Research on the development of country based on Runge-Kutta method

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Abstract. We set up the predicted model of country's development based on Runge-Kutta method to predict the development's trend of a country. When a country is in the absence of policy interference, we regard economy, society and environment as competitors in a biologic chain and obtain the Lotka-Volterra equations. With the Runge–Kutta method of solving equations as well as the initial values,We find that developing countries develop their economy at the expense of the environment from the chart obtained.

1. Introduction

We are discussing the problem basing on the problem D in 2015 MCM. Recent few decades have experienced not only the development of the economy and society, but also the increasing damage to the environment. Therefore the definition of "sustained development" has been put forward to do with it.

Sustainable development is a road-map, an action plan, for achieving sustainability in any activity that uses resources and where immediate and intergenerational replication is demanded. As such, sustainable development is the organizing principle for sustaining finite resources necessary to provide for the needs of future generations of life on the planet. It is a process that envisions a desirable future state for human societies in which living conditions and resource-use continue to meet human needs without undermining the "integrity, stability and beauty" of natural biotic systems.[1]

"Development that meets the needs of the present without compromising the ability of future generations to meet their own needs."[2] Under the principles of the United Nations Charter the Millennium Declaration[3] identified principles and treaties on sustainable development, including economic development, social development and environmental protection. Broadly defined, sustainable development is a systems approach to growth and development and to manage natural, produced, and social capital for the welfare of their own and future generations. The term sustainable development as used by the United Nations incorporates both issues associated with land development and broader issues of human development such as education, public health, and standard of living.

2. Models for predicted development

2.1 Model assumption

1) A project or a policy has a good effect on all the considered factors.
2) The good changes in 10 years could reflect the sustainability level of a project or a policy.

2.2 Model establishing

The Lotka-Volterra equations[4] express an ecological approach to competition modeling [Wolfram Demonstrations Project[5] 2010], which considers the effects of multiple species competing for the same resource :

$$\frac{dD_{EC}}{dt}=\gamma_{EC}D_{EC}\left[1-\frac{D_{EC}+\alpha_{EV}D_{EV}+\alpha_{SO}D_{SO}}{K_{EC}}\right]$$

$$\frac{dD_{EV}}{dt} = \gamma_{EV} D_{EV} \left[1 - \frac{D_{EV} + \beta_{EC} D_{EC} + \beta_{SO} D_{SO}}{K_{EV}} \right]$$

$$\frac{dD_{SO}}{dt} = \gamma_{SO} D_{SO} \left[1 - \frac{D_{SO} + \varepsilon_{EC} D_{EC} + \varepsilon_{EV} D_{EV}}{K_{SO}} \right]$$

$D_{EC}(0) = 0.4615$ $D_{EV}(0) = 0.3077$ $D_{SO}(0) = 0.2308$

Among them,

D_{EC} D_{EV} D_{SO} refers to the economic scores ,the environmental scores, the social scores ;

α_{EV} refers to the correlation coefficient between D_{EV} and D_{EC} ;

α_{SO} refers to the correlation coefficient between D_{SO} and D_{EC} ;

β_{EC} refers to the correlation coefficient between D_{EC} and D_{EV} ;

β_{SO} refers to the correlation coefficient between D_{SO} and D_{EV} ;

ε_{EC} refers to the correlation coefficient between D_{EC} and D_{SO} ;

ε_{EV} refers to the correlation coefficient between D_{EV} and D_{SO} .

Then we solve the system via the Runge-Kutta-Fehlberg method[6], using coeffi-cients derived by Cash-Karp. This method gives efficient solutions without introducing excessive round-off error [Chapra and Canale 2002].Limited data make reliable calibration of the model impossible. The model is therefore best used for running various scenarios and comparing outcomes.

One member of the family of Runge–Kutta[7] methods is often referred to as "RK4", "classical Runge–Kutta method" or simply as "the Runge–Kutta method".

Let an initial value problem be specified as follows :

$$\dot{y} = f(t, y), \quad y(t_0) = y_0.$$

Here, y is an unknown function (scalar or vector) of time t which we would like to approximate; we $\dot{y}$ are told that , the rate at which y changes, is a function of t and of y itself. At the initial time t_0 the corresponding y-value is y_0 . The function f and the data t_0 , y_0 are given.

Now pick a step-size h>0 and define :

$$y_{n+1} = y_n + \tfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$t_{n+1} = t_n + h$$

for n = 0, 1, 2, 3, . . . , using

$$k_1 = f(t_n, y_n),$$
$$k_2 = f(t_n + \tfrac{h}{2}, y_n + \tfrac{1}{2}k_1 h),$$
$$k_3 = f(t_n + \tfrac{h}{2}, y_n + \tfrac{1}{2}k_2 h),$$
$$k_4 = f(t_n + h, y_n + k_3 h).$$ [1]

(Note: the above equations have different but equivalent definitions in different texts) .

Here y_{n+1} is the RK4 approximation of $y(t_{n+1})$, and the next value (y_{n+1}) is determined by the present value (y_n) plus the weighted average of four increments, where each increment is the product of the size of the interval, h, and an estimated slope specified by function f on the right-hand side of the differential equation.

k_1 is the increment based on the slope at the beginning of the interval, using y, (Euler's method) ;

k_2 is the increment based on the slope at the midpoint of the interval, using $y + \tfrac{h}{2}k_1$;

k_3 is again the increment based on the slope at the midpoint, but now using $y + \frac{h}{2} k_2$;

k_4 is the increment based on the slope at the end of the interval, using $y + h k_3$.

In averaging the four increments, greater weight is given to the increments at the midpoint. If the weights are chosen such that if f is independent of y, so that the differential equation is equivalent to a simple integral, then RK4 is Simpson's rule.

The RK4 method is a fourth-order method, meaning that the local truncation error is on the order of $O(h^5)$, while the total accumulated error is order $O(h^4)$.

Tan Delin and Chen Zheng have developed a general formula for a Runge–Kutta method in the fourth-order as follows:

$$y_{n+1} = y_n + \frac{h}{6}\left[k_1 + (4 - \lambda) k_2 + \lambda k_3 + k_4\right]$$
$$t_{n+1} = t_n + h$$

for n = 0, 1, 2, 3, . . . , using

$$k_1 = f(t_n, y_n),$$
$$k_2 = f(t_n + \tfrac{h}{2}, y_n + \tfrac{h}{2} k_1),$$
$$k_3 = f(t_n + \tfrac{h}{2}, y_n + \left(\tfrac{1}{2} - \tfrac{1}{\lambda}\right) k_1 h + \tfrac{1}{\lambda} k_2 h),$$
$$k_4 = f(t_n + h, y_n + \left(1 - \tfrac{\lambda}{2}\right) k_2 h + \tfrac{\lambda}{2} k_3 h).$$

where λ is a free parameter. Choosing $\lambda = 2$, this is the classical fourth-order Runge-Kutta method. With $\lambda = 1, 3, 4, 5$, this formula produces other fourth-order Runge–Kutta methods.

3. Analyzing the predicted model

We utilize the weight of economy, society and environment calculated above as the original value of DEC, DEV and DSO, then we obtain the chart as follows :

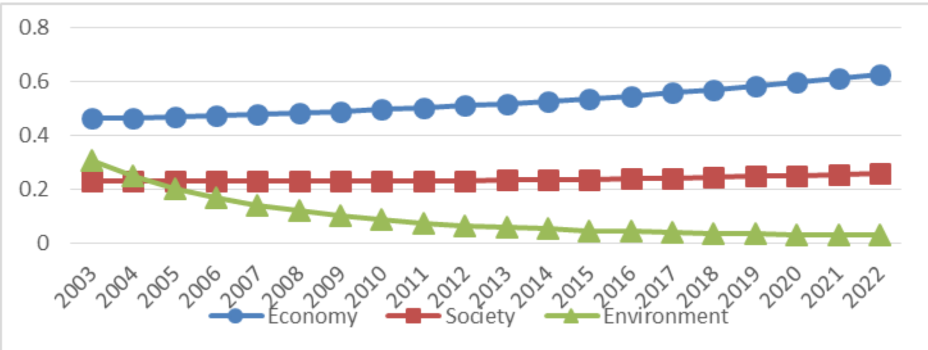

Figure Development Trend

4. Conclusions

From the chart we can find that without any human policies intervention ,employing the competition modeling we can come to the result that with the development of economy, the natural environment as well as the ecosystem will come to an destruction. In our Further investigation, we find that most countries, whether developed countries or developing countries, have the same trand like the chart illustrates. So we can make a conclusion that positive policy is play an important role in sustainable development.

Reference

[1] http://en.wikipedia.org/wiki/WCED

[2]Brundtland Commission (1987). "Report of the World Commission on Environment and Development". United Nations.

[3] http://en.wikipedia.org/wiki/United_Nations_Millennium_Declaration

[4] http://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equation

[5] http://demonstrations.wolfram.com/

[6]http://baike.baidu.com/link?url=R60TW1kXvMNI3hYSKfhdBcYUdNSe8bLP1HHqJ411TO4kQBFlADxQA1k0omg9JABvNliUKj16dWj2ZCYLSeovj_

[7] http://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_methods