All people hail a cab

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Keywords: the waiting time, the rate of the vacant, the zone-to-zone prices, the meter-based prices

Abstract. Our basic model has two parts: to find the relationship between the waiting time and the rate of the vacant taxis arrive, and to find the relationship between the passenger flow volume and the rate of the taxi arriving in the area. In an extended model, we take the Scheduling efficiency into consideration. Finally, we discuss the different methods of taxi ride prices.

We can find the waiting time is related to the rate of the vacant arriving in the area. The correlation of them is negative. We establish the relationship between the passenger flow volume and the rate of the taxi arriving. The total number of cabs is associated with the passenger flow volume and the vacancy rate. Then we can obtain the total number of cabs needed. According to the statistics, we get the total number of cabs needed is 514.

Taking into account of the scheduling efficiency, we find when the scheduling efficiency decreases, the waiting time of unlicensed cabs remains unchanged, but the average waiting time grows. Thus, unlicensed cabs will raise their price in order to earn as much money as possible without impacting on the customer satisfaction greatly.

We calculate the different methods of taxi ride prices. Then we find when the n is equals to 30, the X is 1.56. Still, the X is 1.53 when the n is 50.

We compare analytical and numerical results with reality, using default parameters; we validate that our method is correct and robust.

1.Background

Taxi as public transport supplement, plays a significant role in the transport system. With its convenient, comfortable, fast features, there are parts of populations relies on taxicabs. Nevertheless, some customers are not so pleased with the quality of services by the taxi companies. Passengers always wait for a long time to take a taxi, while the taxis often drive with no passenger. Taxi transportation demand and supply is a pair of interrelated and indivisible concept. Therefore，a balance of supp1y and demand taxi industry is of importance for the optimized allocation of resources. On the other hand, there are many other factors related to the waiting time. We should take all of these into consideration to improve the situation.

In addition, the price of services also causes the complaints at some time. So drafting the method of taxi ride prices is important in a way. The major methods are zone-to-zone prices and meter-based prices nowadays.

The unlicensed companies currently control about 1/7 of all cabs in the city. The governors need to attach great importance to the regulations against unlicensed companies. However, the consequences of strengthening the regulations against unlicensed companies will be a mystery that needs to be explored.

2.Model Overview

The model has two parts: to find the relationship between the waiting time and the rate vacant taxis arrive in the area, and to find how the passenger flow volume varies.

For the first, we focus on the rate of the taxi arriving in the area and the time. The rate is related to the different times of the day. We assume it is a piecewise constant function. So we can obtain the arriving interval of vacant taxis follows an exponential distribution.

Next, we calculate the mathematical expectation of single passenger's waiting time. We take the

arriving interval of vacant taxis into consideration and make a reasonable assumption. Then we have the probability distribution of waiting time. Still, we can get the mathematical expectation of the waiting time.

We estimate the rate at interval j by using maximum likelihood estimation.

To derive an expression for the total number of cabs needed, we introduce the vacancy rate. We analyzed the change of the passenger flow volume varies. Then we can get the total number of cabs needed based on the change and the vacancy.

3.The Model

Step 1

We assume the numbers of taxicabs arriving in the area follow a Poisson distribution. That is to say, the time intervals that connected taxicabs arrive in the area follow an exponential distribution. However, it is not suitable. The reality of the taxi operation is related to the time. From the Poisson distribution, the numbers of empty taxicabs will become lager as the time getting later. Apparently, it is not to the fact.

Recent reports show that the quantities of the taxicabs arriving in the area follow a non-time-homogeneous. The rate (λ) of the taxi arriving in the area is different for distinguish of time. It is changed over time (t), so we derive a function for it.

$$\lambda = f(t) \tag{0.1}$$

We assume it is a piecewise constant function. In a certain period of time, the rate is constant. That is:

$$f(t) = \lambda_i, t_i \le t \le t_{i+1} \tag{0.2}$$

After this period of time, the arriving interval of vacant taxis follows an exponential distribution.

$$p_i(\Delta t) = \lambda_i e^{-\lambda_i \Delta t} \tag{0.3}$$

Step 2

We can calculate the mathematical expectation of single passenger's waiting time based on the probability distribution of the arriving interval of vacant taxis. Taking the time interval j of the day as an example, we assume the arriving interval of vacant taxis follow an exponential distribution. Now, we assume the last taxi pass by at time t and the passenger come taking $f(t_0)$ as probability density at time $t + t_0$.

We can have the probability distribution of waiting time (t_w).

$$p(t_w) = \frac{\int f(t_0)\lambda_j e^{-\lambda_j(t_w+t_0)}dt_0}{\iint f(t_0)\lambda_j e^{-\lambda_j(t_w+t_0)}dt_w t_0} = \frac{\lambda_j e^{-\lambda_j t_w}\int f(t_0)\lambda_j e^{-\lambda_j t_0}dt_0}{\int f(t_0)\lambda_j e^{-\lambda t_0}dt_0 \int \lambda_j e^{-\lambda_j t_w}dt_w} = \lambda_j e^{-\lambda_j t_w} \tag{0.4}$$

Therefore, the **probability distribution** of the waiting time must follow an exponential distribution. The mathematical expectation of the waiting time is:

$$E(t_w) = \int p(t_w)dt_w = \int \lambda_j e^{-\lambda_j t_w} t_w dt_w = \frac{1}{\lambda_j} \tag{0.5}$$

The rate (λ_j) of the time interval j of the day is unknown, we can obtain it based on the historical data. We estimate it by using maximum likelihood estimation.

$$\overset{\wedge}{\lambda} = \arg\max \hat{l}(\lambda \mid \Delta t_1, ..., \Delta t_n) = \arg\max_\lambda \lambda^n e^{-\lambda \sum \Delta t_i} = \sum t_i / n = 1/\mu \tag{0.6}$$

Step 3

We establish an objective function based on the situation. The total number (T) of cabs needed to serve the "Greater Mythical" area is related to the passenger flow volume (k_i) and the
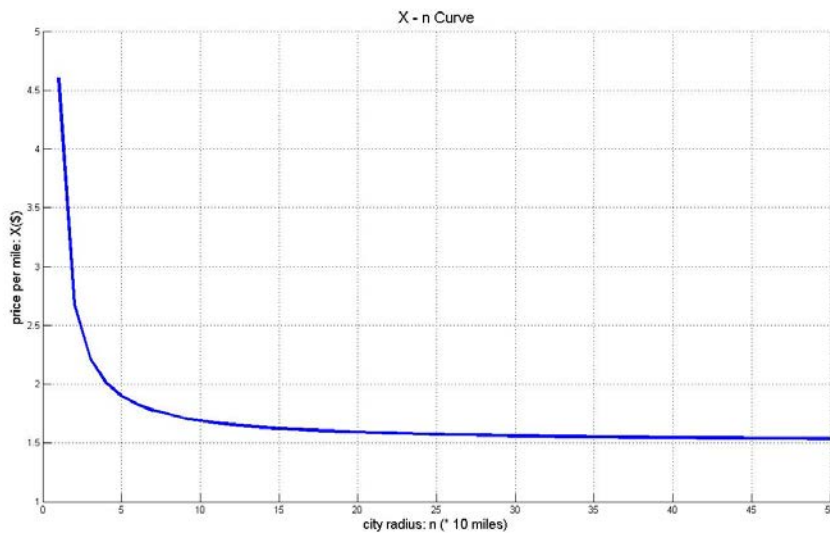
vacancy rate (p_i). We have

$$T = \frac{k_i}{p_i} \tag{0.7}$$

Step 4

We analyzed the relationship between the passenger flow volume and the rate of the taxi arriving in the area, we have

$$k_i = e^{-a\frac{1}{\lambda_j}} k_0 \tag{0.8}$$

4.Solutions to the Requirements

Question **: the total number of cabs needed**

We can find the waiting time is related to the rate of the vacant arriving in the area. The correlation of them is negative. Still, the mathematical expectation of the waiting time is inversely proportional to the rate of the taxi arriving.

The rate is constant in a certain period of time. At the end of this period, the rate is changed over time.

We establish the relationship between the passenger flow volume and the rate of the taxi arriving.

The total number of cabs is associated with the passenger flow volume and the vacancy rate. Then we can obtain the total number of cabs needed.

According to the statistics, we find the vacancy rate is from 40 percent to 70 percent. And we get the total number of cabs needed is 514.

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