The Least Amount Of Water Used By Nonlinear Programming Model

Weiqing Wang

School of North China Electric Power University, Baoding 071000, China;

709226308@qq.com

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Abstract. Firstly, In order to make the temperature of water in tub fixed, thus, the quantity of heat of water from faucet is equal to the heat loss of water in tub . Secondly, according to thermodynamic energy formula, we get that the quantity of heat of water from faucet has relation with its temperature and amount. In this paper, we assume that the temperature of water in tap is certain, so, the quantity of heat per unit time is proportional to the velocity of flow. Thirdly, according to the law of conservation of energy, we get the formula about flow velocity and the size of bathtub. Finally, in order to minimize the amount of water which is equal to velocity of flow? We take the minimum of flow velocity as the goal and the size of tub as constraint condition to establish nonlinear programming model. By solving it, we get the lest amount of water used and the best size of tub. In sensitivity, we analyze the influence of parameters on velocity of flow, the results supports our model is robust.

Introduction

There is a simple bathtub; the bathtub is simply surrounded by a containment vessel, with a faucet on the edge of bathtub. Adding hot water into the bathtub, under natural conditions, the heat of the water will be lost to air, so the temperature will decline. But in the process of bathing, we hope that we can maintain the temperature of the water at a constant temperature. So we need to add hot water through the tap to the bathtub. In order to get the lest amount of water used and the best size of tub,we establish nonlinear programming model.

Nonlinear Programming Model

First, we assume the initial shape and dimension of the bathtub as follows:

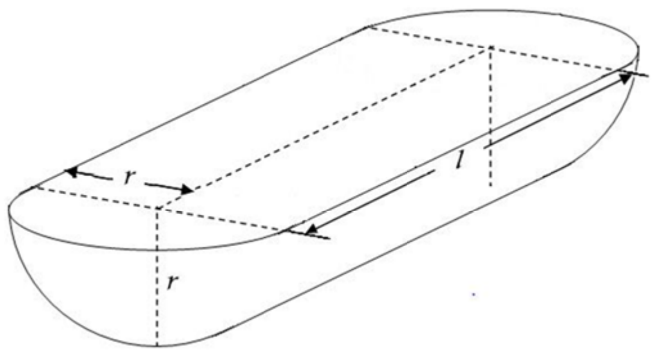

Figure 1 the dimensioning of the bathtub

The effective volume of the bathtub is made up of two one-fourth spherical surfaces and half cylindrical surface.(All the radius are R and the length of column is l.). And the effective volume of the bathtub, we assume, is 0.8.

For a bathtub full of water, R is the maximum height water of the bathtub can be reached. So, we can get that:

The surface area of the water in bathtub:

$$A = \pi R^2 + 2 \times R \times l \tag{1}$$

The area of the bathtub which water in the bathtub contacts with:

$$S = \pi Rl + 2\pi R^2 \tag{2}$$

The quality of water in bathtub:

$$M_w = \frac{1}{2}(\pi R^2 l + \frac{4\pi}{3} R^3)\rho_w \tag{3}$$

Model parameters

Table 1Model parameters

parameter	unit	meaning
M_w	kg	the mass of the water in the bathtub
ρ_w	kg/m^3	the density of water
T_w	K	the temperature of the water in the bathtub
S	m^2	the contact area of the water and the bathtub
A	m^2	the superficial area of the water in the bathtub
h	$Wm^{-2}K^{-1}$	the heat exchange coefficient
T_a	K	the temperature of the air
k	$Wm^{-2}K^{-1}$	the coefficient of heat conduction
d_{faucet}	mm	the caliber of the faucet
A_{faucet}	m	the cross-sectional area of the faucet

Model analysis

In order to maintain a constant temperature in the bath crock, we need to inject hot water constantly into the bathtub. The heat included in the hot water is positive correlation to the injectivity of the hot water. The injectivity of the hot water per unit time is equal to the speed of hot water multiply the cross-sectional area of the faucet. The heat of the water in the bathtub is constant, so the heat contained in the water should be equal to the loss heat in the bathtub.

The objective function is the minimum water consumption per unit time. According to the conservation of heat, we can get the functional relation about the speed of the hot water and the size of the bath crock. Taking advantage of nonlinear programming, we can get the best size of bath crock, which satisfy the minimum water speed.

Assumptions

We assume that:

1. the water just full of the bath, when people enter the bathtub.
2. The man has no effect on the temperature change of the water
3. The initial temperature (T_{w0}) of the water in the bathtub for 42 ℃
4. The temperature(T_{faucet}) of the water comes from the faucet is 55 ℃
5. The volume (V) of bath crock is constant
6. the caliber of the faucet (a) is constant. It is 0.05m
7. Maintain the initial temperature. Add hot water as soon as the people go into the bathtub.

The establishment of the model

We don't take the heat of hot water in the bath into consideration, firstly. Then, the heat loss of water in the bathtub per unit time (Q_S) is as follows:

$$Q_S = hA[T_w - T_a] + k\frac{\Delta T}{d}S \tag{4}$$

The heat which the water contains Per unit time l (Q_i) is as follows:

$$Q_i = C_w \rho_w v T_{faucet} A_{faucet} \tag{5}$$

Where v is the speed of the water in the faucet. And $A_{faucet} = \pi a^2$

We take the condition into consideration that the water just full of the bath, when people enter the bathtub. And the excess water will be run out from the bath when we add the hot water into the bathtub. The displacement of the water per unit time is equal to the speed of hot water multiply the cross-sectional area of the faucet.

Unit time from the heat of hot water are as follows:

The displacement of the water per unit time is

$$Q_p = C_w \rho_w v T_{w0} A_{faucet} \tag{6}$$

Because the temperature of the bathtub is constant, so the heat in the bath is. So the heat the water contains per unit time should be equal to sum of the heat loss of water in the bath and the heat output, i.e.

$$Q_i = Q_S + Q_p \tag{7}$$

So

$$C_w \rho_w v T_{faucet} A_{faucet} = C_w \rho_w v T_{w0} A_{faucet} + hA[T_w - T_a] + k\frac{\Delta T}{d}S \tag{8}$$

Simplify the equation, we can get a function about the speed of water and the crock radius and length of the bath

$$v = \frac{h(\pi R^2 + 2Rl)[T_w - T_a] + k\frac{[T_w - T_a]}{d}(\pi Rl + 2\pi R^2)}{C_w \rho_w (T_{faucet} - T_{w0}) A_{faucet}} \tag{9}$$

Solution of the model

The objective function is the minimum v in the equation (15). The equality constraint conditions is the volume of bath crock, and the actual size of the bathtub. The nonlinear programming model is:

$$\min v = \frac{h(\pi R^2 + 2Rl)[T_{w0} - T_a] + k\frac{[T_{w0} - T_a]}{d}(\pi Rl + 2\pi R^2)}{C_w \rho_w (T_{faucet} - T_{w0}) A_{faucet}}$$

$$s.t. \begin{cases} \frac{2}{3}\pi R^3 + \frac{1}{2}\pi R^2 l = V \\ 0.7 \le R \le 0.8 \\ 1.4 \le 2R + l \le 1.9 \\ R \ge 0, l \ge 0.2 \end{cases} \tag{10}$$

Where the temperature of the water in the faucet is 55℃; the volume of the bathtub is 800L.

We can get the minimum speed of the water is

v=0.0311m/s

So the size of the bathtub is

R=0.7m, l=0.2m

We consider the effect of the temperature of the water in the bathtub when we add bubble into bathtub. We can merely modify the speed function, so

$$\min v = \frac{\alpha h(\pi R^2 + 2Rl)[T_{w0} - T_a] + k\frac{[T_{w0} - T_a]}{d}(\pi Rl + 2\pi R^2)}{C_w \rho_w (T_{faucet} - T_{w0}) A_{faucet}}$$

$$s.t.\begin{cases} \frac{2}{3}\pi R^3 + \frac{1}{2}\pi R^2 l = V \\ 0.7 \le R \le 0.8 \\ 1.4 \le 2R + l \le 1.9 \\ R \ge 0, l \ge 0.2 \end{cases} \tag{11}$$

We can get the minimum speed of the water when we add bubble into bathtub

v=0.028m/s

So the size of the bathtub is

R=0.7 m, l= 0.2m

Sensitivity analysis

In this model, the initial temperature of the water in the bathtub is known by us. Changing it, we analysis the influence of the parameter on the smallest water velocity and the best size. We let the parameter changes within plus or minus 5%.

Change the initial temperature of the water, the results are

Table 2

error	$T_{w0} = 42^\circ C$	$v = 0.0311 m/s$	sensitivity
+1%	$42.42^\circ C$	0.0329m/s	-5.79%
−1%	$41.58^\circ C$	0.0294m/s	5.47%

The initial temperature change , the speed change below 5.79%,

In conclusion, the model is robust

References

[1] Polking, John. Albert, Bogges. Dave, Arnold. Differential Equations. Prentice Hall, 2002