Diffraction of SH Waves by an Elliptic Inclusion with Partially Debonded Region in Bi-Material Half Space

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Abstract: Base on elastodynamics, complex function method with mapping function and Green's function method are used to investigate the scattering of SH waves by an elliptic inclusion with a partially debonded region. The bi-material half space is divided into two parts along the vertical interface to investigate the two right angle plane respectively. Secondly, "image" method and conformal mapping method are employed to construct the scattering wave field in part I. Then the Green's function needed is obtained. Thirdly, with the aid of interface "conjunction" technique, a series of integral equations for determining the unknown force system could be set up through continuity conditions on the interface and Green's function. The method presented in this paper can be used to solve other similar scattering problem by arbitrary shape inclusion with partially debonded region.

Introduction

In the field of composite material, earthquake engineering and geotechnical engineering and other fields, debonded problems between elastic material are of theoretical and engineering application value.

Scattering problems of SH waves[1-7] as the simplest problems in the research of scattering problem, have been studied many researchers, and many valuable results have been obtained. However, the scattering problems for some special structures are still unsolved.The bonding surface of different materials under physical or chemical injury will occur the phenomenon debonding, which will affect the mechanical and chemical properties. This paper mainly studies the effect on the mechanical properties.Many academics have studied the scattering of SH waves by structures with debonded region, but the mathematical model established are mainly concentrated in the whole space model or homogeneous half space. The scattering of SH waves by elliptic inclusion with partially debonded region in bi-material half space are rarely investigated.

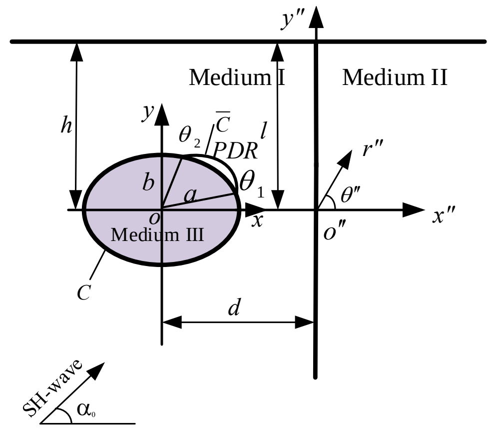

Fig.1 Elliptic inclusion with partially debonded region in bi-material half space

In this paper, complex function method and the idea of "conjunction" are empleoed to study the scattering problems of SH waves by an elliptic inclusion near the vertical interface in bi-material half space.Image method and conformal mapping method are used to study the wave field outside the elliptic inclusion. In the elliptic domain, there are standing waves that satisfy the continuity conditions of displacement and stress on the boundary of domain I and domain III. And the stress continuity condition should be satisfied on the debonded region.

Fig.1 shows a bi-material half-space with an elliptical inclusion with partially dedondeb region (PDR) of half-macroaxis a and half-brachyaxis b. The elliptic inclusion has material properties, shear modulus μ_3 and mass body density ρ_3. The distances from the center of elliptical inclusion to the vertical interface and horizontal surface are d and h, respectively. The incident angle of SH waves is α_0. There are two coordinate systems setting on O and O'' point.

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Basic theory

Conjunction method will be used in this paper, Green's function should be studied before. Green's function used in this paper is an essential solution of displacement field for a quarter-plane with an elliptical elastic inclusion by anti-plane harmonic line source $\delta(z-z_0)$ ($z=x+\mathrm{i}y$) loading at vertical surface. The displacement function G must satisfy the governing equation with omitted the time harmonic factor exp($-\mathrm{i}\omega t$) of the following form:

$$\frac{\partial^2 G}{\partial z \partial \bar{z}} + \frac{1}{4}k^2 G = 0 \tag{1}$$

In which, z and $\bar{z}$ are the complex variables, $k=\omega/c_s$ is wave number, ω and $c_s=\sqrt{\mu/\rho}$ are the disturbing circular frequency and the shear velocity of the media, ρ and μ are the mass body density and the shear modulus of the media, respectively.

The stresses corresponding to Eq.(1) can be written as:

$$\tau_{rz} = \mu\left(\frac{\partial G}{\partial z}e^{\mathrm{i}\theta} + \frac{\partial G}{\partial \bar{z}}e^{-\mathrm{i}\theta}\right), \tau_{\theta z} = \mathrm{i}\mu\left(\frac{\partial G}{\partial z}e^{\mathrm{i}\theta} - \frac{\partial G}{\partial \bar{z}}e^{-\mathrm{i}\theta}\right) \tag{2}$$

For wave scattering problems involving elliptical inclusion in the complex $(z,\bar{z})$ plane, it is possible to map the internal/external region of the elliptical inclusion into the inside/outside region of the circle(in the $(\eta,\bar{\eta})$ plane).

Introducing the mapping function:

$$Z = \omega(\eta) = R\left(\eta + \frac{m}{\eta}\right), \eta = Re^{i\theta} \tag{3}$$

The above mapping function map the outside of the inclusion in the $(z,\bar{z})$ plane into the region $|\eta|>1$. Consequently, the corresponding governing Eq.(1) in $(\eta,\bar{\eta})$ plane takes on the following form:

$$\frac{1}{\omega'(\eta)\overline{\omega'(\eta)}}\frac{\partial^2 G}{\partial \eta \partial \bar{\eta}} + \frac{1}{4}k^2 G = 0 \tag{4}$$

In $(\eta,\bar{\eta})$ plane, Eq.(2) can be written as:

$$\tau_r = \frac{\mu}{R|\omega'(\eta)|}\left(\eta\frac{\partial G}{\partial \eta} + \bar{\eta}\frac{\partial G}{\partial \bar{\eta}}\right), \quad \tau_\theta = \frac{\mathrm{i}\mu}{R|\omega'(\eta)|}\left(\eta\frac{\partial G}{\partial \eta} - \bar{\eta}\frac{\partial G}{\partial \bar{\eta}}\right) \tag{5}$$

Anti-plane SH waves in domain I and domain II

Considering the existence of the free boundary and the interface in bi-material media, the image method is employed to transform right angle space to full space. we can consider the full space model as an equivalent model with multiple wave sources. And the equivalent incident wave can be expressed as follows:

$$W^{(i,e)} = W_0 \exp\{\frac{\mathrm{i}k_1}{2}[(\omega(\eta)-\mathrm{i}h)e^{-\mathrm{i}\alpha_0} + (\overline{\omega(\eta)}+\mathrm{i}h)e^{\mathrm{i}\alpha_0} + (\omega(\eta)-\mathrm{i}h-2d)e^{-\mathrm{i}\gamma_0} + (\overline{\omega(\eta)}+\mathrm{i}h-2d)e^{\mathrm{i}\gamma_0}]\} \tag{6}$$

Where, $\gamma_0=\pi-\alpha_0$, α_0 and W_0 are incident angle and the amplitude of the incident wave respectively.

Similarly, the equivalent reflected wave and refracted wave are:

$$W^{(r,e)} = W_1 \exp\{\frac{\mathrm{i}k_1}{2}[(\omega(\eta)-ih)e^{-\mathrm{i}\alpha_1} + (\overline{\omega(\eta)}+\mathrm{i}h)e^{\mathrm{i}\alpha_1} + (\omega(\eta)-ih-2d)e^{-\mathrm{i}\gamma_1} + (\overline{\omega(\eta)}+\mathrm{i}h-2d)e^{\mathrm{i}\gamma_1}]\} \tag{7}$$

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$$W^{(f,\mathrm{e})} = W_2 \exp\{\frac{\mathrm{i}k_2}{2}[(\omega(\eta)-\mathrm{i}h)e^{-\mathrm{i}\alpha_2} + (\overline{\omega(\eta)}+\mathrm{i}h)e^{\mathrm{i}\alpha_2} + (\omega(\eta)-\mathrm{i}h-2d)e^{-\mathrm{i}\gamma_2} + (\overline{\omega(\eta)}+\mathrm{i}h-2d)e^{\mathrm{i}\gamma_2}]\} \tag{8}$$

In which， $\gamma_1 = \pi - \alpha_1$, $\gamma_2 = \pi - \alpha_2$ ， α_1 and α_2 are the reflection angle and refraction angle respectively.

Standing waves in domain III

Constructing standing wave in the elliptic domain need satisfy the stress free conditions on boundary $\overline{C}$ and continuity condition of displacement and stress on boundary C:

$$\tau_{rz}^{(\mathrm{st})} = \begin{cases} 0 \quad ,\omega(\eta) \in \overline{C} \\ \dfrac{\mu_3 k_3 G_0}{2} \sum_{m=-\infty}^{\infty} C_m [J_{m-1}(k_3|\omega(\eta)| - J_{m+1}(k_3|\omega(\eta)|)][\dfrac{\omega(\eta)}{|\omega(\eta)|}]^m, \omega(\eta) \in C \end{cases} \tag{9}$$

In which, C_m is unknown coefficient, W_{T0} is displacement amplitude of standing wave, W_{T0} =1。

Standing waves in elliptic domain:

$$W^{(\mathrm{st})} = W_{T0} \sum_{m=-\infty}^{\infty} D_n J_n(k_3|\omega(\eta)|)[\frac{\omega(\eta)}{|\omega(\eta)|}]^n \tag{10}$$

In which, D_m is unknown coefficient.

The Fourier expansion of Eq.(10) within the range [$-\pi,\pi$] can be written as

$$\tau_{rz}^{(\mathrm{st})} = \frac{\mu_3 k_3 W_{T0}}{2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_m a_{mn} [J_{m-1}(k_3|\omega(\eta)| - J_{m+1}(k_3|\omega(\eta)|)][\frac{\omega(\eta)}{|\omega(\eta)|}]^n \tag{11}$$

In which:

$$a_{mn} = \begin{cases} \dfrac{\theta_1 - \theta_2 + 2\pi}{2\pi} \quad ,m = n \\ \dfrac{e^{\mathrm{i(m-n)}\theta_1} - e^{\mathrm{i(m-n)}\theta_2}}{2\pi i(\mathrm{m-n})} \quad ,m \neq n \end{cases} \tag{12}$$

In coordinate ($\eta,\overline{\eta}$), θ_1 and θ_2 are the start angel and end angel of debonded region. Comparing Eq. (11) with the stress of Eq. (10) under the condition $|\eta| = 1$

$$D_n = G_0 \sum_{m=-\infty}^{\infty} C_m \frac{J_{m-1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right| - J_{m+1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right|}{J_{n-1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right| - J_{n+1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right|} a_{mn} \tag{13}$$

The standing wave field in domain II can be expressed as：

$$G^{(\mathrm{st})} = W_0 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_m \frac{J_{m-1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right| - J_{m+1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right|}{J_{n-1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right| - J_{n+1}\left|k_3\,\omega(\eta)|_{|\eta|=1}\right|} a_{mn} J_n(k_2|\omega(\eta)|)[\frac{\omega(\eta)}{|\omega(\eta)|}]^n \tag{14}$$

The continuity condition on the boundary of domain III and domain I

$$\begin{cases} W^{(\mathrm{i,e})} + W^{(r,e)} + W^{(\mathrm{s})} = W^{(\mathrm{st})}, \quad \theta \in C \\ \tau_{rz}^{(i,\mathrm{e})} + \tau_{rz}^{(\mathrm{r,e})} + \tau_{rz}^{s} = \tau_{rz}^{(\mathrm{st})}, \theta \in (-\pi,\pi) \end{cases} \tag{15}$$

The Green function we need can be get by solved Eq.(15).

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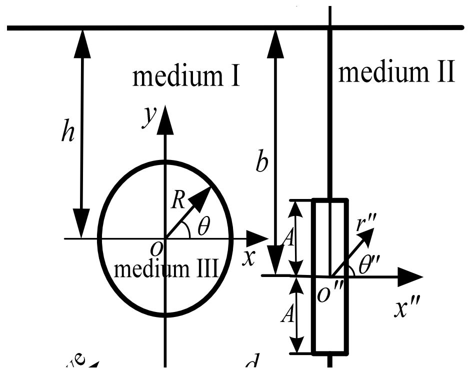

Fig 2. Conjunction model

As shown in Fig.2, unknown force systems f_1 and f_2 are loaded on the conjunction section to satisfy the continuity conditions on the interface. So a series of Fredholm integral equations for determining the unknown forces can be set up .

The total displacements $W^{(\mathrm{I})}$, $W^{(\mathrm{II})}$ and total stresses $\tau^{(\mathrm{I})}_{\theta''z''}$, $\tau^{(\mathrm{II})}_{\theta''z''}$ in the two parts are:

$$\begin{cases} W^{(\mathrm{I})} = W^{(i,e)} + W^{(r,e)} + W^{(s)}, \quad W^{(\mathrm{II})} = W^{(f,e)} \\ \tau^{(\mathrm{I})}_{\theta''z''} = \tau^{(i,e)}_{\theta''z''} + \tau^{(r,e)}_{\theta''z''} + \tau^{(s)}_{\theta''z''}, \quad \tau^{(\mathrm{II})}_{\theta''z''} = \tau^{(f,e)}_{\theta''z''} \end{cases} \tag{16}$$

Where, $W^{(s)}$ is the displacement of the scattering wave. $\tau^{(s)}_{\theta z}$ is the stress of the scattering wave.

The stress continuity condition can be expressed as:

$$\tau^{(\mathrm{I})}_{\theta''z''} \sin\theta''_0 + f_1(r''_0, \theta''_0) = \tau^{(\mathrm{II})}_{\theta''z''} \sin\theta''_0 + f_2(r''_0, \theta''_0) \tag{17}$$

Where r''_0 and θ''_0 are the polar coordinates in the global coordinate system $x''o''y''$ and $z'' = r''\exp(\mathrm{i}\theta'')$, $z'' = z - d$. when $\theta''_0 = \beta_1 = -\pi/2$, $0 \le r''_0 \le \infty$, when $\theta''_0 = \beta_2 = \pi/2$, $0 \le r''_0 \le h$.

According to $\tau^{(i,e)}_{\theta''z''} + \tau^{(r,e)}_{\theta''z''} = \tau^{(f,e)}_{\theta''z''}$, we can get:

$$f_1(r''_0, \theta''_0) = f_2(r''_0, \theta''_0)\,,\ \theta_0 = \beta_1\,,\ \beta_2 \tag{18}$$

The displacement continuity conditions can be written as:

$$W^{(\mathrm{I})} + W^{(f_1)} + W^{(c_1)} = W^{(\mathrm{II})} + W^{(f_2)} + W^{(c_2)} \tag{19}$$

According to $W^{(i,e)} + W^{(r,e)} = W^{(f,e)}$, we can obtain:

$$W^{(\mathrm{s})} + W^{(f_1)} = W^{(f_2)} \tag{20}$$

Where, $W^{(f_1)}$ is the displacement field caused by force system f_1, and $W^{(f_2)}$ is the displacement field caused by force system f_2 .

According to the continuity condition and the Green's function we have obtained, the integral equations with unknown anti-plane forces can be expressed as:

$$\begin{aligned} &\int_A^l f_1(r''_0,\beta_2)[G_1(r'',\beta_1;r''_0,\beta_2) + G_2(r'',\beta_1;r''_0,\beta_2)]dr''_0 + \\ &\int_A^\infty f_1(r''_0,\beta_1)[G_1(r'',\beta_1;r''_0,\beta_1) + G_2(r'',\beta_1;r''_0,\beta_1)]dr''_0 = [-\mathrm{W}^{(\mathrm{S})}]_{\theta''_0=\beta_1} \end{aligned} \tag{21}$$

$$\begin{aligned} &\int_A^l f_1(r''_0,\beta_2)[G_1(r'',\beta_2;r''_0,\beta_2) + G_2(r'',\beta_2;r''_0,\beta_2)]dr''_0 + \\ &\int_A^\infty f_1(r''_0,\beta_1)[G_1(r'',\beta_2;r''_0,\beta_1) + G_2(r'',\beta_2;r''_0,\beta_1)]dr''_0 = [-\mathrm{W}^{(\mathrm{S})}]_{\theta''_0=\beta_2} \end{aligned} \tag{22}$$

In which, G_1 and G_2 are the Green's functions in domain I and II respectively.

Hoop stress around the inclusion can be expresses as

$$\tau^{(\bullet)}_{\theta z} = \tau^{(\mathrm{I})}_{\theta z} + \int_A^l f_1(r''_0,\beta_1)\frac{\mu_1}{r}\frac{\partial G_1(r'',\theta;r''_0,\beta_2)}{\partial\theta}dr''_0 + \int_A^\infty f_1(r''_0,\beta_2)\frac{\mu_1}{r}\frac{\partial G_1(r'',\theta;r''_0,\beta_2)}{\partial\theta}dr''_0 \tag{23}$$

In which, $\tau^{(\mathrm{I})}_{\theta z} = \tau^{(i,e)}_{\theta z} + \tau^{(r,e)}_{\theta z} + \tau^{(s)}_{\theta z}$ 。

Conclusion

In this paper, only theory deduction is presented, analitical results will be given in the follow-up work.

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