Joint Angle-Doppler Estimation With Coprime Sampling On A Uniform Circular Array

Changling Zhou[1, a], Jie Pan[2, b]

[1]School of Information Engineering, Yangzhou University, Yangzhou 225009, China;

[2]School of Information Science and Technology,Nanjing Uiversity of Aeronautics& Astronautics, Nanjing 210000, China.

[a]clzhou521@163.com, [b]434689657@qq.com

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Abstract. This paper is addressed on the joint angle-doppler estimation (JADE) for the uniform circular array (UCA) with coprime sampling. Space-time adaptive processing (STAP) can conveniently estimate direction-of-arrival (DOA) and radial velocities of emitting sources. And UCA can realize the azimuth angle of the 360^o detection without angle ambiguity. In this paper, we extend the coprime sampling for the linear array to UCA by utilizing the real beamspace transformation , we map the UCA manifold vector onto a virtual ULA manifold. And nonuniform time samples at different sensor of the virtual ULA can be employd to generate a full rank autocorrelation matrix by spatial smoothing. Then, we can use MUSIC algorithm to estimate the angle and doppler frequency jointly by applying UCA, coprime samples with STAP framework. Simulations show that making use of UCA also achieve good performance compared to coprime linear array.

Introduction

Space-time adaptive processing (STAP) is a normalized technique in airbone MTI radar signal processing [1]. It refers to the use of two-dimensional (2D) (direction-of-arrivals and Doppler frequencies) spatio temporal adaptive filters. There are many methods to estimate the angle-Doppler information, such as MUSIC algorithms [2], ESPRIT algorithms [3], time-space-time MUSIC [4], and so on.

For coprime sampling [5], two uniform samplers with sample spacings as MT and NT are used, in which M and N are coprime and T is the minimum sampling interval. Coprime sampling can offer an enhanced degree of freedom.

Uniform circular array (UCA) [6] is of interest in many applications, including radar, sonar and navigation. Because it has desirable properties, UCA is capable of providing 360^o azimuthal coverage of the same aperture in any direction and also offers information for source elevation angles. However, uniform linear array (ULA) only in the -60^o to 60^o has a high angle resolution. So we propose the coprime sampling on UCA for estimating joint angle-Doppler [7] at the output of each sensor. Firstly, transforming the beamspace is to map the UCA to ULA; Secondly, taking advantage of spatial smooth is to construct autocorrelation matrix with full rank; Lastly, using MUSIC algorithm is to achieve JADE. By using UCA and coprime sampling, we proved that JADE also can be figured out and the performance is better.

This paper is organized as follows. In Section 2, the signal model is briefly reviewed. In Section 3, real beamspace for uniform circular arrays is introduced in detail. In section 4, we propose beamforming and joint angle-doppler estimation, which are verified by simulations in section 5. The section 6 concludes the paper.

Signal model

Consider a UCA with M sensors in the space, there are D sources emitting electromagnectic wave impinging on the UCA. We take samples at time instant nT in time, T is the minimum sampling interval. At each sensor, two samplers run at sampling rates M1T and N1T, where M1 and N1 are coprime integers, n∈{0,M1, ⋯ (N1 −1) M1,N1, ⋯ (2M1 −1) N1}. Each source is represented by its DOA θ_i and its radial velocity υ_i with λ being the wave length [8]. The observation signal X(m,n) can be modeled as

$$X(m,n)=\sum_{i=1}^{D}A_i e^{j\xi\cos(\bar{\phi}_i-r_m)}e^{j2\pi\bar{f}_i n}+N(m,n), \tag{1}$$

where source signals $\{A_i\}_{i=1}^{D}$ is modeled as zero-mean random variables with the uncorrelated property $\mathbb{E}[A_i^* A_j]=\sigma_i^2\delta_{i,j}$. Here $\bar{f}_i$ is normalized Doppler frequencies defined by $\bar{f}_i=\frac{T}{\lambda}\upsilon_i, \xi=k_0 r\sin\theta$, where $\bar{f}_i\in[-1/2,1/2)$, $r_m=\frac{2\pi m}{M},(m=1,2,\cdots,M)$

$k_0=\frac{2\pi}{\lambda}$. Additive white noise $N(m,n)$ with $\mathbb{E}[N^*(m,n)N(m',n')]=\sigma^2\delta_{m,m'}\delta_{n,n'}$. In there we assume that noise is uncorrelated with the sources $\{A_i\}_{i=1}^{D}$.

Change Eq.(1) into matrix form. Defining a random matrix X =[X(m, n)], Eq.(1) becomes

$$\mathbf{X}=\sum_{i=1}^{D}A_i\mathbf{V}_s(\bar{\theta}_i,\bar{\phi}_i)\mathbf{V}_t^T(\bar{f}_i)+\mathbf{N}, \tag{2}$$

where $\mathbf{V}_s(\bar{\theta}_i,\bar{\phi}_i)$ are spatial steering vectors corresponding to $\bar{\theta}$ and $\mathbf{V}_t(\bar{f}_i)$ denote temporal steering vectors.

Real beamform for uniform circular arrays

In this section, by the beamspace transform [7], we consider the discontinuous circular aperture, and get the following formula: $Q\approx K_0 r$, where Q for the continuous array to be able to stimulate the maximum phase mode and $k_0=\frac{2\pi}{\lambda}$.

There M is the number of sensors, and the weighted vector of the normalized phase excitation function of the Q is defined as

$$\mathbf{W}_q^H=\frac{1}{M}\left[e^{jqr_1},e^{jqr_2},\cdots,e^{jqr_M}\right]. \tag{3}$$

We use The UCA-RB(real beamspace) algorithm which transform element space to beamspace with the beamformer $\mathbf{F}_e^H$.The beamspace transformation $\mathbf{F}_e^H a(\theta)=b(\theta)$, so the UCA manifold vector $a(\theta)$ onto the beamspace manifold $b(\theta)$, here define beamforming matrix $\mathbf{F}_e^H$, ${\mathbf{F}_e}^H=\mathbf{C}_v\mathbf{V}^H$, where

$$\mathbf{C}_v=diag\{j^{-Q},\cdots,j^{-1},j^{0},j^{-1},\cdots,j^{-Q}\}, \tag{4}$$

and

$$\mathbf{V}=\sqrt{M}\left[\mathbf{W}_{-Q}\vdots\cdots\vdots\mathbf{W}_0\vdots\cdots\vdots\mathbf{W}_Q\right]. \tag{5}$$

$\mathbf{W}_q$ corresponds to the UCA mode space of the first q virtual array element. The role of $\mathbf{C}_v$ is to eliminate the **J** in the imaginary part index. The synthesized UCA mode space matrix is:

$$\mathbf{a}_e(\theta,\phi)={\mathbf{F}_e}^H\mathbf{a}(\theta,\phi)=\mathbf{C}_v\mathbf{V}^H\mathbf{a}(\theta,\phi)\approx\sqrt{M}\mathbf{J}_\zeta\mathbf{a}_{\mathrm{UCA}}(\phi), \tag{6}$$

where the expression of $\mathbf{a}_{\mathrm{UCA}}(\phi)$ and ULA are similar. The elevation angle of the signal depends on the ζ as the parameter of the symmetric Bessel matrix $\mathbf{J}_\zeta$.

$$\mathbf{a}_{\mathrm{UCA}}(\phi_i)=\left[e^{-jQ\phi_i},e^{-j(Q-1)\phi_i},\cdots,e^{j(Q-1)\phi_i},e^{jQ\phi_i}\right], \tag{7}$$

and

$$\mathbf{J}_\zeta=diag[J_Q(\zeta),\cdots,J_1(\zeta),J_0(\zeta),J_1(\zeta),\cdots,J_Q(\zeta)]. \tag{8}$$

Beamforming and joint angle-doppler estimation

We apply beamforming in (2).

$$\mathbf{Y}=\mathbf{J}_{\zeta}^{-1}\mathbf{F}_e{}^H X = \sum_{i=1}^{D} A_i \mathbf{V}_{ss}(\bar{\theta}_i,\bar{\phi}_i)\mathbf{V}_t^T(\bar{f}_i) + \mathbf{J}_{\zeta}^{-1}\mathbf{F}_e{}^H\mathbf{N}, \tag{9}$$

where

$$\begin{aligned}\mathbf{V}_{ss}(\bar{\theta}_i,\bar{\phi}_i) &= \mathbf{J}_{\zeta}^{-1}\mathbf{F}_e{}^H\mathbf{V}_s(\bar{\theta}_i,\bar{\phi}_i)\\ &= \mathbf{J}_{\zeta}^{-1}\mathbf{C}_v\mathbf{V}^H\mathbf{V}_s(\bar{\theta}_i,\bar{\phi}_i)\\ &\approx \mathbf{J}_{\zeta}^{-1}\sqrt{M}\mathbf{J}_{\zeta}\mathbf{a}_{UCA}(\bar{\phi}_i)\\ &= \sqrt{M}\mathbf{a}_{UCA}(\bar{\phi}_i).\end{aligned} \tag{10}$$

Vectorizing (9) leads to the following expression

$$\mathbf{x} = vec(\mathbf{Y}) = \sum_{i=1}^{D} A_i \mathbf{V}_{ss,t}((\bar{\theta}_i,\bar{\phi}_i),\bar{f}_i) + \mathbf{n}, \tag{11}$$

where $\mathbf{n} = vec(\mathbf{J}_{\zeta}^{-1}\mathbf{F}_e{}^H\mathbf{N})$, and

$$\mathbf{V}_{ss,t}((\bar{\theta}_i,\bar{\phi}_i),\bar{f}_i) = \mathbf{V}_{ss}(\bar{\theta}_i,\bar{\phi}_i)\otimes\mathbf{V}_t(\bar{f}_i), \tag{10}$$

represents space-time steering vector.

The autocorrelation matrix of x can be estimated as

$$\hat{\mathbf{R}}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^H(k). \tag{11}$$

Next, we study the autocorrelation matrix x. From (11), the ideal autocorrelation matrix of x is found to be

$$\mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right]=\sum_{i=1}^{D}\sigma_i^2\mathbf{V}_{ss,t}((\bar{\theta}_i,\bar{\phi}_i),\bar{f}_i)\mathbf{V}_{ss,t}^H((\bar{\theta}_i,\bar{\phi}_i),\bar{f}_i) + \mathbf{n}', \tag{12}$$

where

$$\mathbf{n}' = \sigma^2\mathbb{E}(\mathbf{I}_p\otimes\mathbf{J}_{\zeta}^{-1}(\mathbf{J}_{\zeta}^{-1})^H). \tag{13}$$

Here $\mathbf{I}_p$ is the $(N_1+2M_1-1)\times(N_1+2M_1-1)$ identity matrix. The term $\mathbf{V}_{ss,t}((\bar{\theta}_i,\bar{\phi}_i),\bar{f}_i)\mathbf{V}_{ss,t}^H((\bar{\theta}_i,\bar{\phi}_i),\bar{f}_i)$ can be rewritten as follows $(\mathbf{V}_t(\bar{f}_i)\mathbf{V}_t^H(\bar{f}_i))\otimes(\mathbf{V}_{ss}(\bar{\theta}_i,\bar{\phi}_i)\mathbf{V}_{ss}^H(\bar{\theta}_i,\bar{\phi}_i))$, which is because of (12) and the property $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC})\otimes(\mathbf{BD})$. The entries in $\mathbf{V}_t(\bar{f}_i)\mathbf{V}_t^H(\bar{f}_i)$ are in the form of $e^{j2\pi\bar{f}_i\tilde{n}}$. Hence, we converts $\mathbf{W}_t(\bar{f}_i) = [e^{j2\pi\bar{f}_i\tilde{n}}], \tilde{n} = -M_1N_1,\cdots,M_1N_1$. So that the new steering vector $\mathbf{W}_t(\bar{f}_i)$ is viewed as the collection of 2M1N1+1 uniform samples. In there $\mathbf{W}_{ss}(\bar{\theta}_i,\bar{\phi}_i)$ can be also regarded as steering vectors $[e^{jQ\tilde{m}}]$, according to Real beamform for uniform circular arrays, the range of $\tilde{m}$ is −2Q to 2Q. From its conversion form we know, $\mathbf{W}_{ss}(\bar{\theta}_i,\bar{\phi}_i)$ is satisfied with centro-Hermitian.

In there $\mathbf{W}_{ss}(\bar{\theta}_i,\bar{\phi}_i) = [e^{jQ\tilde{m}}], \tilde{m} = -2Q,\cdots 2Q$. On the basis of the equivalence of (3) and (4), a matrix $\mathbf{Z}\in\mathbb{C}^{(4Q+1)\times(2M_1N_1+1)}$ is constructed from $\mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right]$ such that

$$\mathbf{Z} = \sum_{i=1}^{D}\sigma_i^2\mathbf{W}_{ss}(\bar{\theta}_i,\bar{\phi}_i)\mathbf{W}_t{}^T(\bar{f}_i) + \mathbf{n}''. \tag{14}$$

where $n'' = \sigma^2 E(e_1e_2^T\otimes J_{\zeta}^{-1}(J_{\zeta}^{-1})^H)$, $e_1 = [\delta_{l,2Q+1}]_{l=1}^{4Q+1}$, $e_2 = [\delta_{l,M_1N_1+1}]_{l=1}^{2M_1N_1+1}$.

Then, spatial smoothing [9]–[11] can estimate the autocorrelation matrix of Z. Finally, the MUSIC algorithm can resolve the DOA and the Doppler frequency information. For spatial smoothing, the submatrix of Z is defined as $\mathbf{Z}_{p,q}\in\mathbb{C}^{(2Q+1)\times(M_1N_1+1)}$, the spatial smoothed matrix $\mathbf{R}_{ss}$ as

$$\mathbf{R}_{ss} = \frac{1}{(2Q+1)(M_1N_1+1)}\sum_{p=0}^{2Q}\sum_{q=0}^{M_1N_2}\mathbf{Z}_{p,q}\mathbf{Z}_{p,q}^H. \tag{17}$$

And then we defined the $\tilde{\mathbf{V}}_t(\bar{f}_i)$ and $\tilde{\mathbf{V}}_{ss}(\bar{\theta}_i,\bar{\phi}_i)$ as

$$\tilde{\mathbf{V}}_t(\bar{f}_i) = [e^{j2\pi\bar{f}_i\tilde{n}}], \tilde{n} = 0, \cdots, M_1N_1, \quad (15)$$

and

$$\tilde{\mathbf{V}}_{ss}(\bar{\theta}_i, \bar{\phi}_i) = [e^{jQ\tilde{m}}], \tilde{m} = 0, \cdots 2Q, \quad (16)$$

so

$$\begin{aligned}\widetilde{\mathbf{W}}_{ss,t}((\bar{\theta}_i, \bar{\phi}_i), \bar{f}) &= \widetilde{\mathbf{V}}_{ss}(\bar{\theta}_i, \bar{\phi}_i) \otimes \widetilde{\mathbf{V}}_t(\bar{f}_i) \\ &= \left[e^{jQ\tilde{m}}\right] \otimes \left[e^{j2\pi\bar{f}_i\tilde{n}}\right].\end{aligned} \quad (17)$$

In the end, $\mathbf{R}_{ss}$ can be utilized in MUSIC algorithms to estimate $((\bar{\theta}_i, \bar{\phi}_i), \bar{f})$, let $\mathbf{U}_n$ denote the noise subspace of $\mathbf{R}_{ss}$, the MUSIC spectrum for the coprime sampling on a uniform circular array is then defined as

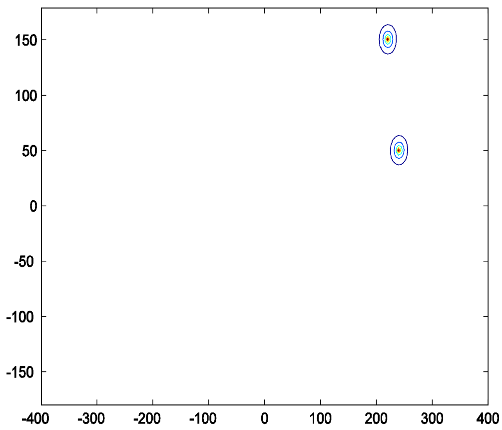

Fig.1Angle-Dopplerpattern using UCA model

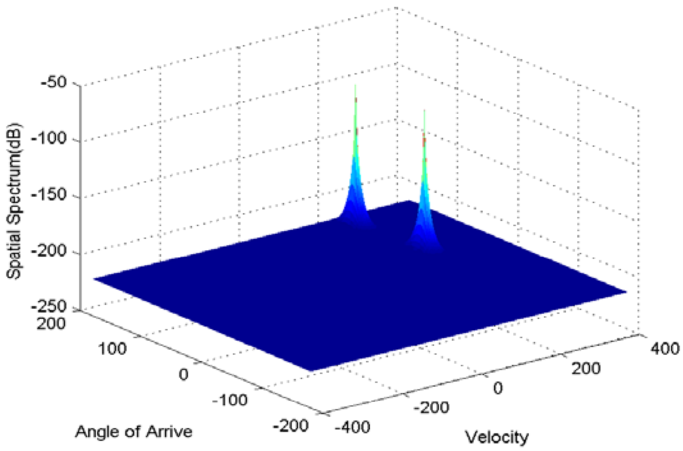

Fig.2 Angle-Doppler pattern's planform

$$P_{MUSIC}((\bar{\theta}_i, \bar{\phi}_i), \bar{f}) = \frac{1}{\left\|\mathbf{U}_n^H \tilde{\mathbf{W}}_{ss,t}((\bar{\theta}_i, \bar{\phi}_i), \bar{f})\right\|_2^2}. \quad (18)$$

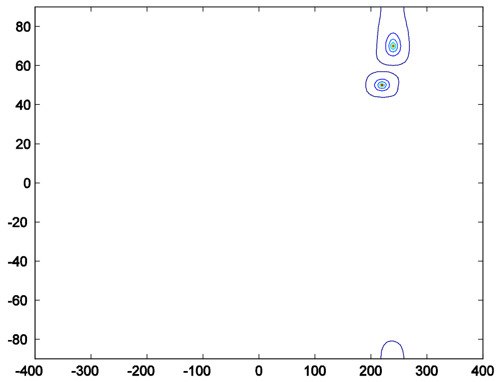

Fig.3 Angle-Doppler pattern using coprime linear array model

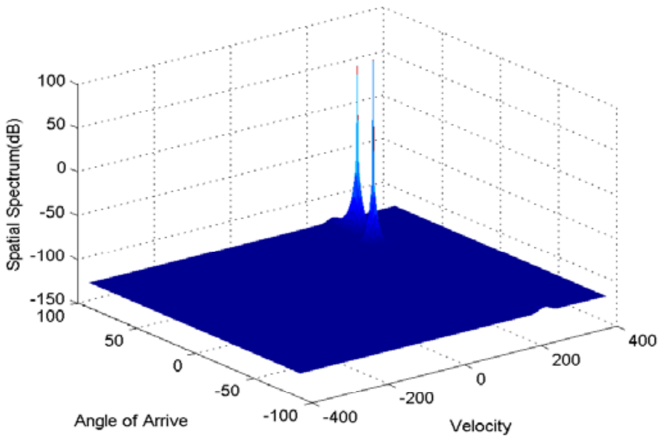

Fig.4 Angle-Doppler pattern's planform

Simulation results

We provide simulation results to evaluate the performances of the co-prime sampling on a uniform circular array approach for Joint angle-Doppler estimation in this section. In this section, we use the MUSIC algorithm with coprime sampling on a uniform circular array. In the simulation, M1= 4 and N1= 5. The number of sensors is 15. According to (13), the autocorrelation matrix of x is estimated from 200 snapshots. The input signal is mixed with additive white Gaussian noise with 0dB SNR. We set the angle and frequency in advance for [150, 220], [50, 240]. Simulation results are shown in Fig.1 and the planform of Angle-Doppler pattern is Fig.2 In Fig.3, we exhibit coprime linear arrays and samplers for estimating the JADE, Fig.4 is Fig.3's planform. Its angle and frequency is [50, 220], [70, 240].

Conclusion

In this paper, we have proposed a coprime sampling for angle-Doppler estimation with the uniform circular array (UCA) model. We have finished real beamspace of UCA, the beamspace manifold whose form is similar to the ULA manifold. So that we established a spatial smoothed matrix. The spatial smoothed matrices are admitted to define the MUSIC spectrum over the angle-Doppler plane. The estimated results were verified great through simulations.

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