right)_{n\times n} \quad 3$$

$$r_{ij}=\frac{1}{p-1}b_{ii}b_{jj},\left(i,j=1,\cdots,n\right) \quad 4$$

What's more, r_{ij} is correlation coefficient of index i and index j.

3. calculate the eigenvalue and eigenvector of correlation coefficient matrix.

if $\left|\lambda I-R\right|=0$,we can calculate the eigenvalue $\lambda_i(i=1,2,\cdots n)$ and eigenvector $h_i=(h_1,\cdots,h_n)$ of the matrix.

4. We can calculate the contribution rate of different principal componen λ_i, and extract the principal component in terms of accumulative contribution rate. The eigenvalue of correlation coefficient matrix R is variance of corresponding principal component $F_i\left(i=1,\cdots,n\right)$.The larger the variance is, the more the information of this principal component is. And this is principal component. Thus we can get the result that the F_i contribution rate of the variance g_i is $g_i=\frac{\lambda_i}{\sum_{i=1}^{n}\lambda_i}$, in general, when accumulative contribution rate of the first W principal component g_i is greater than 85%, we can make sure that the principal component of this system is $F=\left(F_1,\cdots,F_w\right)$.

5. Calculate the synthesis score value of four subsystems of population, resources and environment, economy, and society F_i

$$F_i=g_1F_1+g_1F_1+\cdots+g_wF_w \quad 5$$

Measurement methods of Sustainable development

Measurement model for coordination degrees of the sustainable development components

Most scholars use the distribution function in the membership functions to describe the coordination in the mathematical model[1].

$$C(i/j) = \exp\left[-k\left(u_i - u_{i/j}\right)^2\right] \quad 6$$

Actually this model can be improved by directly calculating the coordination degree between many systems, and the specific model is as follows[2]

$$C_n = \exp\left[-n\sqrt{\prod_{\lambda=1}^{n} k_\lambda \left(u_\lambda - u_{\lambda/\bar{\lambda}}\right)^2}\right] \quad 7$$

In the model, C_n represents the coordination degree among n systems; u_λ represents the actual development level of number $\lambda(\lambda = 1, 2, \cdots n)$ system; $u_{\lambda/\bar{\lambda}}$ represents number λ system`s coordinated development level when number λ system is coordinately developing with its complement system $\bar{\lambda}$

$$k_\lambda = \frac{2}{S_\lambda^2} \quad 8$$

S_λ^2 is the variance of u_λ;if the u_λ is closer to $u_{\lambda/\bar{\lambda}}$,then the coordination coefficient is larger, the coordination degrees $C(i/j)$ is also higher; if u_λ is more far away from $u_{\lambda/\bar{\lambda}}$, that means the smaller the coordination coefficient, coordination of $C(i/j)$ is lower. According to this model, in this article , coordination model about researching on population, resource, environment, economy and society these four subsystems can be represented as

$$C_4 = \exp\left[-4\sqrt{\prod_{\lambda=1}^{4} k_\lambda \left(u_\lambda - u_{\lambda/\bar{\lambda}}\right)^2}\right] \quad 9$$

For calculating $u_{\lambda/\bar{\lambda}}$, we can use the multiple regression analysis, reach to the conclusion that the regression model of number λ system and its complement system $\bar{\lambda}$ to determine, such as in four subsystems, the first system and its complement system's regression model is

$$u1 = a + bu_2 + cu_3 + du_4 \quad 10$$

In this model, in order to achieve coordinated development between the 1st system and number 2,3,4 systems, if u_2, u_3, u_4 change every unit, it requires $u1$ changes $b + c + d$ unit, too, then we can get the value of $u_{1/2.3.4}$, that is

$$u_{1/2.3.4} = b + c + d \quad 11$$

Measurement Model of Sustainability in Sustainable Development

We use CD_t to express the sustainability[3]

$$CD_t(i, j, k, 1) = \left[C_t(i, j, k, 1) u_{i,t}^{\alpha} u_{j,t}^{\beta} u_{k,t}^{\gamma} u_{1,t}^{\delta}\right]^{\frac{1}{4}} \quad 12$$

$CD_t(i, j)$ means the the sustainability of coordinated development of System i and System j at Time t. $C_t(i, j)$ means the coordinated coefficient between System i and System j at Time t. $u_{i,t}$ means the synthesis score of System i at Time t : the comprehensive developing level of System i at Time t. $u_{j,}t$ means the synthesis score of System j at Time t: the comprehensive developing level of System j at Time t.

In the formula, $\alpha, \beta, \gamma, \delta$ mean the proportion $\alpha + \beta + \gamma + \delta = 1$.

Based on the experience, in general, the four subsystem population, resources and environment, economy, society are supposed to have equal effect on the population, resources and environment, economic and social systems. So the each proportion of $\alpha,\beta,\gamma,\delta$ is 0.25.

Evaluation scale

According to the reference[4],we will set the lower interval as [0,0.5),medium range as [0.5, 0.8), the higher interval set to [0.8, 1]. Lower interval belongs to non- acceptable range, medium is bad but acceptable range, higher belongs to the good range.

Empirical Analysis

Among 48 most undeveloped countries of the list UN gives， we choose Madagascar as a sample to study. And we can get the process and result by using the model as the following.

The Result

The index system of population, resources and environment and economy as shown in the Table 2 Original data comes from 'the World Bank'. Use standardized transform method to deal with the initial value of third class indicator， and then get the standardized value of index， as the Table3 shows.

Table 2 Evaluation of Coordination Development Scale

Subsystem	Component	Initial Eigenvalues			Extraction of the quadratic sum		
		total	Variance %	accumulation %	total	Variance %	accumulation %
Population	1	9.177	91.767	91.767	9.177	91.767	91.767
	2	.739	7.387	99.154			
Source	1	5.220	74.571	74.571	5.220	74.571	74.571
	2	1.007	14.386	88.958	1.007	14.386	88.958
	3	.670	9.577	98.534			
Society	1	5.943	59.435	59.435	5.943	59.435	59.435
	2	1.259	12.585	72.020	1.259	12.585	72.020
	3	1.024	10.241	82.261	1.024	10.241	82.261
	4	.781	7.812	90.073			
Economy	1	3.067	38.339	38.339	3.067	38.339	38.339
	2	2.017	25.218	63.556	2.017	25.218	63.556
	3	1.434	17.919	81.476	1.434	17.919	81.476
	4	.817	10.213	91.688			

Table 3 the Standardized Value of Index

Indicator	ZP1	ZPi	ZR1	ZRi	ZS1	ZSi	ZE1	ZEi
2001	-1.4769	…	1.5206	…	-1.1257	…	0.5253	…
2002	-1.2244	…	1.2449	…	-1.1908	…	-2.5721	…
2003	-0.9674	…	0.9693	…	-0.7759	…	1.1639	…
2004	-0.7059	…	0.6936	…	-1.1022	…	0.4183	…
2005	-0.4399	…	0.4180	…	-0.6841	…	0.3159	…
2006	-0.1695	…	0.1413	…	-0.7189	…	0.3921	…
2007	0.1055	…	-0.1353	…	0.7216	…	0.6006	…
2008	0.3861	…	-0.4119	…	1.0940	…	0.7527	…
2009	0.6732	…	-0.6886	…	0.6914	…	-1.0991	…
2010	0.9679	…	-0.9652	…	0.7380	…	-0.3863	…
2011	1.2704	…	-1.2418	…	1.1256	…	-0.1872	…
2012	1.5808	…	-1.5449	…	1.2272	…	0.0757	…

Extract principal component

Analyze the principal component of standardized data on the Table 3 by applying SPSS17.0, and get analytical statement about variance decomposition and extracting the primary component of four systems, seeing the Table2.

According to the rule that initial eigenvalue is greater than 1, we extract two primary component from three systems respectively. It reflects information of their own systems respectively, which has great interpret ability. Divide the data by the eigenvalue correspondent to primary component, the square root of the answer is the coefficient correspondent to each index in two primary components.

Table4

Source	Component		Economy	Component		
	1	2		1	2	3
ZR1	-.188	.030	ZE1	.214	.113	-.401
ZR2	.183	-.074	ZE2	.249	.185	-.046
ZR3	.179	.003	ZE3	.293	.113	-.240
ZR4	-.021	.975	ZE4	.130	.117	.527
ZR5	.188	-.024	ZE5	.088	-.431	-.132
ZR6	.166	.148	ZE6	-.147	.418	.051
ZR7	.166	.116	ZE7	.155	-.239	.327
			ZE8	.250	.070	.270

Population	Component		Society	Component		
	1	2		1	2	3
ZP1	-.109		ZS1	.160	.071	-.013
ZP2	.102		ZS2	.072	.206	.567
ZP3	-.080		ZS3	-.121	.363	-.034
ZP4	.108		ZS4	.125	-.156	-.530
ZP5	-.108		ZS5	.060	.637	-.350
ZP6	.108		ZS6	.104	.073	.441
ZP7	.108		ZS7	.160	.036	.182
ZP8	.102		ZS8	.156	-.185	.003
ZP9	-.109		ZS9	.141	.321	-.084
ZP10	-.108		ZS10	-.152	.202	.127

As Table 4 shows, multiply the eigenvector by the normalized data, then we get the score value of the primary component.

Linear representation about two primary components of population subsystem:

$$F1 = -0.109 \times ZP1 + 0.102 \times ZP2 + \cdots + 0.108 \times ZP10 \quad 13$$

Linear representation about two primary components of resources and environment subsystem:

$$F1 = -0.188 \times ZP1 + 0.183 \times ZP2 + \cdots + 0.166 \times ZP7 \quad 14$$

$$F1 = 0.030 \times ZP1 + 0.074 \times ZP2 + \cdots + 0.116 \times ZP7 \quad 15$$

Linear representation about two primary components of society subsystem:

$$F1 = 0.160 \times ZS1 + 0.072 \times ZS2 + \cdots - 0.152 \times ZS10 \quad 16$$

$$F2 = 0.071 \times ZS1 + 0.206 \times ZS2 + \cdots + 0.202 \times ZS10 \quad 17$$

$$F3 = 0.013 \times ZS1 + 0.206 \times ZS2 + \cdots + 0.202 \times ZS10 \quad 18$$

Linear representation about two primary components of economy subsystem:

$$F1 = 0.214 \times ZE1 + 0.249 \times ZE2 + \cdots + 0.250 \times ZE8 \quad 19$$

$$F2 = 0.113 \times ZE1 + 0.185 \times ZE2 + \cdots + 0.070 \times ZE8 \quad (20)$$

$$F3 = 0.401 \times ZE1 + 0.046 \times ZE2 + \cdots + 0.270 \times ZE8 \quad (21)$$

Comprehensive evaluation

Finally, we use the weights to reach to the principal component comprehensive evaluation model, the weights are from the proportion of eigenvalues each main components of the of the total value of the principal components and features.

$$F_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} F_1 + \frac{\lambda_2}{\sum_{i=1}^{n} \lambda_i} F_2 + \cdots + \frac{\lambda_w}{\sum_{i=1}^{n} \lambda_i} F_w \quad (22)$$

There into, λ_i is the characteristic values of the $i-th$ principal component.

$$F_i = g_1 F_1 + g_2 F_2 + \cdots + g_w F_w \quad (23)$$

The comprehensive evaluation for four subsystems are

Population subsystems: $F = 0.91767 \times F1$;

Resources and environment subsystems: $F = 0.74571 \times F1 + 0.14386 \times F2$;

Social subsystems: $F = 0.59435 \times F1 + 0.12585 \times F2 + 0.10241 \times F3$;

Economic subsystems: $F = 0.38339 \times F1 + 0.25218 \times F2 + 0.17919 \times F3$;

The main component scores and overall scores calculated by the three above systems,Thus, the assess value of the population subsystem and economic subsystem for Madagascar is going down, the value of the resource subsystem and social subsystem increased slightly.

Estimate the Coordination

The regression model using SPSS for each subsystem and its complement system is as followed

$$u1 = 1.163 - 0.857 \times u_2 - 6.271E - 11 \times u_3 + 0.234 \times u_4 \quad (24)$$

$$u2 = 0.821 - u_1 - 4.84E - 11 \times u_3 + 0.144 \times u_4 \quad (25)$$

$$u3 = 1.778E + 10 - 6.736E + 9 \times u_1 - 4.157E + 9 \times u_2 + 3.881E + 9 \times u_4 \quad (26)$$

$$u4 = -1.993 + 0.667 \times u_1 + 0.356 \times u_2 + 1.119E - 10 \times u_3 \quad (27)$$

Reviews of the coordination

The coordination of the population, resources and environment, social and economic calculated by MATLAB is as shown in Table 7.

Table 7 The coordination of the population, resources and environment, social and economic

Year	2001	2002	2003	2004	2005	2006
Coordination	0.36853	0.33187	0.19373	0.20193	0.10878	0.34960
Degree	Imbalance	Imbalance	Imbalance	Imbalance	Imbalance	Imbalance
Year	2007	2008	2009	2010	2011	2012
Coordination	0.23749	0.29666	0.23206	0.39028	0.35713	0.37956
Degree	Imbalance	Imbalance	Imbalance	Imbalance	Imbalance	Imbalance

Reviews of the sustainable development

The coordination trend figure for the Madagascar four subsystems is shown in the fig.1

We can see in the figure that the coordination degree of the sustainable development is serious disorder, and it not have any obvious improvements.

According to the comprehensive development level of each subsystem development above, as well as the calculated coordinate values of population in Guizhou province, resources and environment, economic and social systems, the sustained degree model in accordance with the coordinatedly develop of the population, resources and environment, economic and social system, using Eq.2,we can calculate the Madagascar's sustained degree of population, resources and environment, economic and social systems' coordinated development ,the results are shown in Table 8:

Table 8 The Sustainability of the population, resources and environment, social and economic(us means Unsustainable,u means Sustainable

Year	2001	2002	2003	2004	2005	2006
Sustainability	0.488097467	0.399747485	0.365326289	0.330949562	0.193501244	0.446155116
Degree	us	us	us	us	us	us
Year	2007	2008	2009	2010	2011	2012
Sustainability	0.401206825	0.511515835	0.37118911	0.472473388	0.467740407	0.475687296
Degree	us	s	us	us	us	us

From the above we can see that coordination degree is always imbalanced. And the sustainability arrive the level of sustainable that is 0.5 just in 2008.Therefore, the sustainable state of Madagascar is very terrible. To this, we give the detailed suggests in the Policy to improve the sustainable possibility of Madagascar.

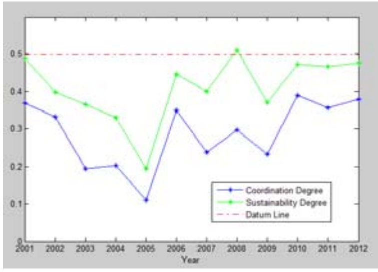

Fig.1

Relational Grade Analysis

Calculation of the correlation coefficient

First, choose a Reference Sequence[5]

$$x_o = x_0(k) \mid k = 1,2,\cdots,n = \left(x_0(1), x_0(2), \cdots, x_0(n)\right) \quad 28$$

and other Sequences

$$x_i = x_i(k) \mid k = 1,2,\cdots,n = \left(x_i(1), x_i(2), \cdots, x_i(n)\right) \quad 29$$

The degrees of correlation from x_i to x_0 are

$$r_i = \frac{1}{n}\sum_{k=1}^{n} \xi_i(k) \quad 30$$

in which, $$\xi_i(k) = \frac{\min_s \min_t \left|x_0(t) - x_s(t)\right| + \rho \max_s \max_t \left|x_0(t) - x_s(t)\right|}{\left|x_0(t) - x_s(t)\right| + \rho \max_s \max_t \left|x_0(t) - x_s(t)\right|} \quad 31$$

Model Improvement

Verhulst Model

We define our existing 2001-2012 processed data as an initial value $X^{(0)}$

$$X^{(0)} = X_1^{(0)}, X_2^{(0)}, X_3^{(0)}, \cdots, X_n^{(0)} \quad 32$$

Than **Verhulst Model** can be used

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \quad 33$$

X_1 cumulatively generates a sequence of operations $X^{(0)}$.Then, we use parameters a and b with **Least Squares Method(LSM)** are

$$\hat{\alpha} = (a,b)^T = \left(B^T B\right)^{-1} B^T Y \quad 34$$

in which, $$B = \begin{bmatrix} -Z_2^{(1)} & 1 \\ -Z_3^{(1)} & 1 \\ \vdots & \vdots \\ -Z_n^{(1)} & 1 \end{bmatrix} \quad Y = \begin{bmatrix} X_2^{(0)} \\ X_2^{(0)} \\ \vdots \\ X_2^{(0)} \end{bmatrix}$$

$$Z_k^{(1)} = 0.5(X_k^{(1)} + X_{k-1}^{(1)}) \quad 35$$

Time response series of Verhulst Model

$$X^{(0)}_{k-1} = \left(X^{(0)(1)} - \frac{b}{a} \right) e^{-ak} + \frac{b}{a} \qquad k = 1,2,3,\cdots n-1 \tag{36}$$

Then

$$X^{(0)}_{k-1} = X^{(1)}_{k+1} - X^{(1)}_{k} \tag{37}$$

Therefore, we use r_i to describe the degree of correlation between x_i and x_0, that is x_i changes as x_0 changes. According to the weights of four subsystems calculated in the first question, we use MATLAB to calculate.

Validation of the Model

We use residual analysis to test our model, The definition of sequence grey prediction is

$$\hat{X}^{(0)} = \left(\hat{X}^{(0)}_1, \hat{X}^{(0)}_2, \cdots, \hat{X}^{(0)}_n \right) \tag{38}$$

Then, the residual sequence is

$$\varepsilon^{(0)} = \left(X^{(0)}_1 - \hat{X}^{(0)}_1, X^{(0)}_2 - \hat{X}^{(0)}_2, \cdots, X^{(0)}_n - \hat{X}^{(0)}_n \right) \tag{39}$$

The relative error sequence is

$$\Delta = \left(\left| \frac{\varepsilon_1}{X^{(0)}_1} \right|, \left| \frac{\varepsilon_1}{X^{(0)}_2} \right|, \cdots, \left| \frac{\varepsilon_1}{X^{(0)}_n} \right| \right) \tag{40}$$

In the end, we get the relative error sequence of model.

$$\bar{\Delta} = \frac{1}{n} \sum_{k=1}^{n} \Delta k \tag{41}$$

Results and conclusions

So, we can get the results shown in Fig.2.

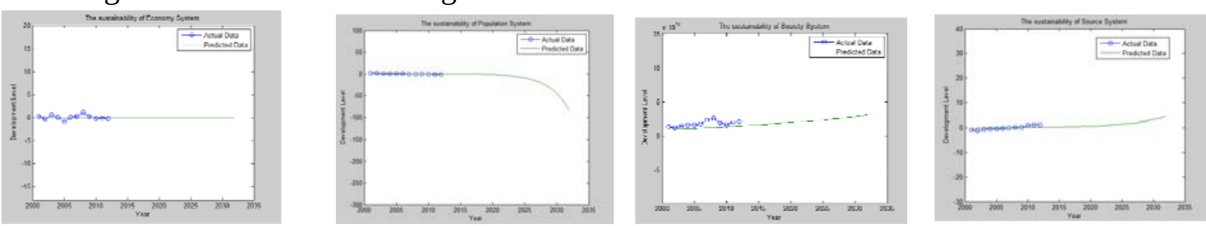

Fig.2

Conclusions

1. The indicators of the entire country are classified into four subsystems in the evaluation by the principal component of each system , resulting in the overall degree of coordination and sustainability ,making the whole model hierarchy clearer and more rigorous.
2. Analysis of the correlation between the various subsystems makes the assessment of the entire system more persuasive .
3. By processing the data with some fuzzy sets methodologies, we built the Measurement Model of Sustainable Development to determine the level of sustainable development of one country. Highly coordinated and good sustainability is obtained if the Estimation Intervals of Coordination degree and Sustainability are both in [0.8,1]; and if they are both in [0,0.5), imbalance and unsustainable situations happen. And the rest are generally coordinated and sustainable cases.
4. According to the Empirical Analysis, it is concluded that environment and society of Madagascar improved slightly , but the population and the economy deteriorated. Based on these analysis, policy suggestions are given on four areas including the population, the economy, the environment and the society.

5.We improved the GM(1,1) Model in Grey Theory, comparing to the general Grey prediction models, which is have the higher accuracy. In the Empirical Analysis of Madagascar,the model predicts each index precisely.
6.In the whole modeling process, we give full consideration to the validity, the feasibility and the efficiency of our model.

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