Water temperature distribution of bathtub

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Abstract. we use the governing equation and continuity equation to build a two-dimensional model of water temperature on micro element theory. Because the partial differential equation is difficult to obtain its analytical solution, we use finite difference method to work out numerical solution of the model.

Introduction

In the bathtub, water temperature is determined by heat exchange between the interior and exterior parts of water. The heat exchange between the water and exterior environment is determined by the temperature brought by inflowing hot water and that taken away by overflow.The heat exchange in the interior of water including:The water vertical flowing caused heat exchange and the heat exchange caused by water density convection.

Problem analysis

There are many other factors influence water temperature in the bathtub, mainly including:

The liquidity of water: including the heat conduction and diffusion, the geometry of a water, and the location of overflow drain, etc.

The heat exchange at the water surface*:*

- *Convection:* Water and air heat transfer between two different media mainly by convection
- *Heat conduction:* If the air temperature and water temperature is different, energy exchange can occur through heat conduction.
- *Evaporation:* On the border of water and air, when some water molecules have enough energy, they will escape to the air.

Water and energy exchange in the bathtub: the heat exchange of bathtub with water is by solid heat conduction.

Assumption

- The initial air temperature and humidity are fixed value in the bathroom.
- The bathroom is an independent system.
- Per unit time, the addition of hot water is constant.
- The temperature of the water flowing into bathtub is 45℃.
- Initial water temperature in the bathtub is 39℃.

Water Temperature Distribution of Bathtub

We consider the distribution of temperature in the water and we use the finite element method to analyze the temperature distribution in the water in this model.

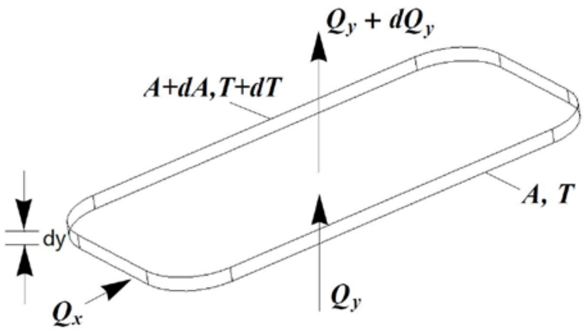

Fig.1 The infinitesimal

Assuming that bathtub isothermal surface is horizontal upward, the water temperature structure in bathtub composed of different temperature of countless isothermal surfaces [1]. From figure 1, the infinitesimal of hierarchical model is presented. We take out an infinitesimal body along the constant-height surface.

According to the heat conduction differential equations,

$$\frac{\partial T}{\partial \tau}=\frac{\lambda}{\rho c}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\phi}{\rho c}$$

Where ϕ represents the heat flux.

Then we analyze the Heat through infinitesimal. Considering the heat movement include,

i.The isothermal surface is on horizontal direction, So In horizontal direction, total heat passing the infinitesimal is zero.

$$Q_4=0$$

ii. Water flows along the vertical direction: Changes of infinitesimal heat per unit time caused by vertical flow is,

$$Q_5=-\frac{\partial}{\partial y}\left(c\rho Q_y T\right)dy$$

Where Q_y represents the net flow passing side y per unit time, T represents the temperature the infinitesimal.

Changes of heat will result in changes of water temperature,

$$Q_6=-c\rho\frac{\partial T}{\partial \tau}Adxdy$$

Where Q_6 represents the changes of water temperature, A represents the area of infinitesimal in horizontal direction.

Because total heat is balanced, we reason out following equation of heat balance,

$$Q_4+Q_5+Q_6=0$$

By the above equation we can obtain the partial differential equation below:

$$\frac{\partial T}{\partial \tau}=\frac{1}{c\rho A}\frac{\partial^2}{\partial y^2}\left(c\rho Q_y T\right)=0$$

For water at ambient temperature and pressure conditions, in the calculation of water temperature, effects of pressure changes on density can be ignored -- water is assumed not to be compressed [2]. The relationship between the density ρ and temperature T is presented,

$$\rho=\rho_0+aT^2+bT$$

Where ρ_0 represents the density of the freezing water, a represents a constant and b

represents a constant too.

We assume that the water flow according to the density of water into the same water layer. As a result, the three-dimensional model can be simplified into a two-dimensional model as follows:

$$\begin{cases} \dfrac{\partial T}{\partial \tau} = -v\dfrac{\left(\rho_0 + 3aT^2 + 2bT\right)}{\rho_0 + aT^2 + bT}\left(\dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial x^2}\right) = 0 \\ \dfrac{\partial \rho}{\partial x} + \dfrac{\partial \rho}{\partial y} = 0 \end{cases}$$

We adopt with finite difference method, and use *MATLAB* to get the numerical solution of this model.

Table 1 Parameters of model three

Physical Quantity	Value	Unit
a	-0.0067	
b	0.07	
L	1.7	m
H	0.6	m
λ	0.635	W/ (m ·K)
h	169	W/(m^2·K)
c	4200	J/(kg · K)
ρ	1000	kg/m^3
A	1.36	m^2

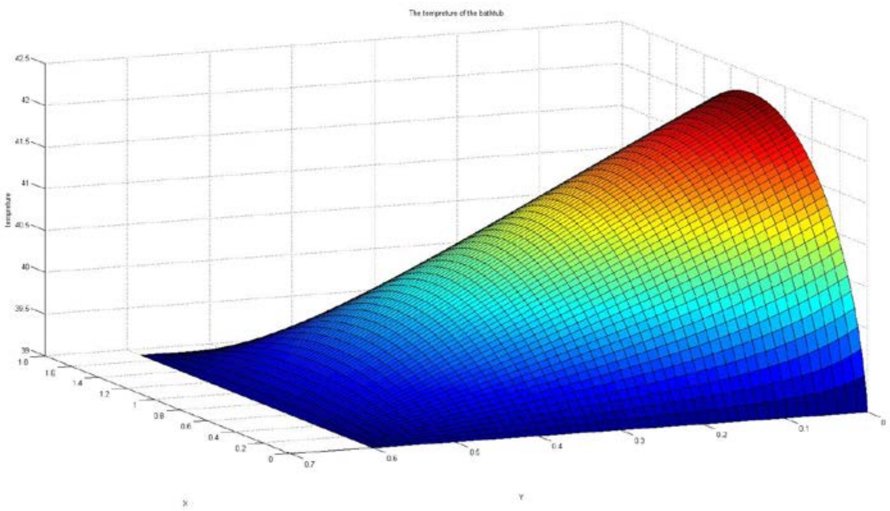

Fig.2 The temperature distribution of the water

Summary

Through the analysis of the water heat distribution in the bathtub affected by different factors.We discuss the influence of bathtub shapes on temperature and find the Optimal Shape of Bathtub.For the best thermal insulation effect, we make the bathtub shape into a cylinder. This makes the size of bathtub as large as possible and contact area minimum to reduce heat loss furthest.

References

[1].Shiming, Yang and Wenquan, Tao. heat transfer theory.