Water Calculations in Billings

Shuo Zhang

North China Electric Power University(Bao Ding) ,Hebei 071000,China

735359890@qq.com

Keywords: water-recycle grey model cost-plus mode

Abstract. It is the purpose of the paper to develop a water-recycling plan for Billings.In this paper, I make two essential models and an extended model. I first develop grey model to predict the data of water use in the future 30 years at Billings. Grey model can well predict uncharted data and get an obvious trend.. Besides, I develop a cost-plus model to calculate the costs of water treatment both the non-potable and potable recycling systems. At last, I provide some suggestions about the water consummation at Billings.

Introduction

Billings is located in the south-central portion of Montana and is the county seat of Yellowstone County, also the biggest city in Montana. Billings has a semi-arid continental climate with dry, very warm summers, and cold, dry winters.[1] In recent years, due to the increase of the local population, global warming, El Nino and many other reasons, Billings is in deep trouble of water short. Beyond that, many other cities also have experienced a spate of dryness, especially in southwest America, which presents a challenge to the sustainability of current water use by human and natural systems in the region.[2]

I develop a water-recycling plan for Billings and there are three parts in the original problem statement.

Assumptions

Domestic water and industrial water are the mainly use in Billings. I neglect other kinds of water use.

Steps of getting non-potable water and potable are as follows: raw water is treated with flocculation aid and the coagulant. Next, the solids are allowed to precipitate. With the filtration of the filter tank and disinfection, I get non-potable water. Then, the non-potable water are filtered and disinfected further and potable water is produced. I neglect transportation costs.

The Model 1: Grey Model

Step1. Data examine and Processing: To guarantee the feasibility of the model, it is necessary to exam the known data. I have $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),...,x^{(0)}(n))$, then calculate transmission $\lambda(k)$:

$$\lambda(k)=\frac{x^{(0)}(k-1)}{x^{(0)}(k)},k=2,3,...,n$$

If all $\lambda(k)$ falls on $\Theta=(e^{-\frac{2}{n+1}},e^{\frac{2}{n+2}})$, $x^{(0)}$ can be used as the data of the GM(1,1) model. Or else, $x^{(0)}$ will be transformed to make itself fall on Θ. We are supposed to find suitable constant c to make translation:

$$y^{(0)}(k)=x^{(0)}(k)+c,k=1,2,...,n$$

Get $\lambda_y(k)=\dfrac{y^{(0)}(k-1)}{y^{(0)}(k)}\in\Theta,k=2,3,...,n$

Step2. Building Model: Building GM(1,1) as equation(1), I can get the predicted values:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}, k = 0,1,\cdots,n-1,\cdots$$

And $$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), k = 1,2,\cdots,n-1,\cdots$$

Step3. Testing the Predicted Value:

(1) Testing Residue: Calculate $\varepsilon(k)$

$$\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}, k = 1,2,...,n,$$

Where $\hat{x}^{(0)}(1) = x^{(0)}(1)$, if $\varepsilon(k) < 0.2$, it can be considered that the final results achieve accuracy generally. If $\varepsilon(k) < 0.1$, it meets the high precision.[3]

(2)Testing Deviate Value: Calculate $\rho(k)$

$$\rho(k) = 1 - \left(\frac{1-0.5a}{1+0.5a} \right) \lambda(k),$$

If $\rho(k) < 0.2$, it can be considered that the final results achieve accuracy generally. If $\rho(k) < 0.1$, it meets the high precision.[4]

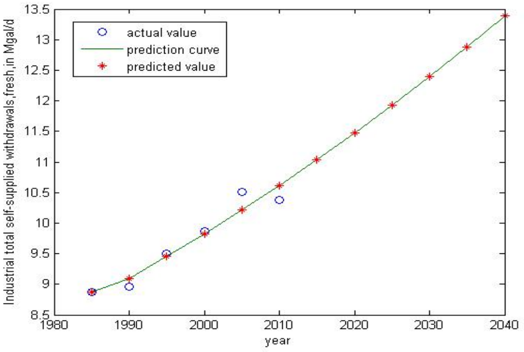
Calculation Results of the Model

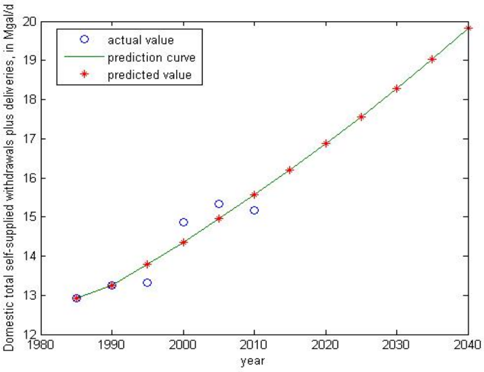
The original data is into GM(1,1), then we can get results as follows:

Table 1

Data fitting

year	The demand of domestic water(Mgal/d)	The demand of industrial water(Mgal/d)
2016	16.33364	11.1211
2017	16.4657	11.2074
2018	16.59882	11.2945
2019	16.73301	11.3822
2020	16.86829	11.4706
2021	17.00467	11.5596
2022	17.14215	11.6494
2023	17.28074	11.7399
2024	17.42045	11.831
2025	17.56129	11.9229
2026	17.70326	12.0155
2027	17.84639	12.1088
2028	17.99067	12.2029
2029	18.13612	12.2976
2030	18.28275	12.3931





Model 2: Costs of water recycling

Table 2.Model 2 parameters

Parameter	Meaning
Q_1	The demand for non-potable water of reclaimed water
Q_2	The demand for potable water of reclaimed water
S_1	The non-potablewater plantsinvestment costs
S_2	The potable waterplantsinvestment costs
C_1	Annual depreciation charge for Construction project investment of recycle water
C_2	Reclaimed water management fee per year
k_1	Fixed assets depreciation rates
k_2	Correction coefficient
k_3	Cost factor of repair
E_1	Power bill
E_2	Pharmaceutical costs
E_3	Repair and maintenance costs
E_4	Wages and benefits
M_1	Costs of producing non-potable water for each unit
M_2	Costs of producing potable water for each unit
a_i	Dosage of the i th chemical
b_i	The price of drug i unit dose
n	Types of chemicals needed

Step1: Investment of reclaimed Wastewater Treatment Plant

$$C_1 = k_1 Q^{\alpha}$$

Step 2: Costs of water management

$$C_2 = E_1 + E_2 + E_3 + E_4$$

Where $E_1 = k_2 eQH$, $E_2 = \dfrac{365Q}{10^3}\sum_{i=1}^{n}(a_i b_i)$, $E_3 = k_3 E_1$, $E_4 = (C_1 + E_1 + E_2 + E_3) \times k_4$

The costs of reclaimed water is S

$$S = C_1 + C_2$$

Step 3: Costs of both water:

$$M_1 = \frac{Q_1}{Q_1 + Q_2} S_1$$

$$M_2 = \frac{Q_2}{Q_1 + Q_2} S_1 + S_2$$

Sensitivity

Based on the results of Model 1, the water use in 2015 is into Model 2 to calculate C_1, E_1, E_2, E_3, E_4 and S , where $k_1 = 0.05, k_2 = 1.15, k_3 = 0.025, k_4 = 0.15, e = 1.00$. [5]Then I modified parameter values by 10% and get the following results.

Table 3.Sensitivity of costs of reclaimed water to $\pm 10\%$change in parameter values

Parameter	Percentage change in costs of reclaimed water in	
	+10%in parameter	-10%in parameter
Q	9.98%	-9.98%
k_1	0.0977%	-0.0977%
k_2	9.72%	-9.72%
k_3	0.24%	-0.24%
k_4	1.3%	-1.3%

Conclusion

From the figure and table, I know our model can well fit the raw data. Both the domestic and industrial water use present an increasing trend. Billings will consume more and more water if it develops still at current situation.According to exact data, I can get the costs of two kinds of water systems. And by the analysis of sensitivity, I know our model has a good stability.

Reference

[1] https://en.wikipedia.org/wiki/Billings,_Montana

[2] Daniel R. Cayan, Tapash Das, David W. Pierce, Tim P. Barnett, Mary Tyree, and Alexander Gershunov. Future dryness in the southwest US and thehydrology of the early 21st century drought.United StatesGeological Survey, La Jolla, CA 92093-0224.

[3]Jeffery Forrest, Zaiwu Gong, Tianxiang Yao.Grey Systems: Theory and Application. Emerald Group Publishing Limited

[4] Erdal Kayacan, Baris Ulutas, Okyay Kaynak. Grey system theory-based models in time series prediction. Expert Systems with Applications 37 (2010) 1784 – 1789. http://dx.doi.org/10.1108/20439371211197622.

[5] Colette L. Meehan. Fixed Price Vs. Cost Plus. http://smallbuisiness.chron.com/fixed-price-vs-cost-plus-2220.html.