Initial solution to the problem of shadow localization ——to solve the shadow length with latitude and longitude

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Abstract. This article analysis the variation laws of the shadow length on each parameter, and obtain the shadow length curve of a known straight rod. The shadow length is directly related to solar altitude angle and the length of rod. The factors that affect solar altitude angle directly are: solar declination, geographical latitude, time angle. Among them, solar declination is determined by date, angle by the local time, and local time is related to latitude and time. According to the relationship between the three factors and the solar altitude angle, the change law of the solar altitude angle is obtained, and the variation curves of the parameters are deduced.

1. Introduction

How to determine the location and date of video filmed is an important aspect of video data analysis. Sun shadow positioning technology is a method to solve this problem through solar shadow variation of objects in video analysis. In this paper, the mathematical model of shadow length is established, and analyze influence about the variation law of its length. Then we obtain a change curve about a straight bar's shadow length in Tiananmen Square according to the established model.

2. Changing Laws about Shadow Length along with Each Parameter

By the relevant physics and geography knowledge, the factors that influence the length of the shadow has the following several points: pole position (longitude and latitude), time (date and time) and rod lengths. Then, we will analyze the influence of the five factors on the shadow length.

- **Rod lengths:** According to the principle that light travels along the line, we can know that when the time is certain, the shadow length increases with the increase of the length of the rod, that is, the positive correlation between the two. It no longer go through other algorithms.
- **Longitude:** We know that each difference of 15 degrees longitude, time difference of 1 hours.[1] Therefore, while several points with different longitudes and the same latitude locates in the same regional time, if rod length is same, and so does the shadow length. In other words, when the latitude, rod length are certain values, variation laws of shadow length with different longitudes is only influenced by regional time. Therefore, if a certain place is fixed value, it is meaningless to study laws of shadow length under different longitude. Because the sun is not possible to beam every corner of the earth at the same time.
- **Time:** For a fixed location, solar altitude angle varies with time. Therefore at different times in a day, the shadow length is changing with the height of the sun angle. The following article studies the variation law of shadow length with a situation of a 3-meter rod in the Tiananmen Square in a period of 9:00-15:00 at GMT +8, October 22, 2015.
- **Latitude:** Solar altitude angle is different with latitude at the same regional time while its longitude is a fixed value. Therefore, shadow length of a rod varies with different latitude. Then the variation law will be discussed in the following article.

According to the data [2], the formula for calculating the height of the sun can be obtained as following:

$$\alpha = arcsin(\sin\varphi * \sin\delta + \cos\varphi * \cos\delta * \cos t) \quad (1)$$

In order to study the variation laws of solar altitude angle with latitude, it is knows to other parameters. That is to say, it take time at 3:00 GMT+8, the October 22nd, longitude 23 degrees 116 minutes 29 seconds. The variation law of shadow length changing with latitude is shown in the following figure:

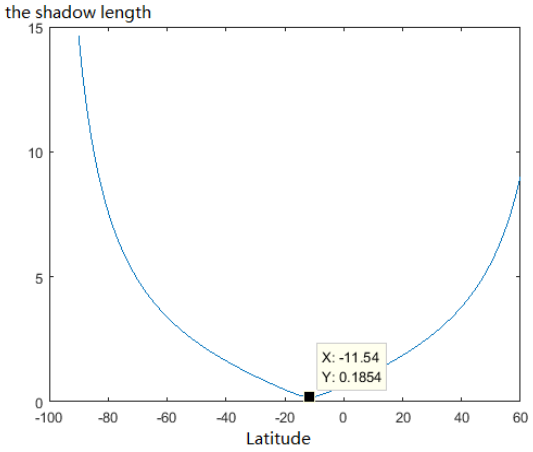

Fig.1 the Variation Law of Shadow Length Changing with Latitude

It is known to us in the figure that the coordinates of the smallest shadow length is (0.1854, -11.54). That is to say, the subsolar point is 11.54 degrees north latitude at that time. According to the data [2], the theoretical value is close to the actual value.

- **Time:** it is obvious to know according to the formula (1) that in the same place, the shadow length is correspond to the number of days. So we can use the same idea with the last factor, and draw the shadow length curve of this rod within a year.

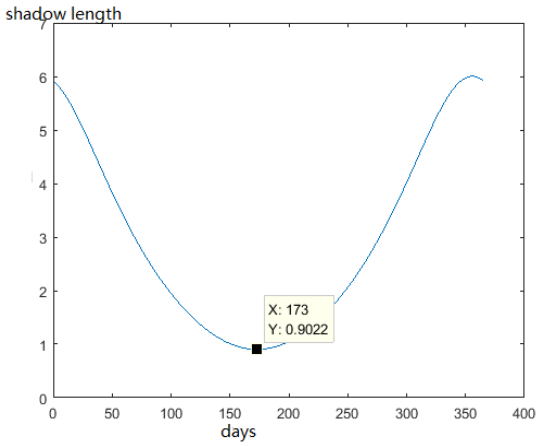

Fig.2 the Shadow Length Curve within a Year

Just as the figure shows, the smallest shadow length is in the number 173 day. For example in 2015, today is June 22ed—the summer solstice. According to the knowledge of geography, the time that shadow of the northern hemisphere is shortest (that is to say, solar altitude angle is max) is the summer solstice. Therefore, analysis is credible.

3. The Shadow Curve of a Rod in the Tiananmen Square

According to the model established above, the assumed date is October 22nd, that is to say $N=295$. Then we obtained the variation curve of solar altitude angle changing with time as follows:

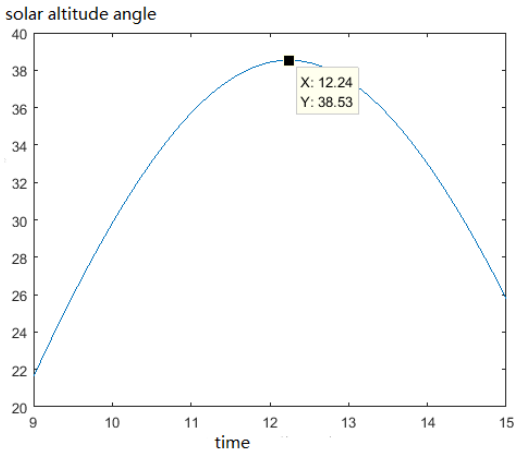

Fig.3 the Variation Curve of Solar Altitude Angle Changing with Time

Then, we can obtain the shadow length according to the relationship between the solar altitude angle and the shadow length. The schematic diagram is shown below.

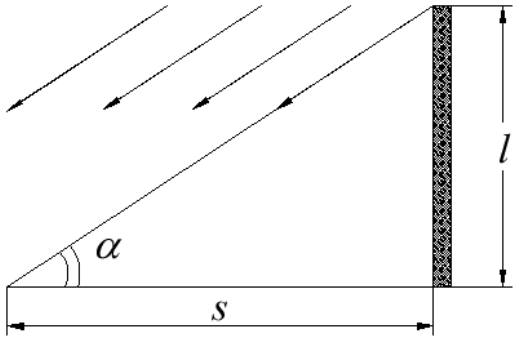

Fig.4 the Schematic Diagram Solving Shadow Length

According to the relationship between the edge and the angle of the triangle, we can know:

$$s = l / tan\,\alpha \tag{2}$$

Then we can obtain the shadow length of a 3-meter rod at each time in the period of 9:00-15:00 GMT+8 in the Tiananmen Square. And the variation curve can be drew with MATLAB as follows.

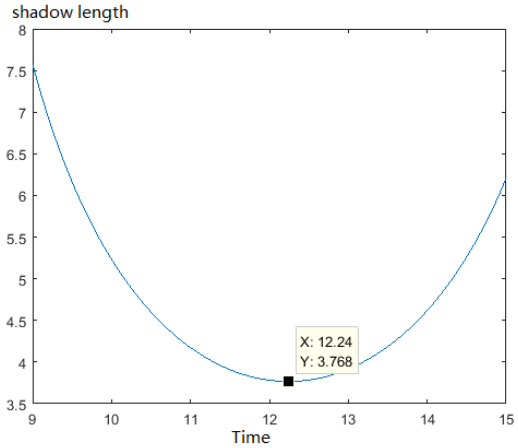

Fig.5 the Shadow Length Curve for a Period

As is known to us, the biggest shadow length is 7.573 meters at 9:00 GMT+8, and the smallest shadow length is 3.768 meters at 12:14 GMT+8.

4. Conclusion

This article obtained the related factors that affect the length of shadow through the

combination of mathematics, physics and geography, and analyzed the relationship between them, which lays the foundation for the next step to solve the other quantity according to the length of shadow.

Reference

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[2] Hongli, Jiang. Mathematical deduction and analysis of the solar direct point latitude [J]. Mathematical Bulletin, 2007, 46(9): 39-40