Four Economic Indicators of Space Debris Removal schemes

Jiali Chen
NORTH CHINA ELECTRIC POWER UNIVERSITY Baoding
Jialichen6666@163.com

Keywords: space debris mitigation schemes, cost, benefits, risk, interference.

Abstract. In this paper, four significant factors including the costs, the risks, the benefits, and the interference to other satellites, are taken into account by transferring them to the specific commercial indexes. The transferring calculation is on the basis of a series of integral-differential equations which are set up in advance.

Introduction

Space debris is considered as a serious problem which obstructs humanities efforts at space exploration.[1] There are some previous studies analyzed space debris removal in the aspect of technology and economy. However, many researches are limited in predicting the development trend of space debris or only putting forward the cost and risk caused by space debris.[2] None of them give a commercial strategy to space debris removal schemes.

Four Economic Indicators

Space debris is a variable which ranges in size and mass, and yet, its size changes over time, so space debris changes over time either. Based on this, we determine a time-dependent model.

Risk Analysis. Collision between satellites and space debris happen occasionally, which causes a great loss to satellites. The probability of collision is an important physical quantity of risk assessment. For a given cross-sectional area of space systems, the probability of collision is connected with space debris collision probability, assemble density (the number of space debris per volume), relative speed and running time in one specific area. The average number of collisions c is indicated in Eq. (1).

$$c = v_{ret} AD\Delta t \tag{1}$$

Where v_{ret} is relative speed, A is cross-sectional area, D is space debris assemble density and Δt is running time.

$$F = v_{ret} D \tag{2}$$

We assume that flux F, the number of debris through unit cross-sectional area in unit time, is a variable connected with the size and assemble density of space debris.

Based on Poisson statistical distribution, we obtain the probability of collision P when the collision frequency is at the zero time and N time, as indicated in Eq. (3).

$$\begin{cases} P_{i=n} = (c^n / n!)e^{-c} \\ P_{i=0} = e^{-c} \end{cases} \tag{3}$$

Where e is the Euler number, n is nature number.

Based on Eq. (3), we can obtain that the probability of at least a collision through the area of space systems in the running time Δt, as showed in Eq. (4).

$$P_{i\geq 1} = 1 - e^{-c} \tag{4}$$

For the value of the parameter c is very little, we can consider $1-e^{-c}$ as c. That is to say, $1-e^{-c} \approx c$. As a result, the probability of at least a collision through the area of space systems is indicated as follows.

$$P_{i\geq 1} = c = FA\Delta t \tag{5}$$

According to Eq. (5), for F is a variable connected with the size and assemble density of space debris, we can draw the conclusion that collision probability is proportional to the size and assemble density of space debris. Next, the risk of debris removal can be obtained as long as we calculate the collision frequency.

Costs and Benefits Analysis. The cost decreased with the increasing removal and the degree of the risk is proportion to the increasing quantity of space debris.

Assume B_{t-1} is the benefit of a private firm, and it can be expressed as,

$$B_{t-1} = \sum_t \alpha^{t-1} B_t \{(1-\delta_t)V_t\} \tag{6}$$

Where α is the discount factor to express future benefits in today's terms, δ_t is the proportionate reduction in a spacecraft's lifetime productivity due to risk of debris collisions, and V_t is the sizes of satellites at the start of a period of fixed duration *t*.

Assume D_t is debris accumulates which is considered as,

$$D_t = (1+\lambda)D_{t-1} + \varphi_t V_t \tag{7}$$

Where φ_t is primary debris generated from a spacecraft in period *t*, λ is the rate of debris growth through coalitional breakup debris.

Then, primary debris generated from a spacecraft in period t is given by Eq. (8),

$$\begin{cases} \varphi_t = r^m + r^f \bar{D}_t \\ \bar{D}_t = \varepsilon D_t + (1-\varepsilon)D_{t-1}, 0 < \varepsilon < 1 \end{cases} \tag{8}$$

Where r^m is mission related debris, including launch vehicles, r^f is fragmentation debris per spacecraft in period *t*.

Interference Analysis. The productivity loss (externality) *C* satisfies the following equation:

$$C = B\gamma \sum_t \alpha^{t-1} V_t \frac{d\bar{D}_t}{dV_1}, 0 < \gamma < 1 \tag{9}$$

Assume $\Delta\tau$ is the interference to the satellites, and we obtain the following equations:

$$\Delta\tau = \begin{cases} \zeta t_w + (1+\beta)\gamma, 0 < d < r \\ \zeta t_w + (1+\beta)\Delta, d = 0 \\ 0, d > r \end{cases} \tag{10}$$

Where ζ , β and γ are the influence coefficients, *r* is the working radius of the system, t_w is the continuous working time, Δ is associated with the mass, volume and energy of the removal equipment. However, the energy is related to mass and volume. So Δ can be described as

$$\Delta = \beta' m' + (1-\beta') v_E' \tag{11}$$

Where the reduction coefficient of β is β', m' is the reduction coefficient of the mass, and v_E' is the reduction coefficient of the volume.

Based on the equations above, we can calculate the cost and benefit.

Conclusion

Analysis of the Quantity of Debris. The quantity of space debris has an impact on the evaluation results. As indicated in Fig.1, at one point of time, the larger the number of debris is, the higher the synthesis score is.

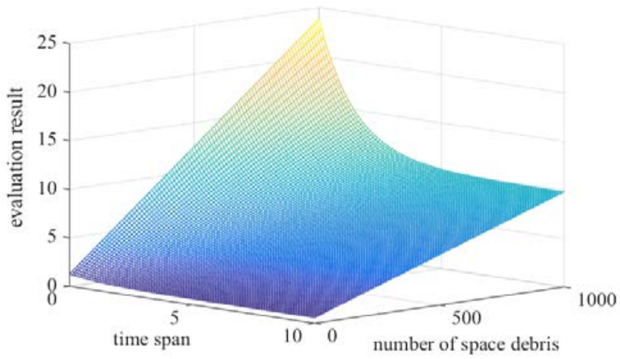

Fig. 1 the sensitivity analysis of the quantity of debris

Analysis of Initial Decline Rate. As shown in Fig.2, similarly, the synthesis score change little with a long time pan and decreasing of initial decline rate. The synthesis score changes apparently with small span of time or large initial decline rate.

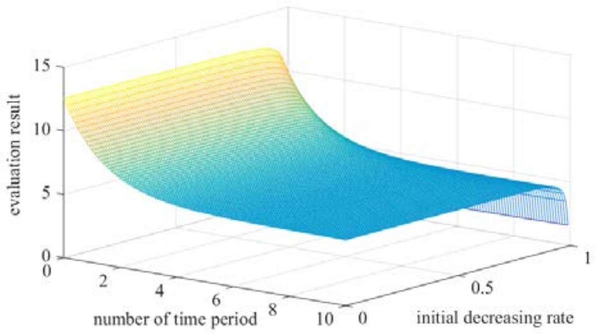

Fig. 2 Analysis of initial decline rate

Analysis of Initial Increase Rate. Fig.3 shows the costs have few connect with the change of initial increase rate. Only if the initial increase rate is high, the influence on evaluation result is obvious. This illustrates that the costs are lower than benefits in space debris removal and it is consistent with the actual situation.

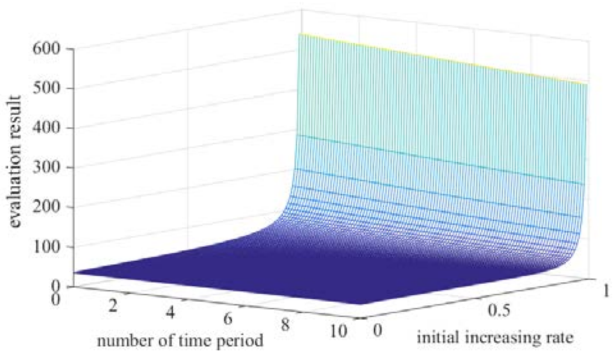

Fig. 3 Sensitivity analysis of initial increase rate

References

[1] Minghe Shan, Jian Guo, Eberhard Gill, Review and comparison of active space debris capturing and removal methods, Progress in Aerospace Sciences, Volume 80, January 2016, Pages 18-32, ISSN 0376-0421, http://dx.doi.org/10.1016/j.paerosci.2015.11.001.

[2] Molly K. Macauley, The economics of space debris: Estimating the costs and benefits of debris mitigation, Acta Astronautica, Volume 115, October–November 2015, Pages 160-164, ISSN 0094-5765, http://dx.doi.org/10.1016/j.actaastro.2015.05.006.