

# Study on the airspace window firing method of shipboard gun

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**Abstract:** Warship defense is very important, especially close-in air-defense. Many researchers study methods of defending anti-ship missile. In those methods, the firing method of future airspace window is a promising method. Although it has many problems need to solve. In this paper using the "soft" and "hard" way, two methods of future airspace window are studied and analyzed deeply, one is the firing data distribution method and another is preplacing gun barrel angle method. Study conclusion shows our method is s a kind of effective way to improve the high-speed and complicated maneuvering anti-ship missile attacking.

## 1. Introduction

In a modern naval surface ship defending missile, shipboard gun is an important part and the last barrier. With the development of missile technology and tactics, the ship defending missile attacking faces new challenges. Firstly we put forward higher requirements on weapon and equipment, such as weapon system has higher precision and faster response, better target maneuver detection and more comprehensive recognition<sup>[1]</sup> technology, etc. Secondly the traditional using mode has also been challenged. At present, all of the close-in gun weapons defending missile is single point tracking firing in advance to reduce the projectile distribution. With the missile technology developing and the maneuver mode becoming complicated, tracking an anti-ship missile is more difficult. When the target maneuver become more complex, the cause error significantly will sharply increase in the ship fire control system. The firing data errors reduce the effectiveness of defending a missile sharply. In order to improve shipboard gun anti-missile effectiveness. There are two main ways to solve this problem. One is to improve the calculating precision of firing data and application of new technology. Another is to optimize the use of shipboard gun and firing methods to improve the kill probability, such as the airspace window firing method<sup>[2]</sup> etc.

The basic ideas of airspace window firing method is to optimize the firing data configuration. This can enlarge the coverage of the projectile, and improve the tolerance in error of firing data and improve the firing effectiveness.

The airspace window firing method has two ways. One is "soft" way based on the distribution of the firing data, another is "hard" way through the barrel scattered distribution center.

## 2. The attacking characteristics and influence of high speed complex maneuvering target

In order to achieve penetration, anti-ship missiles use many tactics maneuvers such as low sea-skimming flight, the attack end jumped dive attack, proportional guidance attack. The precision of shipboard gun firing data and anti-missile attacking effectiveness will be influenced.

### 2.1 Increase obviously errors of firing data

Effectiveness of shipboard gun firing depends on the precision of firing data. Calculating the trajectory of the firing data depends on the basis of the target points, schedule of target motion and non-standard ballistic meteorological conditions. Because before the launch, how target move is not possible, calculating the schedule of target motion only adopts the hypothesis method. In fact, the target motion law may have difference with assumption. Especially the anti-ship missile at end, its motion law have large deviation with assumed target motion law. This will lead to increase error of firing data. Shipboard gun firing theory also is known as the target distribution increases. Though the simulate calculation, the jumped subduction of anti-ship missile will make the error of firing data increase more than 3 times<sup>[3]</sup>.

Secondly, considering that the speed of the target is high, even that hypothesis consistent with target motion law, due to the observation error of high speed moving target increases, the error factors of target motion will also greatly increase. At the same time due to the target speed is high, the schedule of settlement error will increase during target motion in projectile flight time.

### 2.2 Reduce Firing effectiveness

Complex maneuvering anti-ship missile attacking effectiveness at high speed is declining. The firing projectile spread is a normal distribution in the early point of prediction. When the goal deviate from the early point of prediction is larger, the probability of projectile hitting target is very small.

The projectile and target distribution exists in the direction, height and distance, which have the same nature and law. Take the direction for example to analysis for convenience. Figure 1 is distribution diagram in the direction of the projectile and target when firing a target for the single point. Solid line is projectile distribution probability density. Dotted line is the target distribution of probability density.

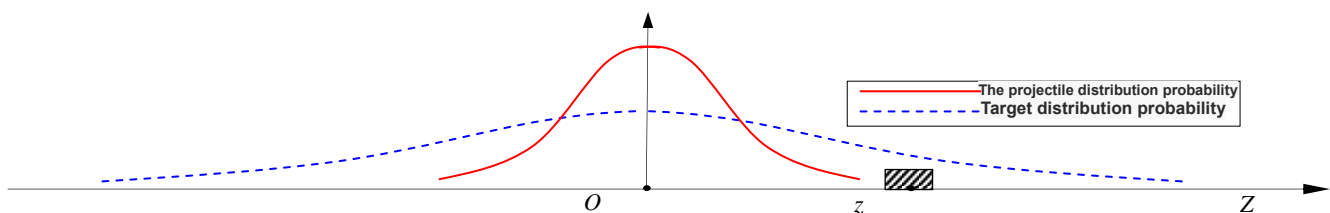


Figure 1 single point firing bullets and target distribution diagram

Assume that there is no systematic error (the third group error), the projectile dispersion center and target distribution center point are in superposition on the point  $O$ . The possible location  $z$  obeys the normal distribution. Seeing from the diagram, only target locates in the scope of the projectile distribution, it is likely to hit target. When the target is beyond the scope of the projectile spread, even more of the projectile, it is impossible to hit target. Because the high-speed and complicated maneuvering of anti-ship missiles, which results in increasing target distribution. This will make the effectiveness of shipboard gun firing reduce, and even lead to failure of the fight against.

## 3. Main factors in the airspace window firing method

In order to realize the airspace window firing method, we should optimize firing range and reasonably distribute probability density of the projectile.

### 3.1 Optimize the firing area

In order to improve the hit probability, when the target distribution (error of firing data) is larger, adopt the method of factitious spread makes the projectile cover possible area. It will solve the problems caused by the target distribution is larger. Factitious spread range depends on the error of firing data.

Due to missile attacking must ensure enough fire power density, when low speed shipboard gun missile attacking, firing the calculated elements for single point is the only choice. With close-in defense shipboard gun firing rate increasing, it is possible that cover for a wider range on premise of meeting the requirements of power density.

In order to ensure the firing area can meet the requirements of power density, area fire of shipboard gun should have high emissivity. We should optimize factitious spreading area accord to the rate and the density of firepower.

### 3.2 Reasonably distribute the area projectile probability density

As to airspace window firing, the projectile should cover area of firing and consider in the probability distribution of firing area and the probability density distribution of projectile. No matter adopt what kind of factitious spreading method, considering the feasibility of the project implementation, it should regard kill probability as firing effectiveness index. According to the probability characteristics of target distribution, we should reasonably choice the projectile distribution and optimize the distribution of projectile.

#### 4. Firing data distribution method of airspace window firing

Airspace window firing method essentially changes single point to the firing area. It converts traditional single point firing to multiple points. This can make up the defects of error of firing data in high-speed and complicated maneuvering anti-ship missile attacking and enhance the probability on the lake.

##### 4.1 Kill probability of single point firing

The effectiveness of the shipboard gun firing method has decisive significance on the hitting probability. But the effectiveness of firing method to choose is decided by target distribution error and shipboard gun spread error<sup>[4]</sup>. For convenience, it analyzes the kill probability of the single point firing.

Assume that the error probability spreading in the direction and high respectively are  $E_{zj}$ 、 $E_{hj}$ . In order to simplify the calculation, we put the  $E_{zj}$  as the unit of measuring on the direction and put the  $E_{hj}$  as the unit of measuring on the discretion. Set the value of 1. Target distribution probability of error on the direction and discretion respectively are  $E_{zx}$ 、 $E_{hx}$ , also we put the  $E_{zj}$ 、 $E_{hj}$  as the unit. They both obey the normal distribution.

$f(z)$  and  $f(h)$  respectively represent that spread in the direction of the projectile and the distribution of high and low density. There are:

$$f(z) = \frac{\rho}{\sqrt{\pi}} e^{-\rho^2 z^2} \quad f(h) = \frac{\rho}{\sqrt{\pi}} e^{-\rho^2 h^2}$$

$\varphi(z)$  and  $\varphi(h)$  respectively represent that the target distribution in the direction and the distribution of high and low density. There are:

$$\varphi(z) = \frac{\rho}{E_{zx} \sqrt{\pi}} e^{-\rho^2 \frac{z^2}{E_{zx}^2}} \quad \varphi(h) = \frac{\rho}{E_{hx} \sqrt{\pi}} e^{-\rho^2 \frac{h^2}{E_{hx}^2}}$$

The variables  $z$  and  $h$  take  $E_{zj}$  and  $E_{hj}$  as the unit

From the above description, the projectile distribution rules and the target distribution rules are the same on the direction and height. Therefore, we take discussion only in the direction as follows.

We take kill probability as the firing effectiveness indicators of firing effect. Assume the target under the condition that hit by  $m$ , damage probability is  $G(m)$ .  $G(m)$  obeys or approximate obeys index damage law, which is:

$$G(m) = 1 - \left(1 - \frac{1}{\omega}\right)^m$$

$\omega$  is expectations of mutilate target missiles number needed to hit. Its value depends on the caliber of shipboard gun and the target type. Under such circumstances, launch  $S$  projectile and the damage probability of goals ( $R_s$ )

$$R_s = 1 - \left(1 - \frac{P}{\omega}\right)^s$$

$P$  is round hitting probability.

Assume that launch  $S$  projectile and set  $D$  for the projection width of target direction ( $E_{zj}$  as the unit). When the target is located in position  $Z$ , as shown in figure 1, the probability  $P$  of firing a single hit is

$$P = F\left(Z + \frac{D}{2}\right) - F\left(Z - \frac{D}{2}\right)$$

The kill probability is:

$$R_S = 1 - \left(1 - \frac{P}{\omega}\right)^S$$

$$= 1 - \left[1 - \frac{F\left(Z + \frac{D}{2}\right) - F\left(Z - \frac{D}{2}\right)}{\omega}\right]^S$$

If considering the target distribution, the probability of that target located in  $Z$  is:

$$\varphi(z)dz = \frac{\rho}{E_{fX} \sqrt{\pi}} e^{-\rho^2 \frac{z^2}{E_{fX}^2}} dz$$

The probability  $R_S$  of damage target on the direction is:

$$R_S = \int_{-\infty}^{+\infty} \varphi(Z)R_S(Z)dz$$

$$= \int_{-\infty}^{+\infty} \frac{\rho}{E_{fX} \sqrt{\pi}} e^{-\rho^2 \frac{z^2}{E_{fX}^2}} \left[1 - \frac{F\left(Z + \frac{D}{2}\right) - F\left(Z - \frac{D}{2}\right)}{\omega}\right]^S dz$$

From the formula,  $R_S$  is connected with target size, the number of the projectile, the projectile and target distribution. When target size, the number of the projectile, the projectile distribution are unitary,  $R_S$  decreases with increasing of the target distribution, which has shown in figure 2.

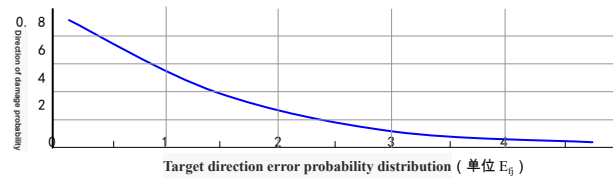


Figure 2 direction relationship full probability and target distribution

From the above analysis on the direction, with the the target distribution increasing, the kill probability will fall sharply in single point firing for goals. It can be thought as a normal distribution of firing near projectile spread in the prediction of the future and spread relatively small. The projectile hit probability is very small when target deviates from the prediction of the future.

In order to improve the hit probability, when the target distribution (error of firing data) is bigger, we take factitious distribution methods. It makes the projectile cover larger area where goal may appear. It will solve the problems that missile failure due to the target distribution is larger. Then the question is how to conduct factitious distribution of the projectile that firing in the space area, namely the effective firing of the future window airspace method.

#### 4.2 Distributing the firing data

Effective firing method can use the markings and direction of ladder to describe on the direction. Assume firing data adopt uniformly distribution and discusses only on the direction.

Due to the target distribution is symmetrical, an odd number of the markings configurate in symmetrical target distribution center. It is beneficial. Markings correspond with the target distribution center. As shown in figure 3, we take target distribution center as central location, adopt  $(2l+1)$  effective marking position and set up  $R$  ladder quantity( $E_{ff}$  as the unit).

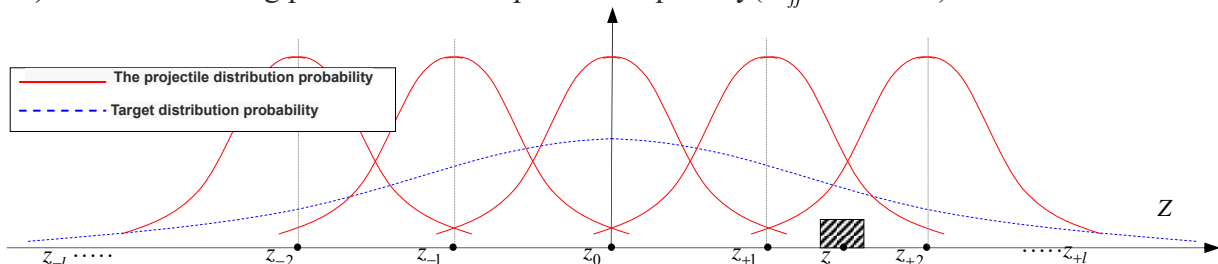


Figure. 3 more firing projectile and target distribution diagram

From the above analysis, using the same method can get full probability of mutilate goal  $R_s$  whe adopting factitious distribution on effectiveness firing the markings.

$$R_s = \int_{-\infty}^{+\infty} \varphi(Z) R_s(Z) dz$$

$$= 1 - \int_{-\infty}^{+\infty} \frac{\rho}{E_{fX} \sqrt{\pi}} e^{-\rho^2 \frac{z^2}{E_{fX}^2}} \prod_{-l}^{+l} \left[ 1 - \frac{F(Z - iR + \frac{D}{2}) - F(Z - iR - \frac{D}{2})}{\omega} \right]^{\frac{S}{2l+1}} dz$$

Given target distribution probability error ( $E_{fX}$ ), the target direction projection width ( $D$ ), total emission( $S$ ) and damage the average hit projectiles ( $\omega$ ) under certain conditions, the size of the damage probability ( $R_s$ ) depends on effective two-dimensional variable ( $l, R$ ).

We can set different target distribution probability error ( $E_{fX}$ ) under the same condition. Calculate the optimum solution of the maximum ( $l^*, R^*$ ). SO as to determine the optimal allocation method of effective firing markings under the circumstance of different target distribution.

The calculating model express the relationship between the optimal effective firing method ( $l^*, R^*$ ) and target distribution  $E_{fX}$ , target size  $D$ , the number of the average projectile that damage the target  $\omega$ , total emission  $S$ .

For example, when a certain shipboard gun fires a certain type of anti-ship missile, the shipboard gun parameter, target size, the number of the firing projectile and the number of the average projectile that damage the target play certain conditions. Then we can calculate the relationship between the number of optimal effective firing markings and error probability of the target distribution. As shown in figure 4.

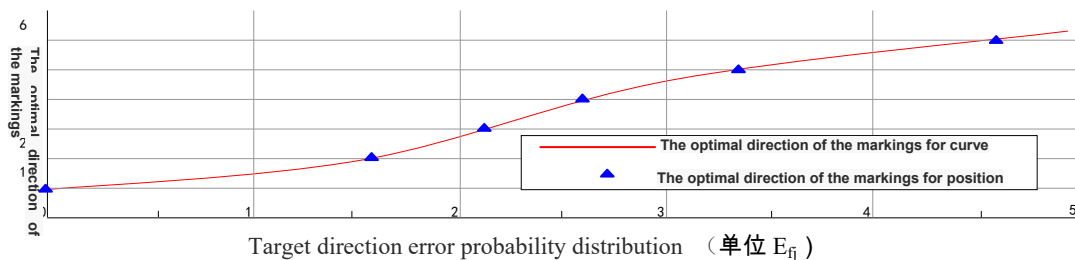


Figure 4 the optimal number of the markings and target distribution relationship

We can come to the conclusion that when other conditions are certain, the optimal effective firing method changes with the size of target distribution. When the target distribution is small, the optimal effective firing method is single point firing. When the target distribution increases, the optimal effective firing method also increases. The optimal type of ladder  $R^*$  changes with the value of  $E_{fX}$ ,  $D$ ,  $S$  and  $\omega$ . Its value changes between  $0.75 \sim 3.37 E_{ff}$ .

The optimal effective firing method ( $l^*, R^*$ ) changes with  $E_{fX}$ ,  $D$ ,  $\omega$  and  $S$ .  $D$ ,  $\omega$  depend on the target type. When the target type is confirmed,  $D$ ,  $\omega$  are confirmed. Total emission( $S$ ) relates with the target location and movement situation. Shipboard gun weapon system can obtain after confirming target.  $E_{fX}$  depends on the parameters of the shipboard gun system and target maneuver mode. When determining the target type, the target of maneuver mode can be confirmed. Then we can get  $E_{fX}$  through observable data of target. So when the target type is confirmed, using the calculation model to determine the optimal effective firing method is feasible.

## 5. Preplacing gun barrel angle method of airspace window firing

Airspace window firing of preplacing gun barrel angle method. Namely through each barrel of the ROF(Rate Of Fire) multi-tube shipboard gun presetting certain angle, makes each barrel dispersion center reasonably distribute in the region of the firing according to the optimization.

### 5.1 Spread the distribution center method

When using the firing data distribution, the location and number of firing data can be more easy to implement according to the optimized configuration. When preinstalling the barrel angles to factitious spread, the distribution of each tube dispersion center is restricted by the number of barrels. Because ROF multi-tube gun generally is revolving barrel gun, the allocated distribution center cannot be greater than the number of gun tube. When the number of dispersion center is restricted, using the circular symmetric configuration as shown in figure 5 can better match the low of target distribution. It is a main choice.

Circular symmetric configuration can better realize the overlap of projectile spread scope. Through setting the range of overlap, it can achieve the requirement of the power density and the projectile spreading probability density.

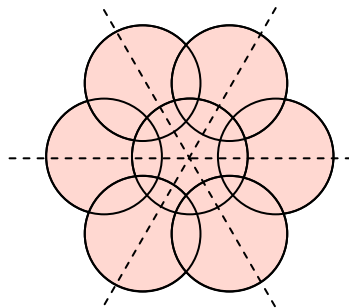


Figure 5 each tube dispersion center configuration diagram

Assume that the shipboard gun tube is  $n$  revolving barrel gun. Factitious spreading way uses the circular symmetric configuration of tube dispersion center. In order to reach the goal of factitious spread, when each barrel arrives at launch position, the axis of gun tube should point to the corresponding dispersion center. Because the close-in shipboard gun firing is close and flat trajectory is straight, we analysis without considering the fall ballistic. In figure 6,  $M_i$  is on the surface vertical position of the gun barrel  $i$ . Corresponding to the preset angle of gun tube  $M_i$ , we can use two angles to determine, which are angle of barrels preset angle offset direction  $\lambda_i$  and the angle between barrel and firing data  $\beta_i$ . The preset angle is determined by the calculating  $\lambda_i$  and  $\beta_i$  of each tube.

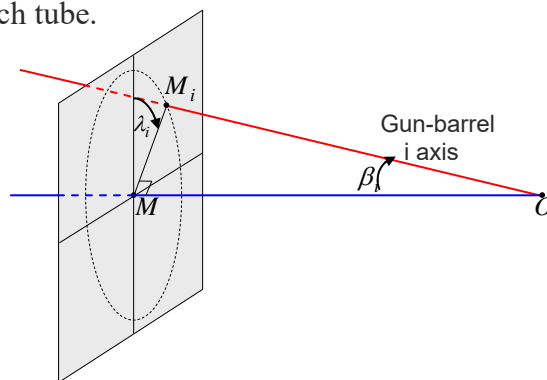


Figure 6 barrels preset angle diagram

### 5.2 Preplace gun barrel angle direction

As shown in figure 7, assume that the number of barrel is  $n$  and the numerical order tube is  $i$ ,  $i=1、2、3 \dots n$

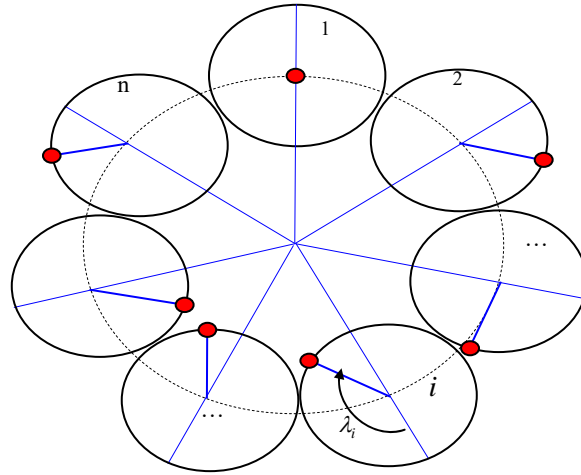


Figure 7 barrel preset angle setting direction sketch map

According to the previous analysis of the circular symmetric configuration, firing data position should have corresponded with barrel. The rest of barrels equably distribute on a circle according to the offset angle of uniform interval. According to the configuration, the direction angle  $\lambda_i$  of the gun tube  $i$  is :

$$\lambda_i = (i-1) \cdot \frac{2\pi}{n}$$

### 5.3 Preplace gun barrel angle

When making factitious distribution, in order to ensure that the the projectile in the distribution area equably spread. The probability density of projectile spreading is more consistent in the scope of the position.

Due to the single shipboard gun spread with restriction, we analyze the spreading probability density on the direction for two barrels for convenience. When multi-tube firing, the result of the discussion is also applicable.

Assume the probability error of barrels spread in the direction is  $E_{ff}$ . Deference of dispersion center is  $a$ . Take connection point of the two spreading center as a benchmark,  $E_{ff}$  as the unit. When the projectile from two gun go through the surface vertical position located on the point, direction probability density respectively are  $\varphi_1(x)$  and  $\varphi_2(x)$  :

$$\varphi_1(x) = \frac{\rho}{\sqrt{\pi}} e^{-\rho^2 \left( \frac{x+a}{2} \right)^2}, \quad \varphi_2(x) = \frac{\rho}{\sqrt{\pi}} e^{-\rho^2 \left( \frac{x-a}{2} \right)^2}$$

When the numbers of the projectile from two gun are equal, the probability density of projectile spreading  $\varphi(x)$  is:

$$\begin{aligned} \varphi(x) &= \frac{1}{2} \varphi_1(x) + \frac{1}{2} \varphi_2(x) \\ &= \frac{\rho}{2\sqrt{\pi}} \left[ e^{-\rho^2 \left( \frac{x+a}{2} \right)^2} + e^{-\rho^2 \left( \frac{x-a}{2} \right)^2} \right] \end{aligned}$$

As show in figure 8, two dotted lines respectively represent the density curve of spread probability for  $\varphi_1(x)$  and  $\varphi_2(x)$ . Solid line is  $\varphi(x)$ . We can come to the conclusion that the density function of spread probability  $\varphi(x)$  is associated with deference dispersion center  $a$ . When  $a \neq 0$ , spread of the projectile no longer obeys the normal distribution. With the increase of deference dispersion center  $a$ ,  $\varphi(x)$  will be transformed from a unimodal function to a bimodal function.

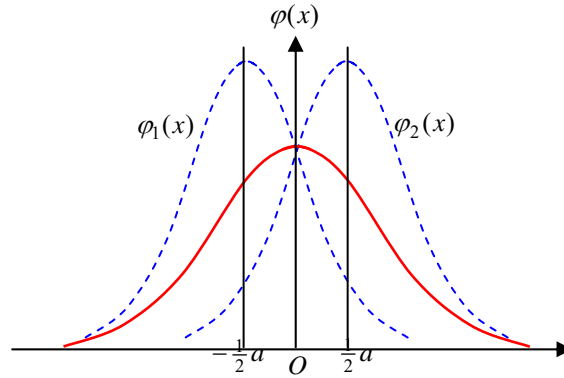


Figure 8 projectile spreading probability density diagram

Through calculation and analysis, when  $a \in [0, 4E_{ff}]$ , we can obtain good spreading evenness. Considering that the spreading of various tube projectile has the overlap and distribute uniformity, The maximum of  $a$  is  $4E_{ff}$ .

This is line analysis though the surface vertical position of the gun barrel located on the point. In order to realize the setting of preset angle, the conclusion should be converted into angle. The error probability of projectile dispersion related to the firing range. Because close-in defense gun firing distance is closer, muzzle velocity is higher and the ballistic is straighter, the probability error of spreading that described by the ballistic angle is not obvious in the firing range. Assume that the probability error of angle on direction and altitude are same in single tube. Common firing distance average error probability of single tube spread angle is  $E_{\beta_j}$ . Then the angle value of preplacing gun barrel angle deviation firing data  $\beta$  is:

$$\beta \in [0, 4E_{\beta_j}]$$

When  $\beta$  is confirmed, the number of barrel, the influence of the emissivity for the fire density and the demand scope of the factitious factors should be considered.

Through optimizing the preset angle of gun tube  $\lambda_i$  and  $\beta$ , we can realize that area firing of uniformly spread. This method is applicable to ROF close-in defense shipboard gun that is used to defense anti-ship missile as the main mission. In theory, this method can contain more error of firing data and ascend the firing effectiveness for high speed complex combat maneuvering anti-ship missile.

## 6. Conclusion

With the anti-ship missile rapid development, there are new requirements for shipboard gun. The solutions are that developing equipment and studying on the use methods. But they must be based on operational needs and be studied on theory unceasingly. At last, they must be applied to practice.

From the perspective of shipboard gun firing theory, this paper bases on the influence of the shipboard gun for high-speed complex maneuvering anti-ship missiles firing. From "soft" and "hard" way, this paper explores the firing data distribution method and airspace window firing of preplacing gun barrel angle method. In this paper, although the method is feasible from theory, we have a lot of work to do on. Hope this paper can forward with his valuable contributions and provide the reference for study and use of ships close-in defense system.

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