Undershoot Reduction in Discrete-Time ADRC of NMP Plant by Parameters Optimization

Tong Wu
School of Automation and Electrical Engineering,
University of Science and Technology Beijing,#30 Xueyuan Rd,Haidian District,
Beijing,100083,P. R. China

Weicun Zhang
School of Automation and Electrical Engineering,
University of Science and Technology Beijing, #30 Xueyuan Rd,Haidian District,
Beijing,100083,P. R. China
E-mail: xiaobulu1992@163.com, weicunzhang@263.net
www.buaa.edu.cn

Abstract

Undershoot phenomena caused by unstable zeros of non-minimum-phase (NMP) plant is difficult to deal with in active disturbance rejection control (ADRC). This paper proposes a new method to reduce undershoot by optimizing the controller parameters of discrete-time ADRC system. Simulation results are given to verify the effectiveness of the proposed scheme.

Keywords: ADRC, NMP System, Discrete Control, Parameters Optimization

1. Introduction

Active disturbance rejection control (ADRC), first proposed by Han Jingqing, as a novel control design method has been applied to solve various control problems since 2000s[1]. The ADRC technique, including Extended State Observer (ESO), mentioned in Han's book[2] is a kind of non-linear technique which shows great advantages in dynamic performance, robust performance, and disturbance rejection performance. But it confronts with much trouble in finding appropriate tuning method to deal with ADRC parameters for different plants. Gao Zhiqiang focuses on the linearized ADRC and gives a bandwidth-parameterization method to solve the linear ADRC tuning problem. [3] The number of ADRC parameters to be tuned can be decreased to two in this method which largely simplifies the task of ADRC design.

The discrete control technology as an emerging technology has been widely applied in control system. Various discrete techniques have been proposed to solve control problems. [4, 5] In This paper, a discrete ADRC is designed to achieve the control task for NMP system which having the undershoot problem due to the unstable zeros.[6, 7] A modified method of using appropriate non-linear function for parameters optimization to reduce undershoot will be discussed in the following sections. For simplicity, the subject to be discussed in this paper is the second order system.

2. Discrete Active Disturbance Rejection Control

2.1. *Discrete Extended State Observer*

The linear extended state observer (LESO)[3] can be constructed as

$$\begin{aligned}\dot{z} &= Az + Bu + L(y-\hat{y}) \\ \hat{y} &= Cz\end{aligned} \tag{1}$$

where

$$A=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}, B=\begin{bmatrix}0\\b_0\\0\end{bmatrix},$$

$$C=\begin{bmatrix}1&0&0\end{bmatrix}, L=\begin{bmatrix}\beta_1\\\beta_2\\\beta_3\end{bmatrix} \tag{2}$$

Using ω_0 -Parameterization method mentioned in Ref. 3, the observer gain vector L can be get as

$$\beta_1 = 3\omega_0, \beta_2 = 3\omega_0^{\ 2}, \beta_3 = \omega_0^{\ 3} \tag{3}$$

where ω_0 is the bandwidth of the observer. And the value of b_0 falls in the range from 20 to 100. The extended state observer can be discretized as[8, 9]

$$\begin{aligned}z(k+1) &= \Phi z(k) + \Gamma u(k) + L_p(y(k)-\hat{y}(k)) \\ \hat{y}(k) &= Hz(k)\end{aligned} \tag{4}$$

where

$$\Phi = e^{Ah}, \Gamma = (\int_0^h e^{At}dt)B, H = C, L_p = \begin{bmatrix}\beta_1\\\beta_2\\\beta_3\end{bmatrix} \tag{5}$$

h is the sample period. And the estimator gain vector L_p is determined by the equation

$$\lambda(z) = \left|zI - (\Phi - L_p H)\right| = (z-\beta)^3 \tag{6}$$

Combine equation (2) and (5), and solving (6) for L_p yields[8]

$$\Phi = \begin{bmatrix}1 & h & \frac{h^2}{2}\\ 0 & 1 & h\\ 0 & 0 & 1\end{bmatrix}, \Gamma = b\times\begin{bmatrix}\frac{h^2}{2}\\ h\\ 0\end{bmatrix},$$

$$H = \begin{bmatrix}1 & 0 & 0\end{bmatrix}, L_p = \begin{bmatrix}3-3\beta\\ \frac{(\beta-1)^2(\beta+5)}{2h}\\ -\frac{(\beta-1)^3}{h^2}\end{bmatrix} \tag{7}$$

$$\beta = e^{-\omega_0 h} \tag{8}$$

where b is a constant.

2.2. *Discrete ADRC control law*

The parameterized control law can be constructed as[3, 8]

$$u_0(k) = k_p(r(k) - z_1(k)) - k_d z_2(k) \tag{9}$$

$$u(k) = \frac{u_0(k) - z_3(k)}{b} \tag{10}$$

where

$$\omega_0 = 2 \sim 5\omega_c, k_p = \omega_c^{\ 2}, k_d = 2\omega_c \tag{11}$$

3. Optimization of ADRC Parameters

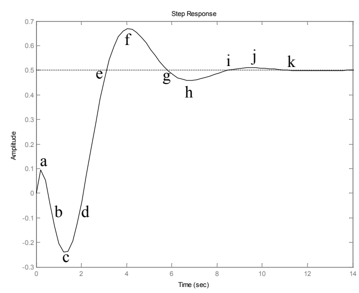

Figure 1 step response of NMP system

For the controller design, we need a bigger proportional gain at a, c, f, h and j, and a smaller one at b, e, g, i and k in Fig. 1. A bigger differential gain between a and b, c and d, f and g, h and i and j and k, and a smaller one between b and c, e and f, g and h and i and j in Fig. 1[10]. Generally this is not realizable where controller parameters are found being fixed. But we can choose appropriate non-linear functions to optimize the proportional gain and differential gain of the controller as a solution to this problem[10].

We assume y_d as the derivative of the output, y_{d2} as the second derivative of the output, and $e(k)$ as the error between the reference and the output of the system. Using non-linear function, controller parameters can be constructed as

$$e(k) = r - z_1(k) \tag{12}$$

$$y_d(k) = \frac{z_1(k) - z_1(k-1)}{h} \tag{13}$$

$$y_{d2}(k) = \frac{y_d(k) - y_d(k-1)}{h} \tag{14}$$

$$K_p = K_{pr} \times (1 + \frac{2}{e^{y_d} + e^{-y_d}}) \tag{15}$$

$$K_d = \frac{K_{dr}}{(1 + e^{-2sign(y_d)sign(y_{d2})|e(k)|})} \tag{16}$$

where K_{pr}, K_{dr} is the original proportional gain and original differential gain of the controller which can be settled using the method mentioned in (9) and (11). And $z_1(k)$ is the state of the extended observer which is used to estimate the output of the plant.

4. Design Example

A simulation of NMP system is used to demonstrate the control design procedure and its resulting performance. The plant is modeled as

$$G(\mathrm{s}) = \frac{-s + 2}{s^2 + 40s + 400} \tag{17}$$

Its corresponding z transfer function and difference equation is shown as (18) and (19)

$$G(z) = \frac{-0.004501z + 0.004546}{z^2 - 1.81z + 0.8187} \tag{18}$$

$$\begin{aligned} y(k) &= 1.81y(k-1) - 0.8187y(k-2) \\ &-0.004501u(k-1) + 0.004546u(k-2) \end{aligned} \tag{19}$$

The sample period h is taken as 0.005s. The discrete extended state observer can be constructed as

$$\begin{aligned} z(k+1) &= \Phi z(k) + \Gamma u(k) + L_p(y(k) - \hat{y}(k)) \\ \hat{y}(k) &= Hz(k) \end{aligned} \tag{20}$$

As is mentioned in Ref. 2, the extended state is fed back to cancel the model uncertainty. ω_0 is selected as 100 rad/sec and using bandwidth-parameterization, parameters of discrete ESO are chosen as

$$u(k) = \frac{u_0(k) - z_3(k)}{b} \tag{21}$$

$$\Phi = \begin{bmatrix} 1 & 0.005 & 1.25\times10^{-5} \\ 0 & 1 & 0.005 \\ 0 & 0 & 1 \end{bmatrix}, \ \Gamma = b \times \begin{bmatrix} 1.25\times10^{-5} \\ 0.005 \\ 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \ L_p = \begin{bmatrix} 1.2 \\ 86.8 \\ 2436.6 \end{bmatrix} \tag{22}$$

where $b = 120$.

Finally the controller can be designed as (23) to control augmented system where r is the setpoint. The value of ω_c is 25 rad/sec .

$$\begin{aligned} u_0(k) &= K_p(r - z_1(k)) - K_d z_2(k) \\ K_p &= \omega_c^2 \times (1 + \frac{2}{e^{y_d} + e^{-y_d}}) \\ K_d &= \frac{2\omega_c}{(1 + e^{-2sign(y_d)sign(y_{d2})|e(k)|})} \end{aligned} \tag{23}$$

The result for the simulation mentioned above is shown in figure 2. The dash line represents the step response of regular discrete ADRC and the solid line represents the step response of modified discrete ADRC. Intuitively, the undershoot of modified discrete ADRC is lower than that of regular discrete ADRC.

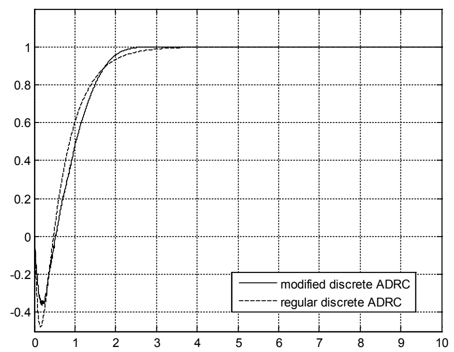

Figure 2 Step response of regular discrete ADRC and modified discrete ADRC

5. Conclusion Remarks

In this paper, a method of ADRC parameters optimization using nonlinear function has been discussed. It is shown that the modified discrete ADRC has smaller undershoot than the regular one. Simulation result has been shown to verify the effectiveness of the proposed scheme.

Acknowledgements

The author would like to thank the anonymous reviewers for their constructive and insightful comments for further improving the quality of this work. This work was supported by the Major State Basic Research Development Program (973 Program) (No. 2012CB821200).

References

1. Z. Gao, On the Foundation of Active Disturbance Rejection Control, *Control Theory & Applications*, **30**(12) (2013) 1498-1510.
2. J. Han, *Active Disturbance Rejection Control Technique-the technique for estimating and compensating the uncertainties* (National Defense Industry Press, Beijing, 2008).
3. Z. Gao, Scaling and Bandwidth- Parameterization Based Controller Tuning, *American Control Conference*, (USA, Denver, 2003), pp. 4989-4996.
4. D. V. Stallard, Discrete Optimal Terminal Control, with Application to Missile Guidance, *IEEE Transactions on Automatic Control,* **18**(4) (1973) 373-377.
5. D. Zhou and C. Gao, Variable Structure Control of Discrete System with Time-delay, *Systems and Control in Aerospace and Astronautics*, (China, Shenzhen, 2008), pp. 1-5.
6. S. Zhao and Z. Gao, Active Disturbance Rejection Control for Non-minimum Phase Systems, *Chinese Control Conference*, (China, Beijing, 2010), pp. 6066-6070.
7. M. Tang and P. Wu. Control of Non-minimum Phase Servo System Based on Auto Disturbance Rejection, *Radar Science and Technology*, 11(**3**) (2013) 335-340.
8. R. Miklosovic, A. Radke, Z. Gao, Discrete Implementation Generalization of the Extended State Observer, *American Control Conference*, (USA, Minneapolis, 2006), pp. 2209-2214.
9. C. Zhang, J. Zhu, Y. Gao, Active Disturbance Rejection Controller Based on Deadbeat Observer Design and Performance Analysis, *Control Theory & Applications*, **32**(1) (2015) 29-34.
10. D. Zhang, K. Liu, B. Fang, Non-minimum Phase System Control Based on Nonlinear PID, *Microcomputer Development*, **14**(12) (2004) 4-6.