Strategies of Adding Hot Water when Bathing

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Abstract. To keep the water temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water, we establish a mathematical model of the temperature of the bathtub water in space and time to determine the best strategy the bather can adopt when bathing. On the basis of the model, we propose two water adding strategies. The first strategy is that the faucet keeps open, namely there is constant trickle of hot water added into the bathtub all the time. The second is that the bather turns on the faucet only when water is cooler. We use the heat conduction equation in strategy I. With Alternating Direction Implicit (ADI) Method, we get the numerical solution to the heat conduction equations. Then, we calculate the water consumption in strategy II with improved Genetic Algorithm. After comparing the consumption of water with the condition of stationary bath time, we discover that adding water intermittently can save more water.

1. Introduction

When we take a bath, we would like to keep the water warm all the time. In this paper, we make strategies which we can use to bath comfortably without wasting too much water. We construct models on the basis that the temperature of water is in a specific range. After that, we calculate the consumption of the water used in every method.

Convective heat transfer is the transfer of heat from one place to another by the movement of fluids and also involves the combined processes of conduction and advection. [2]The heat equation is a parabolic partial differential equation that describes the distribution of heat (or variation in temperature) in given region over time. [3] And Newton came up with Newton's cooling law [1701] by experiment.

We come up with two ways of adding water. The first one is that there is constant trickle of hot water added into the bathtub all the time. The second is that the bather turns on the faucet only when water is cooler. With ADI method, we solve the heat equation to analyze the distribution of temperature in water specifically. To choose a better strategy, we calculate the water consumption of the strategies respectively.

2. Assumption and Justifications

- **When the bather is in the water, the temperature of his body is same everywhere and is constant.**

People is homothermal animal, the temperature of their skin changes not so much.

- **The temperature of the bathtub's inwall is close to the water's.**

After adding water in the bathtub, the bathtub's inwall gets warm in a short time and is close to the water's.

- **We only take into account the heat conduction between the air and water, ignoring the heat convection at the surface of the bathtub.**

The air doesn't flow. During bathing, there isn't rapid flow, so the heat dissipating capacity is much smaller than in heat conduction.

- **The temperature of the room doesn't change and is lower than the water in the**

bathtub.

To simplify the model, we assume the temperature of the room is constant.

3. The Mathematical Model

In this section, we assume that:

- The shape of the bathtub is a cuboid.
- The faucet at an end of the bathtub, and the overflow drain at the other.
- The temperature changes only in the direction of Y-axis and Z-axis as the following Figure 1.

The bathtub is shown in Figure 1.

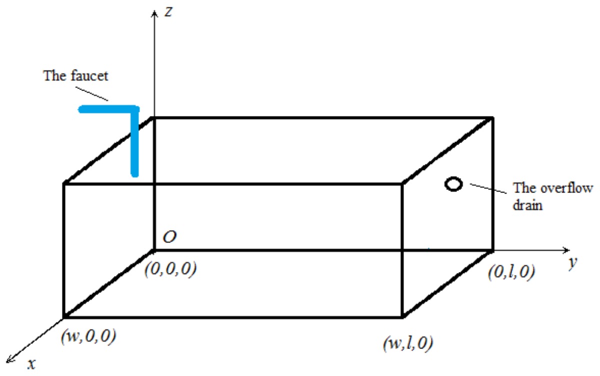

Figure 1. The sketch map of the bathtub

3.1 Strategy I: Adding hot water continuously

The first strategy is to keep the faucet open. In this way, there is constant trickle of hot water entering into the bathtub; the temperature at one location doesn't change with time going.

Since the speed of the water flows is slow, we assume there is only thermal conduction in water's heat transmission.

We analyze the temperature distribution in the water quantitatively by the heat conduction equation.For the function *T(y, z, t)* of the two spatial variables *(y, z)* and the time variable *t*, the heat equation is

$$\frac{\partial T}{\partial t} - \alpha^2 \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = 0 , \qquad (1)$$

where

- k is heat transfer coefficient
- k_1 is heat exchange coefficient
- $\alpha = \frac{k_1}{k} > 0, t > 0$

We specify the third boundary condition (2) for T to solve the equation.

$$\begin{cases} -k \frac{\partial T}{\partial z}\Big|_{z=h} = \alpha (T - T_a) \\ T(y,z,t)\Big|_{y=0} = T_s \end{cases} \qquad (2)$$

where h is the depth of the water.

The initial condition is

$$T(y,z,0) = T_{\_} \qquad (3)$$

where $T_{\_}$ is the temperature of the water when the person feels noticeably cooler.

According to the equation (1), (2) and (3), we can get the boundary value problem (BVP) of two-dimension heat equation as follows:

$$\begin{cases} \dfrac{\partial T}{\partial t} - \alpha^2 \left(\dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} \right) = 0 \\ -k \dfrac{\partial T}{\partial z}\Big|_{z=h} = \alpha (T - T_a) \\ T(y,z,t)\Big|_{y=0} = T_s \\ T(y,z,0) = T_- \end{cases} \tag{4}$$

To solve equations (4), we invoke **Alternating Direction Implicit (ADI) Method** [4] came up with by Peaceman and Rachford in 1955.

The results of the heat equations are numerical value of numerous discrete points; we can fit a curve in MATLAB according to the solutions as follows in figure 3.

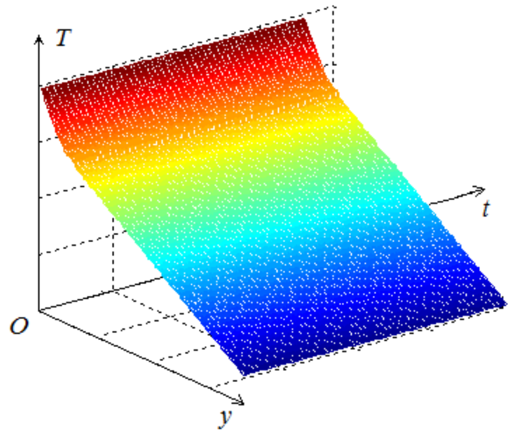

Figure 3. The solution of the temperature distribution

A function *T(t, y)* can be obtained from Figure 3.

$$T = 70.11 - 0.6145t + 2.87\times10^{-14} y - 1.853\times10^{-15} ty - 0.0001161t^3 + 3.566\times10^{-17} t^2 y$$
$$+5.813\times10^{-7} t^4 - 2.62\times10^{-19} t^3 y - 1.096\times10^{-9} t^5 + 6.506\times10^{-22} t^4 y \tag{12}$$

To calculate the consumption of water, we invoke Newton's cooling law and the solution formula of specific heat capacity.

$$\frac{dQ}{dt} = [T(y,t) - T_a] hwdy \tag{13}$$

where

- Q is the quality of heat in per unit time;
- h is the heat transmission coefficient.

$$c\rho q(T_h - \overline{T}) = Q \tag{14}$$

where

- c is the specific heat capacity of water;
- ρ is the density of water.

According to the equations (13) and (14), we can get the function $q = f(t)$

$$q = \frac{w}{c\rho(T_n - \overline{T})}(70.11yt - 3.0725\times10^{-14} t^2 y + 1.435\times10^{-14} y^2 t$$
$$-4.6325\times10^{-16} t^2 y^2 - 2.9025\times10^{-5} t^4 y + 5.943\times10^{-18} t^3 y^2$$
$$-1.1626\times10^{-7} t^5 y - 3.275\times10^{-20} t^4 y^2 - 1.827\times10^{-10} t^6 y$$
$$+6.506\times10^{-23} t^5 y^2 - T_a ty) \tag{15}$$

Choosing a series of values from the equation (15), we fit a curve with least square method as follow in Figure 4.

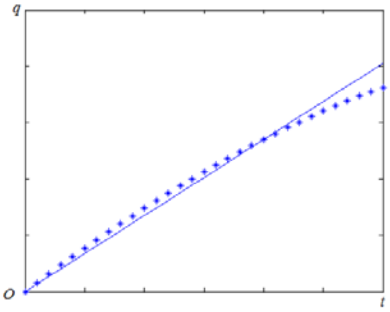

Figure 4. The simulation of the solution

From the Figure 4, we can know the consumption of water q is proportional to the time t. So we can get the slope k from the figure and the reduced form of equation (15) can be gained.

$$q = kt \tag{16}$$

where the value of k is 0.704.

3.2 Strategy II: Adding hot water intermittently

We also come up with the second strategy: the man only turns on the faucet when the water gets cooler.

3.2.1 Calculation of water consumption

We assume that the flow of the faucet is the biggest. So the amount of water added is $q_m t_i'(T_h - T_i)$. The parameter T_i is the temperature after the bather adds water. We assume the bather stirs the water while he adds hot water, so we treat the temperature of the water even.

In the i^{th} time, we let t_i represent the time of bath after adding water , $t_i^{'}$ represent the time that adding water needs, x represent the times of adding water when bath ends, V_i represent the water consumption, t_w represent the time of bathing

Our purpose is by controlling the values of $t_i^{'}$ and n, to gain the minimum value of V_i, when $\sum_{i=1}^{n} t_i \geq t_w$.

There are mainly two processes:

- The bather stands up to add water
- The bather takes bathing

The quantity of heat transfer to the air is $Q_{a_{i+1}}$.

$$Q_{a_{i+1}} = (T_- - T_a)hAt_{i+1}^{'} \tag{17}$$

The quantity of heat taken out of by the motion of the bather is Q_{p_i}.

$$Q_{p_i} = Q_m + Q_{s_i} \tag{18}$$

where,

- Q_m is the quantity of heat taken out with the bather's motion
- Q_{s_i} is the quantity of heat relative to t_i

$$Q_{s_i} = (T_- - T_a)\rho c V_s \left(1 - \frac{1}{e^{\frac{hs}{cm}} t_{i+1}^{'}}\right) \tag{19}$$

The value of t_{i+1} depends on that of V_i and t_i ; the value of V_{i+1} depends on that of $t_{i+1}^{'}$. There are functions

$$t_{i+1} = f(V_i, t_{i+1}) \tag{20}$$

$$V_{i+1} = f(t_{i+1}^{'}) \tag{21}$$

where

$$t_{i+1} = \ln\left(\frac{T_{i+1} - T_a}{T_- - T_a}\right) \quad (22)$$

$$T_{i+1} = \frac{(V_i T_- + q_m t_{i+1}' T_h)\rho c - Q_{p_{i+1}} - Q_{a_{i+1}}}{V_i \rho c + q_m t_{i+1}'} \quad (23)$$

$$V_{i+1} = V_i + q_m t_{i+1}' \quad (24)$$

To gain the minimum value of V_i, we invoke the improved Genetic Algorithm. According to our specific problem, we do some changes to the algorithm.

Based on the deduction above, we obtain:

$$t_{i+1} = f(V_i, t_{i+1})$$

$$V_{i+1} = f(t_{i+1}', V_i)$$

We assume that our feasible strategy includes n times adding of water and the time is separately $t_1', t_2', \cdots, t_n'$. If the solution meets the condition $\sum_{i=1}^{n} t_i \ge t_w$, namely the time of the bath is at least t_w. Each feasible solution is correspond to a value of V_n. When V_n is the smallest, we select the solution as the best solution.

The results are shown in Table 1.

Table 1. The results of Genetic Algorithm

iteration times	*x*	*1*	*2*	*3*	*4*	*5*	Water Consumption
500	5	4.43	3.88	4.24	5.05	3.43	21.03
1000	4	5.25	4.82	4.98	5.60	-	20.65

4. Results

Taking the common bathtub as example, the length is 1.7m, the width is 0.8m and the height is 0.7m. We assume the time of bath is 30 min.In the strategy I, the water consumption is 21.12L. And in the strategy II, the water consumption is 21.03L when the iteration times are 500. However, it is 20.65L when there are 1000 times.

The amount of water used of the two methods is similar so if the bather does not want to add frequently, he will prefer to continuously adding. Moreover, the intermittent method is a little difficult to control because of the requirement of the amount that is added each time.

References

[1] Wikipedia. 2016a. Hot tub. https://en.wikipedia.org/wiki/Hot_tub

[2] Wikipedia. 2016b. Convective heat transfer . https : // en. wikipedia. Org / wiki / Convective _heat _transfer

[3] Wikipedia. 2016c. Heat _ equation https : // en. wikipedia. Org /wiki /Heat_ equation# Three-dimensional_problem

[4] Wang Y H, Sun R X. An ADI Method to Solve Two-dimensional Parabolic Equation with Changeable Coefficient [J]. Journal of Jiamusi University, 2006, 24(4):589-591

[5] Wikipedia. 2016d. Bubble bathhttps://en.wikipedia.org/wiki/Bubble_bath