The Analysis of Water Supply Capacity Based on Analytic Hierarchy Process and Fuzzy Comprehensive Evaluation

Xinge LI

School of North China Electric Power University, Baoding 071000, China;

122102894@qq.com

Keywords: AHP fuzzy comprehensive evaluation water supply capacity

Abstract. Proposing solution to clean water shortage is of great importance to all citizens of the world. In this paper, we analyze and evaluate water supply capacity in a region based on building mathematical model by analytical hierarchy process (AHP) and fuzzy comprehensive evaluation. First of all, hierarchical structure consists of three layers, including target layer, criterion layer and sub-criterion layer. The weight matrixes of the first two layers are decided through AHP. Secondly, we describe factors in five degrees for determining five levels of capacity assessment. Fuzzy comprehensive evaluation is applied to acquiring the membership degree of five grades and judge the level of supply capacity rely on principle of maximum membership. Therefore, we are able to acquire water supply ability.

1. The determination of weight by AHP

As water supply capacity is involved by various indexes, we firstly analyze various indexes and get the weight of each index based on Analytic Hierarchy Process. Considering there are many indexes to the demand for water, AHP shall be a great way for this issue. Here we take Ukraine as an example to illustrate its water supply capacity.

Making level analysis of each index layer as figure 1 follows.

According to AHP and evaluation index system in figure 1, we suppose A1 to denote water supply capacity. Meanwhile, three indexes in criterion layer (including nature, social and economic development, cultural factor as well) are denoted by B1, B2, B3 respectively. Besides, we suppose C_{ij} to express the importance of $C_{ij}$to B_i. Lastly, we suppose a_{ij} to express the relative importance B_i compared to B_j (or C_{ki} compared to C_{kj}), As a result, we can get the equation below:

$$D = (a_{ij})_{n\times n}, a_{ij} > 0, a_{ji} = \frac{1}{a_{ij}} \tag{1-1}$$

Here, D denotes that pairwise comparison judgment matrix.

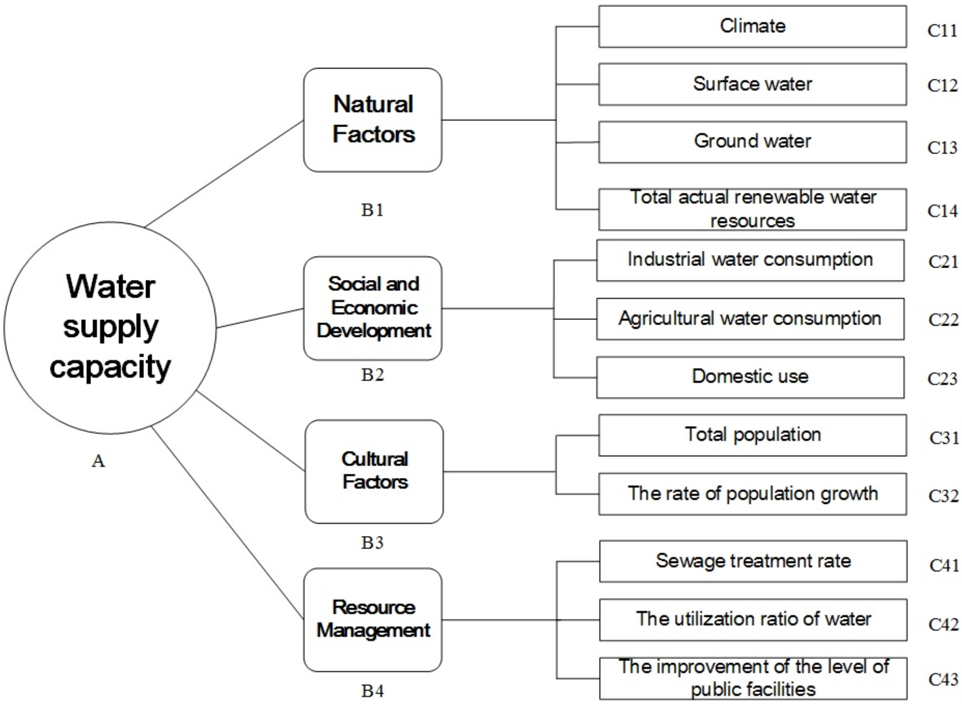

Figure 1: Hierarchy structure in the model

In order to build pairwise comparison judgment matrix, we create important degree valuation as table 1 below according to 1-9 proportion scale.

Table 1 1-9 scale method

1	Equally important
3	Slightly more important
5	Clearly more important
7	Strongly more important
9	Extremely more important
2,4,6,8	Intermediate values

- **Between criterion layer and target layer**

When $\lambda = \lambda_{max,}$ we can get the weight vector $^{(2)}=(w_1{}^{(2)},\ w_2{}^{(2)},\ w_3{}^{(2)},\ w_4{}^{(2)})^T$. Where $^{(2)}$ is the weight of B_i to A.

$$W^{(2)} = (0.5385, 0.2196, 0.1210, 0.1210)^T$$

We should make sure the degree of consistency is in an allowable range so as to let X reasonably express the weight of . Here we define CI to measure the consistent degree of matrix, and define RI to express the range allowable.

$$CI = \frac{\lambda - n}{n-1} \tag{1-2}$$

$$CR = \frac{CI}{RI} \tag{1-3}$$

Notes: n is equal to the number of characteristic roots of matrix;

λ is equal to the maximum among the characteristic roots of matrix ;

CR means the consistency ratio.

If CR<0.1, the consistency of comparison is acceptable.

- **Between criterion layer and secondary layer**

With the same method, we can get the weight vector $^{(3)} = (w_i{}^{(3)})^T$. Where $^{(3)}$ is the relative importance of C_{ki} to B_k.

2. Supply Capacity Evaluation

In consideration of the water situation, convenience to collect data and information, as well as intuitive analysis, we divide into five levels to evaluate water supply capacity and determine the membership functions that show the degree of index close to five levels defined.

Table 2 grading standard on evaluation indicators

Criterion layer	**Index layer**	Ⅰ **Absolute-weak**	Ⅱ **Significant-weak**	Ⅲ **Weak**	Ⅳ **Slight-weak**	Ⅴ Strong
Natural Factors	**Climate (mainly effected by temperature)**	$\bar{t}$< -5℃, dry air	-5℃<$\bar{t}$<0℃, little humidity with less wind	0<$\bar{t}$<5℃, normal humidity with strong wind	5<$\bar{t}$<10℃, greater humidity but less wind	$\bar{t}$>10℃, much greater humidity but less wind
	Surface water	≤ 10	10~50	50~100	100~200	>200
	Ground water	≤ 10	10~50	50~100	100~200	>200
	Total actual renewable Water resources	<10	10~100	100~500	500~1000	>1000
Social and economic development	**Industrial water consumption**	>50	30~40	20~30	10~20	<10
	Agricultural water consumption	>70	50~70	30~50	10~30	<10
	Domestic use	>20	15~20	10~15	5~10	<5
Cultural factors	**Total population**	Densely populated (>2000)	Large populations (1000~2000)	Moderated populated (500~1000)	Small populations (200~500)	Sparsely populated (<200)
	The rate of population growth	Dramatically increase (11%~13%)	Rapidly increase （9%~11%）	Steadily increase (7%~9%)	Gradually increase (5%~7%)	Slowly increase (<`5%)
Resource Management	**Sewage treatment rate**	Substandard rate (<40%)	Acceptable rete (40%~55%)	General rate (55%~70%)	Favorable rate (70%~85%)	Satisfied rate (>85%)
	The utilization ratio of water	Substandard ratio (<30%)	Acceptable ratio (30%~50%)	General ratio (50%~70%)	Favorable ratio (70%~90%)	Expected ratio (>90%)
	The improvement of the level of public facilities	Poor level	Low level	Common level	Well level	Excellent level

Notes:

1. Precipitation, Industrial, agricultural consumption and domestic use are

summarized in billion cubic kilometers annually.
2. Total volume of water is summarized in billion cubic kilometers.
3. Denotes the annual average temperature and s expresses forest coverage.
4. The meaning of grading standards are following showed:

Ⅰ——Absolute-weak, the ability to supply water is close to zero.

Ⅱ——Significant-weak capacity, it is crucial to seek foreign water.

Ⅲ——Weak capacity, water resource cannot satisfy requirement.

Ⅳ——Slight-weak capacity, it is possible to meet the demand for water.

Ⅴ——Strong capacity, it is adequate for reaching the demand for water.

Quantitative index quantification

For the factors such as precipitation, the greater of its value, the better effect on the capacity.

Qualitative index qualification

On the purpose of that supply evaluation model is adapted to common regions, we give an accurate value to qualitative index respectively for universal evaluation, that is, establish grading standard on qualitative index and membership functions.

Table 3 grading standard on qualitative index

Index value	**0~0.2**	**0.2~0.4**	**0.4~0.6**	**0.6~0.8**	**0.8~1**
judgment	Ⅰ	Ⅱ	Ⅲ	Ⅳ	Ⅴ

Considering grading standard and determine index value so as to obtain the membership by its function.

In the implementation process the parameter x in membership functions is determined by the true value of each index. After determining the membership functions, the membership degree of factor C_{ki} to grading B_k is evaluated and hence the fuzzy appraisal matrix R_k is obtained.

When making a comprehensive evaluation, the influence of each factor on the corresponding grading should be considered. Form the result of weight determination, we regard four weight vector (3) (k=1,2,3,4) as fuzzy subsets which denotes index sets, and the appraisal result is M_k.

$$M_k = {}^{(3)} \cdot R_k \qquad (1\text{-}4)$$

M_k decides preliminarily that evaluations on the main four criterions, for evaluating the water supply, we should get the appraisal result M with the same method above.

$$M = {}^{(2)} \cdot R \qquad (1\text{-}5)$$

Where (2) is the relative importance of B_i to A in weight determination section, R is supposed of M_k (k=1,2,3,4).

As the supply capacity is described by four criterions and this ability is judged by five standards, we can obtain the final result is that

$$M = (m_1, m_2, m_{3,4}, m_5) \qquad (1\text{-}6) \quad m_{a=} \{m_1, m_2, m_{3,} m_4, m_5\} \qquad (1\text{-}7)$$

Where m_i expresses the relative significance to supply capacity, m_a is the maximum of all. Choose corresponding standard as final evaluation on water supply capacity in accordance with maximum membership principle.

3. Verification of the model

Here we choose Ukraine, from the UN water scarcity map, as our object. According to Ukraine's situations of all aspects and specific data in 2010 from Ukraine's national bureau of statistics, we decide the pairwise comparison judgment matrixes of factors in each layer based on equation 1-1 and table1. The following shows pairwise comparison judgment matrix of the criterion layer:

$$D = \begin{bmatrix} 1 & 3 & 4 & 4 \\ 1/3 & 1 & 2 & 2 \\ 1/4 & 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 & 1 \end{bmatrix}$$

According to this, we carry out the weight of factors in each layer.

$$W^{(2)} = (0.5385, 0.2196, 0.1210, 0.1210)^T$$

Then, attaining assessment of water supply capacity in Ukraine by fuzzy integrated evaluation. The results are as follows:

$$M = (0.1219, 0.3498, 0.2635, 0.3026, 0.1189)$$

We can see apparently from the result, water supply capacity in Ukraine belongs to level 2 in table 3, which means Ukraine is equipped with significant weak supply capacity. The result is in accordance with the situation in reality.

4. Conclusion

By analyzing different factors which have an effect on water resources, we can clearly find which is more important to the water supply capacity of the region than others, that is the foundation of taking preventive measures. After having an analysis on Ukraine, we Verify the feasibility of the model.

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