

Advanced Methods of 3D Stress Intensity Factor Assessment

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Abstract. This paper explains and demonstrates how to get stress intensity factors using different evaluation methods for a 3D plate with a straight through-thickness crack. The methods include interaction integrals, J-integral evaluation and energy-release rate. All results from these methods are compared with others, and some useful information about the accuracy and the applicability of these methods obtained.

Introduction

Structural design concepts traditionally use a strength-of-material approach for designing a structure. This method does not consider the elevated stress levels due to the cracks. The presence of such stresses can lead to catastrophic failure of the structure.

Fracture mechanics accounts for the cracks or flaws in a structure. The fracture mechanics method to design of structures includes crack size as one important variable, which can anticipate the adverse elevated stress levels and avoid the unexpected collapse accident.

Fracture analysis is typically carried out either using the energy criterion or the stress-intensity-factor criterion. The advanced approaches include interaction integrals, J-integral evaluation too. All these method are used to get stress intensity factors for a 3D plate with a straight through-thickness crack in this paper, and the results are compared with others, and some useful information about the accuracy and the applicability of these methods provided.

Evaluation approaches of stress intensity factors(SIFs)

General Theory. For a linear elastic material the stress and strain fields ahead of the crack tip are expressed as:

$$\begin{aligned}\sigma_{ij} &= -\frac{K}{\sqrt{r}} f_{ij}(\theta) \\ \varepsilon_{ij} &= -\frac{K}{\sqrt{r}} g_{ij}(\theta)\end{aligned}\quad (1)$$

where K is the stress-intensity factor, r and θ are coordinates of a polar coordinate system. These equations apply to any of the three fracture modes.

For a Mode I crack, the stress field is given as:

$$\begin{aligned}\sigma_x &= \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left(1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right) \\ \sigma_y &= \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left(1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right) \\ \sigma_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)\end{aligned}\quad (2)$$

For center cracked plate geometry, the stress intensity factor for this problem was calculated by Isida as equal to:

$$K_I = 1.334\sigma\sqrt{\pi a}\quad (3)$$

Energy-Release Rate. The energy-release rate G is defined in elastic materials as the rate of change of potential energy released from a structure when a crack opens.

The energy-release rate is given by:

$$G = \frac{\pi\sigma^2 a}{E} \quad (4)$$

At the moment of fracture, G is equal to the critical energy-release rate G_c , a function of the fracture toughness. The value of G_c for a material can be determined via a relatively straightforward set of crack experiments. For a single-fracture mode, the stress-intensity factor and the energy-release rate are related by: $G = \frac{K^2}{E'}$,

where G is the energy-release rate, $E' = \frac{E}{1 - \nu^2}$ for plane strain, and $E' = E$ for plane stress. (E is the material Young's modulus, and ν is the Poisson's ratio.)

J-Integral. J-Integral is one of the most widely accepted parameters for elastic-plastic fracture mechanics, and it can be looked as a Stress-Intensity Factor too. Hutchinson and Rice and Rosengren independently showed that the J-Integral characterizes the crack-tip field in a nonlinear elastic material. They each assumed a power law relationship between plastic strain and stress.

The J-Integral evaluation is based on the domain integral method by Shih. The domain integration formulation applies area integration for 2-D problems and volume integration for 3-D problems. The J-Integral is defined as follows :

$$J = \lim_{\Gamma \rightarrow 0} \int_{\Gamma_0} \left[(W + T)\delta_{ij} - \sigma_{ij} \frac{\partial u_j}{\partial x_i} \right] n_i d\Gamma \quad (5)$$

where W is the strain energy density, T is the kinematic energy density, σ represents the stresses, u is the displacement vector, and Γ is the contour over which the integration is carried out.

For a 2-D problem, body forces within the integration of area, and pressure on the crack surface, the domain integral representation of the J-Integral is given by:

$$J = \int_A \left[\sigma_{ij} \frac{\partial u_j}{\partial x_i} - w \delta_{ij} \right] \frac{\partial q}{\partial x_i} dA \quad (6)$$

Where

$$\omega = \omega^e + \omega^p = \int_0^{\varepsilon_{ij}^e} \sigma_{ij} d\varepsilon_{ij}^e + \int_0^{\varepsilon_{ij}^p} S_{ij} d\varepsilon_{ij}^p$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$$

where σ_{ij} is the stress tensor, u_j is the displacement vector, w is the strain energy density, δ_{ij} is the Kronecker delta, x_i is the coordinate axis, and q is referred to as the crack-extension vector.

$$q(X_i) = \sum_{l=1}^m N_l q_l \quad (7)$$

$$\frac{\partial q(X_i)}{\partial X_i} = \sum_{l=1}^m \sum_{p=1}^{2 \text{ or } 3} \frac{\partial N_l}{\partial \eta_p} \frac{\partial \eta_p}{\partial X_j} q_l \quad (8)$$

Equation (6) can be the following equation when Gaussian integral method used:

$$J = \sum_A \sum_{p=1}^p \left\{ \left[\left(\sigma_{ij} \frac{\partial u_j}{\partial x_i} - w \delta_{ij} \right) \frac{\partial q}{\partial x_i} \right] |J| \right\}_p W_p \quad (9)$$

Wherein: p is integration points of a element, $|J|$ is the Jacobian and W_p is the weight of Gaussian integration points.

For the 3-D problem, domain integral representation of the J-Integral becomes a volume integration, which again is evaluated over a group of elements, and the principal is similar to the 2-D problem. And there is equation(10) for J integration:

$$\bar{J}\Delta L = \int_V \left\{ \left[\sigma_{ij} \frac{\partial U_j}{\partial X_i} - \omega \delta_{ij} \right] \frac{\partial q}{\partial X_i} \right\} dV \quad (10)$$

Interaction Integrals. Similar to the domain integral method for J-Integral evaluation, the interaction integral method for stress-intensity factors calculation applies area integration for 2-D problems and volume integration for 3-D problems. The interaction integral is defined as

$$J = - \int_V q_{i,j} \left(\sigma_{ki} \varepsilon_{ki}^{aux} \delta_{ij} - \sigma_{kj}^{aux} u_{k,i} - \sigma_{kj} u_{k,i}^{aux} \right) dV / \int_S \delta q_n ds \quad (11)$$

where σ_{ij} , ε_{ij} , u_i are the stress, strain and displacement respectively, σ_{ij}^{aux} , ε_{ij}^{aux} , u_i^{aux} are the stress, strain and displacement of the auxiliary field, and q_i is the crack-extension vector.

The interaction integral is associated with the stress-intensity factors as

$$I = \frac{2}{E^*} \left(K_1 K_1^{aux} + K_2 K_2^{aux} \right) + \frac{1}{\mu} K_3 K_3^{aux}$$

where K_i ($i = 1,2,3$) are the mode I, II and III stress-intensity factors, K_i^{aux} ($i = 1,2,3$) are the auxiliary mode I, II and III stress-intensity factors, $E^* = E$ for plane stress and $E^* = E / (1 - \nu^2)$ for plane strain, E is the Young's modulus, ν is the Poisson ratio, and μ is the shear modulus.

Modelling and calculation of SIFs

Modelling of a center cracked plate. For center cracked plate geometry is shown in Fig. 1. Given the symmetry, only a quarter of the panel was modeled by a structure analysis software, and got the SIFs from the different approaches through the post process. The main stress S1 is shown in Fig.2.

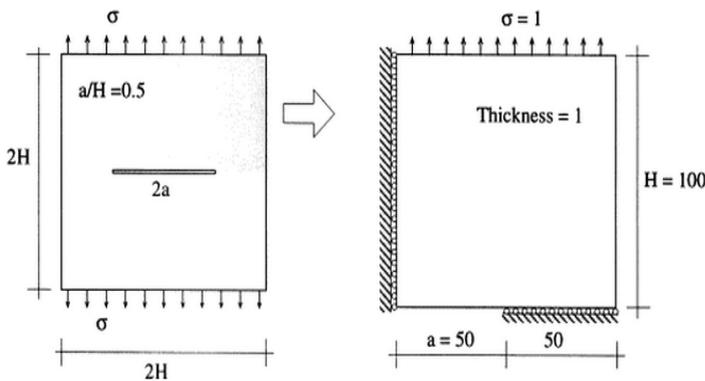


Fig.1 Center cracked plate

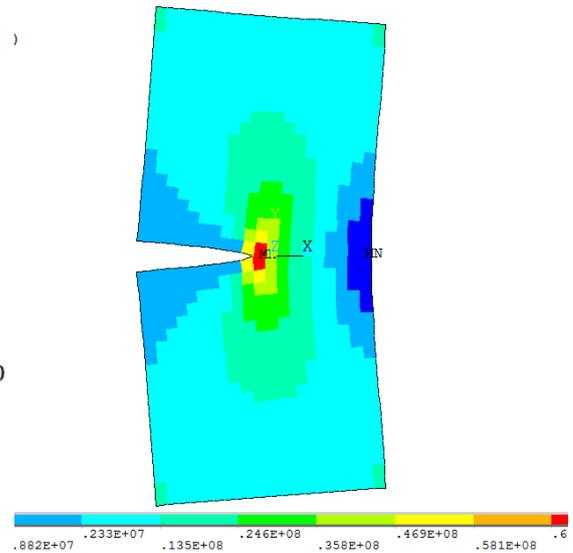


Fig.2 The main stress S1

The results of SIFs. All results in 10 contours are listed as follows, and the SIFs obtained too.

J-integral

***** POST1 J-INTEGRAL RESULT LISTING *****

CrackID = 2

Crack Front Node = 46

Contour Values = -398.61 11450. 12489. 12572.

Contour Values = 11295. 11187. 11124. 11093.

Contour Values = 11081. 11079.

The maximum 11876 is the J-integral value.

In the case of plane stress state, the K2 can be obtained by:

$$K = \sqrt{EJ} = \sqrt{2.1 \times 10^{11} \times 12575} = 4.994 \times 10^7$$

In the case of plane stress state, the K2 can be obtained by:

$$K = \sqrt{\frac{E}{1 - \nu^2}} = 5.235 \times 10^7$$

Interaction integral

Results of K in 10 contours via interaction integral method:

***** POST1 K1 RESULT LISTING *****

CrackID = 1

Crack Front Node = 46

Contour Values = -0.75948E+06 -0.15925E+07 -0.25411E+07 -0.35253E+07

Contour Values = -0.21803E+07 -0.21879E+07 -0.20679E+07 -0.18833E+07

Contour Values = -0.16648E+07 -0.14371E+07

The maximum value is -0.35253E+07.

***** POST1 K2 RESULT LISTING *****

CrackID = 1

Crack Front Node = 46

Contour Values = 0.21107E+07 0.42911E+07 0.33014E+07 0.31757E+07

Contour Values = 0.29090E+07 0.26357E+07 0.23779E+07 0.21282E+07

Contour Values = 0.19054E+07 0.16770E+07

The maximum value is 0.42911E+07.

Where in regular method, K1= -0.215030E+07; K2= 0.427962E+07

Conclusions

Area and volume integrals offer much better accuracy than contour integral and surface integrals, and are much easier to implement numerically. In comparison to the traditional method, the interaction integral method offers better accuracy, fewer mesh requirements, and ease to use.

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