Thermal transmission of the static water in three-dimension

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Keywords: Numeric calculation, thermal transmission, Gauss-Seidel.

Abstract. We study the thermal transmission of the static water. First we divide the water into lots of little grids in three-dimensional space and study the temperature distribution of the static water. To make accurate numerical calculation, we then discuss the thermal transmission of the typical grids and establish relevant discretions equations. Finally, we calculate relevant date by the iterative method of Gauss-Seidel and get the temperature distribution in space.

Introduction

The thermal transmisson of the static water in two-dimensional can be calculate by traditional model, but the thermal transmisson in three-dimensional is too complex to calculate in traditional way. So we consider extend formal model to three-dimensional to study the thermal transmission at the base of the laws of thermodynamics.

Analysis of the thermal transmission

Considering the temperature on the vertical plane changes with the height, the temperature distribution of the water should be described in three-dimensional space. Thus the grid in two-dimensional model is not suitable enough and we consider establishing the three-dimensional grid.

We divide the whole water into grids in three-dimensional space, and the inner grid will exchange heat with all the six grids next to it. The sketch map is as following:

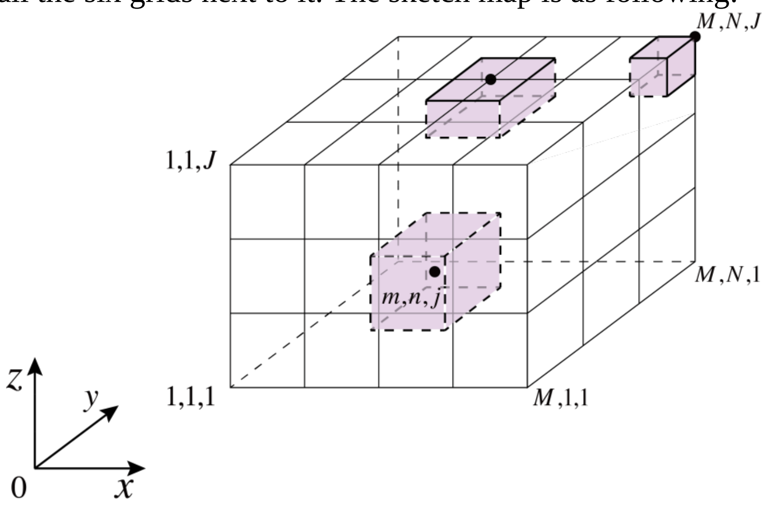

Fig. 1 The distribution of the grids in three-dimensional space

As is showed on Fig. 1, we know that the grids in three-dimensional space has extra thermal transmission along the height of the bathtub comparing with the two-dimensional model. And the thermal transmission of the inner grid is showed by following figure.

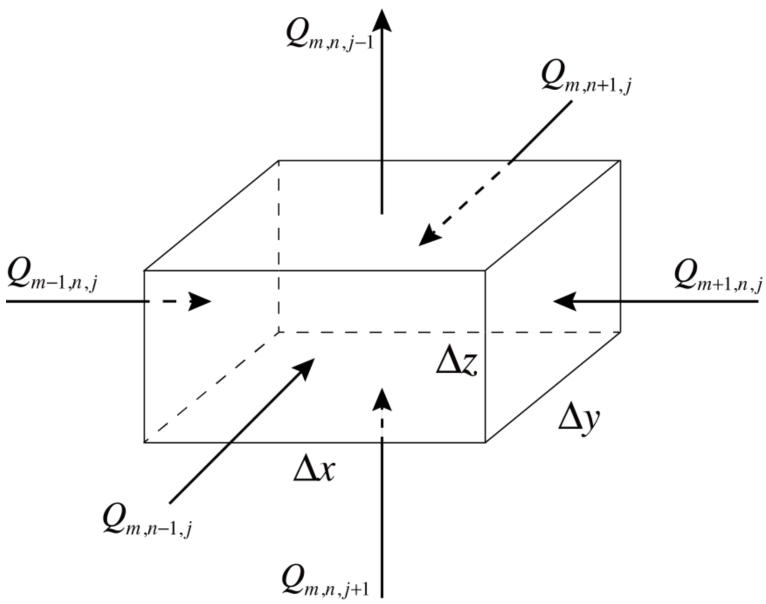

Fig. 2 The thermal transmission of the inner grid in space

As is showed in Fig. 2, the thermal transmission is from six directions. The grid exchanges heat mainly by thermal conduction if we assume the water is static. What's more, the grids in boundary also have the thermal convection with the environment, and we neglect the influence of thermal radiation to the whole thermal transmission.

Numerical Calculation

Table 1 Symbol table

Symbol	Meaning	Value/Units
$t^{i}_{m,n,j}$	The temperature of the grid (m,n,j) at the time of i.	℃
λ	The ability of conducting heat	W/(m·K)
h	The surface coefficient of heat transfer	$W/(m^2 \cdot K)$
ρ	The density of the water	kg/m^3
c	Specific heat capacity of the water	J/(kg·K),

Considering the law of the conservation of energy, we can establish the equations about the thermal transmission of inner grids.

$$\begin{aligned}&\lambda\left(t^{i}_{m-1,n,j}+t^{i}_{m+1,n,j}+t^{i}_{m,n-1,j}+t^{i}_{m,n+1,j}+t^{i}_{m,n,j-1}+t^{i}_{m,n,j+1}-6t^{i}_{m,n,j}\right)\\&=c\rho\Delta x^{2}\left(t^{i+1}_{m,n,j}-t^{i}_{m,n,j}\right)\end{aligned} \tag{1}$$

The equation of the grids on boundary is:

$$\begin{aligned}&\lambda\left(\frac{t^{i}_{m-1,n,j}}{2}+\frac{t^{i}_{m+1,n,j}}{2}+\frac{t^{i}_{m,n-1,j}}{2}+\frac{t^{i}_{m,n+1,j}}{2}+t^{i}_{m,n,j-1}\right)+h\Delta x\left(t_0-t^{i}_{m,n,j}\right)\\&=\frac{c\rho\Delta x^{2}}{2}(t^{i+1}_{m,n,j}-t^{i}_{m,n,j})\end{aligned} \tag{2}$$

And the thermal transmission of the grids at corner follows:

$$\frac{\lambda}{4}\left(t^{i}_{m-1,n,j}+t^{i}_{m,n-1,j}+t^{i}_{m,n,j-1}\right)+\frac{3}{4}h\Delta x\left(t_0-t^{i}_{m,n,j}\right)=\frac{c\rho\Delta x^{2}}{8}\left(t^{i+1}_{m,n,j}-t^{i}_{m,n,j}\right) \tag{3}$$

To get the numeric needed more quickly, we consider to calculate our equations by the iterative method of Gauss-Seidel, and the temperature of each boundary is the same as the temperature of environment. Then we write the code at the basis of the equations in our three-dimensional model.

Since the temperature distribution of the water in three-dimensional space is difficult to be described directly, we consider to use the Fluent to show the original temperature distribution of the water. We assume that there is a heat source in an oblong vessel with heat water flowing to it, and we make the equilibrium state as the original state and get rid of the heat source. The temperature distribution of the water is showed as follows:

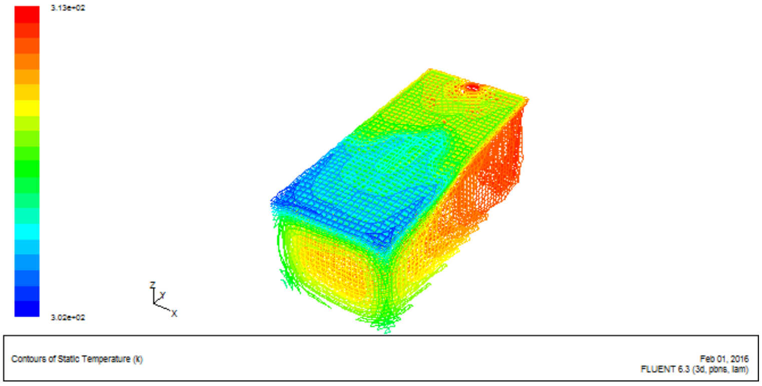

Fig. 3 Original temperature distribution of the water by Fluent

Summary

If we seem the fluent water as the open system which follows the laws of thermodynamics, we can make good numeric calculation and get the distribution of the water in space and time.

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