An Adaptive UKF Method for Geomagnetic/Celestial Autonomous Navigation

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Abstract—**An adaptive unscented Kalman filtering(UKF) method for geomagnetic/celestial autonomous navigation is presented to solve the problem that the conventional UKF is sensitive to the initial value and declines in accuracy and further diverges due to the system model inaccuracy. Firstly, the residual sum of squares of observations is taken as the criteria to determine the trend of filter divergence. When the residual sum of squares of observations is more than theoretical prediction, an adaptive factor is chosen to adjust the con-variance matrixes of state vectors and observed vectors adaptively in order to reduce the impact on filtering caused by initial value and inaccuracy of the system model. Simulation results show that adaptive UKF can restrain filter divergence efficiently, ensure convergence and stability of the filter and improve the precision of the autonomous navigation method using geomagnetism/celestial information under the condition of large initial errors or system model inaccuracy.**

Keywords-geomagnetic filed; navigation precision; geomagnetic/celestial autonomous navigation; adaptive UKF

I. INTRODUCTION

Because of the simple principle, low cost, high reliability and no accumulation of position error over time, Celestial navigation based on star sensor/infrared horizon sensor, has been successfully applied to autonomous navigation for satellites, by taking the elevation angle of star as the measurable variable. However, for near-surface aircrafts whose flight altitude is much lower than that of satellites, infrared horizon sensors, which apply to high altitude area above 300km, can not sense horizon accurately, so the celestial navigation based on star sensor/infrared horizon can not be applied. Meanwhile, because the motion of aircraft is not enough to change the point of starlight in the inertial coordinate system, autonomous navigation can not be realized only by star sensor.

There are abundant geomagnetic resources around the orbit of near-surface aircrafts, and the geomagnetic field intensity vector is the function of the aircraft position. Navigation information of the aircraft can be achieved by matching the geomagnetic field intensity vector measured by magnetic sensor in real time and the known regional geomagnetic model.

In order to be able to carry on a high precision navigation to the near-surface aircrafts, Geomagnetic/Celestial autonomous Navigation, which combines geomagnetic field intensity vector and starlight vector is applied in this paper. Geomagnetic/celestial autonomous navigation system is a highly nonlinear system, which is usually estimated by unscented Kalman filtering(UKF). However, UKF filtering algorithm is very sensitive to the initial value of the state vector and variance matrix, at the same time, a larger state estimation error also can be produced when system model is inaccurate, and it could even cause filter divergence. In order to solve the problems above, the paper, based on the UKF algorithm, fuses the adaptive thinking, which regards the measurement of residual sum of squares as a judgment basis of the divergence trend of filter. Once the filter divergence happens, the adaptive factor is chosen to adjust the weights between the measured values and the predicted values to suppress filter divergence, so that the stability and the convergence of filtering can be ensured and the navigation performance can be improved.

II. GEOMAGNETIC MODEL

IGRF is an international general model of global geomagnetic. In the Passive area of the near-earth space, the scalar magnetic potential of the main magnetic field originating from the earth interior by spherical harmonic series can be expressed as [2]:

$$V(r, \lambda, \varphi') = \mathrm{Re} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \cdot$$
$$\left[\left(\frac{\mathrm{Re}}{r} \right)^{n+1} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m (\cos(\varphi')) \right] \quad (1)$$

Where, n is the order of spherical harmonic series, m is the degree of spherical harmonic series, the item of n=1 accounts for 80%~85% of all of the magnetic field, which represents the main characteristics of spatial and temporal distribution of the geomagnetic field. g_n^m and h_n^m are Gauss coefficient, $P_n^m(\cos(\varphi'))$ is n-order and m-degree Schmidt normalized associated Legendre function. $\mathrm{Re} = 6378.137 km$ is the equatorial radius of the earth reference ellipsoid, r is geocentric distance, λ is geographical longitude, φ' is geographical colatitude, $\varphi' = 90^\circ - \varphi$, φ is geographical latitude.

The geomagnetic field intensity vector is the negative gradient of the magnetic potential of the geomagnetic field. in

the geographic coordinate system, the three axis component of the geomagnetic field intensity is:

$$
\begin{cases}
B_e = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\dfrac{\mathrm{Re}}{r}\right)^{n+2}\left(g_n^m \sin m\lambda - h_n^m \cos m\lambda\right)\cdot \\
\qquad \dfrac{m}{\cos\varphi} P_n^m(\sin\varphi) \\
B_n = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\dfrac{\mathrm{Re}}{r}\right)^{n+2}\left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right)\cdot \\
\qquad \dfrac{\partial P_n^m(\sin\varphi)}{\partial\varphi} \\
B_u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\dfrac{\mathrm{Re}}{r}\right)^{n+2}(n+1)\left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right)\cdot \\
\qquad P_n^m(\sin\varphi)
\end{cases}
\tag{2}
$$

Formula(2) indicates that the geomagnetic field intensity is the function of the position parameters φ, λ, r of the aircraft. The position of the aircraft can be obtained by measuring the intensity of geomagnetic field, and then the aircraft can be navigated

III. PRINCIPLE OF GEOMAGNETIC/CELESTIAL AUTONOMOUS NAVIGATION

For satellites in high-altitude area, celestial autonomous navigation can be realized by taking the elevation angle of star between the starlight vector measured by star sensor and the tangential direction from the satellites to the edge of the earth detected by infrared horizon sensor as the measurable variable. Likewise, for near-surface aircrafts, geomagnetic/celestial autonomous navigation can be realized by taking the included angle between the starlight vector measured by star sensor and the geomagnetic field intensity measured by magnetometer as the measurable variable.

A. State equation of Geomagnetic/ Celestial Navigation System

The near-surface aircrafts do not satisfy orbital dynamics equations, so we establish a kind of dynamic equation which is suitable for general aircrafts. The aircraft can be treated as a particle. Geographic latitude, geographic longitude, height and three axis velocities in the geographical coordinate system of the aircraft are selected as state variables to establish dynamic equation, which is taken as the state function of Geomagnetic/Celestial autonomous navigation system. The state function is as follows:

$$
\begin{cases}
\dfrac{d\varphi}{dt} = \dfrac{v_n}{R_m + h},\ \dfrac{d\lambda}{dt} = \dfrac{v_e}{(R_n + h)\cos\varphi},\ \dfrac{dh}{dt} = v_u \\
\dfrac{dv_e}{dt} = a_e + w_{ae},\ \dfrac{dv_n}{dt} = a_n + w_{an},\ \dfrac{dv_u}{dt} = a_u + w_{au}
\end{cases}
\tag{3}
$$

Where, φ is geographical latitude, λ is geographical longitude, h is height. v_e, v_n and v_u are eastward velocity, northward velocity and vertical velocity respectively. a_e, a_n and a_u are the accelerations of three axis of the coordinate system respectively. w_{ae}, w_{an} and w_{au} are acceleration noise respectively. R_m and R_n are earth reference ellipsoid radius of curvature in meridian and radius of curvature in prime vertical. Suppose $X = [\varphi \quad \lambda \quad h \quad v_e \quad v_n \quad v_u]^{\mathrm{T}}$ is state variable, $w = [0 \quad 0 \quad 0 \quad w_{ae} \quad w_{an} \quad w_{au}]^{T}$ is zero-mean process noise. System state equation can be written as:

$$
\dot{X}(t) = f[X(t), t] + w(t)
\tag{4}
$$

B. Measurement equation of Geomagnetic/Celestial Navigation System

When the aircraft works, the platform always tracks the geographic coordinate system. The geomagnetic field intensity of the direction of east, north and vertical measured by three axis sensor fixed on the platform is $B = [B_e \quad B_n \quad B_u]^T$. Meanwhile, after the navigational stars are observed by the star sensor on the aircraft, the unit starlight vector in the J2000 horizontal equatorial coordinate system can be obtained by star map recognition, and it can be expressed as: $S_i = [\cos\alpha\cos\delta \quad \sin\alpha\cos\delta \quad \sin\delta]^T$. Where, α and δ are right ascension and declination of the measured star in the J2000 horizontal equatorial coordinate system respectively.
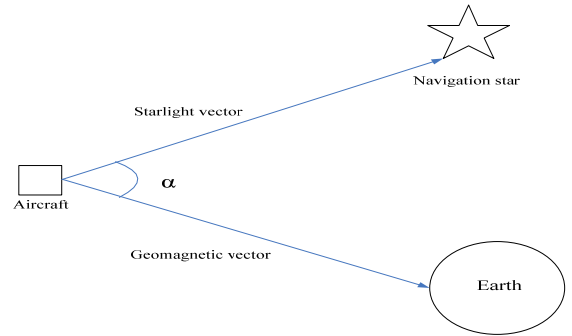

FIGURE I. PRINCIPLE OF GNS/CNS

When starlight vector is transformed from the inertial coordinate system to the geographical coordinate system. There will be an included angle between the starlight vector and the geomagnetic vector. When the motion state of the aircraft is changed, this included angle will also be changed. The motion state information can be reflected by observing the included angle. We use the position and velocity of the aircraft as state variables to establish the dynamic equation and take the included angle between the starlight vector and the geomagnetic vector as measurable variable. By using the appropriate filtering algorithm, the position and velocity of the

aircraft can be estimated and the autonomous navigation can be achieved [2].

In the geographic coordinate system, the included angle between starlight vector obtained by the star sensor and geomagnetic field intensity vector obtained by magnetic sensor is taken as the measurable variable of the autonomous navigation system. The measurement equation is as follow:

$$Z(t) = \arccos(\frac{S \cdot B}{\sqrt{B^T B}}) + v(t)$$

$$= \arccos\left[\frac{(C_i^g S_i) \cdot B}{\sqrt{B^T B}}\right] + v(t) = h[x(t), t] + v(t)$$

$$(5)$$

Where, B is the magnetic field intensity vector measured by the magnetic sensor. S_i is the unit vector of the direction of navigational star in J2000 geocentric equatorial coordination. $v(t)$ is measurement noise, which is zero-mean Gauss white noise. C_i^g is the transformational matrix from the J2000 geocentric equatorial coordinate system to the geographic coordinate system. Seeking the coordinate transformational matrix is the key to the measurement equation.

The transformational matrix from the geocentric inertial coordinate system to geographic coordinate system can be expressed as:

$$C_i^g = C_e^g \cdot C_i^e$$

$$(6)$$

Where, C_e^g expresses transformational matrix of coordinates from earth fixed coordinate system to geographic coordinate system, which is related to the latitude and longitude of the aircraft. Its expression is given by formula(7). C_i^e expresses the transformational matrix of coordinates from J2000 geocentric equatorial coordinate system to earth fixed coordinate system, which is mainly related to the rotation of the earth, but also is affected by the pole shift , precession and nutation. Its expression is given by formula(8) [3], [4].

$$C_e^g = \begin{bmatrix} -\sin\lambda & \sin\varphi\cos\lambda & \cos\varphi\cos\lambda \\ \cos\lambda & \sin\varphi\sin\lambda & \cos\varphi\sin\lambda \\ 0 & \cos\varphi & \sin\varphi \end{bmatrix}$$

$$(7)$$

$$C_i^e = [Q(t)R(t)W(t)]^T$$

$$(8)$$

Where, $Q(t)$ and $W(t)$ are the pole shift and precession and nutation matrix respectively, $R(t)$ represents the transformational matrix because of the earth rotation.

$$R(t) = R_z(-\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0. \\ 0 & 0 & 1 \end{bmatrix}$$

$$(9)$$

Where, θ represents earth rotational angular, whose change is not uniform actually. Its expression is given by formula(10) [3], [4].

$$\theta(Tu) = 2\pi \cdot (0.7790572732640 + 1.00273781191135448Tu)$$

$$(10)$$

Where, $UT1 = UT + (UT1 - UT)$, Tu is the difference between Julian Day of the world time $UT1$ and 2451545.0.

IV. ADAPTIVE UKF ALGORITHM

The state equation and the observation equation of the system model established in this paper are nonlinear. The nonlinear filtering method is more commonly used and effective. UKF approximates nonlinear function with nonlinear probability density distribution. Under Kalman filtering framework, UKF generates a small number of Sigma sampling points by UT transform, then realizes the nonlinear transfer of mean value and covariance by transformed sampling points. UKF does not have truncation error, so it dose not need to seek the Jacobian matrix. So it has smaller amount of calculation and higher precision than EKF.

Compared with other nonlinear filters, the UKF algorithm has a better filtering effect. However, the application of UKF algorithm still exists some limitations. UKF filter is very sensitive to the selection of initial value. The offset of the system state and the initial value of variance will directly affect the filtering mean value and filtering estimation, and then causes a certain initial error of optimal observation. In addition, due to the existence of system model error, external disturbance and nonlinear transfer error, the predicted value of UKF time status update also exists error. The above factors will directly affect the accuracy of the filtering algorithm, and even lead to the divergence of the filter [7], [8].

The divergence of the filter means that the actual estimation error exceeds the error of theoretical prediction. The sum of squared residuals between the real measurement value received by the filter and the measurement value calculated by the estimation can be used as the criterion to determine the trend of filter divergence. Once the filter exits the trend of divergence, in order to reduce the impact on filtering caused by initial value and inaccuracy of the system model and refrain filtering divergence, the adaptive principle is adopted to balance the weight ratio between the Forecast information of state function and the Observation information by selecting suitable adaptive factors to adjust the con-

variance matrixes of state vectors and observed vectors adaptively [8] [9].

The specific implementation steps of adaptive UKF are as follows:

A. Initialization

$$\hat{X}_0 = E[X_0], P_0 = E[(X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^T] \quad (11)$$

B. Calculation of Sigma Sampling Point

$$\chi_{k-1,i} = \left[\hat{X}_{k-1}, \hat{X}_{k-1} + \gamma(\sqrt{P_{k-1}})_i, \hat{X}_{k-1} - \gamma(\sqrt{P_{k-1}})_i\right] \quad (12)$$

$$\gamma = \sqrt{n+\lambda} \quad (13)$$

$$\lambda = \alpha^2(n+\kappa) - n \quad (14)$$

Where, n represents the dimension of the state vector, α is scale parameter, which determines the distribution state of the Sigma point, and it usually takes a very small positive number. κ is usually set to 0 for state estimation, which is set to for system parameter estimation, $(\sqrt{P_{k-1}})_i$ represents the value of column i of the root mean square.

Calculate weight:

$$\omega_0^c = \frac{\lambda}{n+\lambda} + 1 - \alpha^2 + \beta$$
$$\omega_0^m = \frac{\lambda}{n+\lambda}$$
$$\omega_i^c = \omega_i^m = \frac{1}{2(n+\lambda)} \quad (15)$$

For Gauss distribution, suppose $\beta = 2$.

C. Time Update

$$\chi_{(k|k-1),i} = f(\chi_{k-1,i}) \quad (16)$$

$$\hat{X}_{k|k-1} = \sum_{i=0}^{2n} \omega_i^m \chi_{(k|k-1),i} \quad (17)$$

$$P_{k|k-1} = \sum_{i=0}^{2n} \omega_i^c \cdot \left[\chi_{(k|k-1),i} - \hat{X}_{k|k-1}\right] \cdot \left[\chi_{(k|k-1),i} - \hat{X}_{k|k-1}\right]^T$$
$$+ \hat{Q}_k \quad (18)$$

$$\gamma_{(k|k-1),i} = h(\chi_{k-1,i}) \quad (19)$$

$$\hat{Z}_{k|k-1} = \sum_{i=0}^{2n} \omega_i^m \gamma_{(k|k-1),i} \quad (20)$$

D. Measurement Update

Judgment of divergence trend and selection of adaptive factors:

$$\sigma_k = \begin{cases} 1 & , \tilde{Z}_k^T \tilde{Z}_k \leq tr[E(\tilde{Z}_k \cdot \tilde{Z}_k^T)] \\ \dfrac{tr[E(\tilde{Z}_k \cdot \tilde{Z}_k^T)]}{\tilde{Z}_k^T \tilde{Z}_k} & , \tilde{Z}_k^T \tilde{Z}_k > tr[E(\tilde{Z}_k \cdot \tilde{Z}_k^T)] \end{cases} \quad (21)$$

$$E(\tilde{Z}_k \cdot \tilde{Z}_k^T) = \sum_{i=0}^{2n} \omega_i^c \cdot \left[\gamma_{(k|k-1),i} - \hat{Z}_{k|k-1}\right] \cdot \left[\gamma_{(k|k-1),i} - \hat{Z}_{k|k-1}\right]^T \quad (22)$$

Where, $\tilde{Z}_k = Z_k - \hat{Z}_{k|k-1}$ is denoted by innovation sequence, which is the difference between the real measurement value received by the filter and the measurement vector calculated by the optimal estimation, including the information of actual estimation error. In addition, it plays an important role in evaluating the nature of the valuation property. It is also an important variable to judge the trend of the divergence of the filter.

The state vector and the covariance matrix of the measurement vectors can be adjusted by adaptive factor σ_k adaptively to restrain filter divergence:

$$P_{ZZ} = \sigma_k \cdot \sum_{i=0}^{2n} \omega_i^c \cdot \left[\gamma_{(k|k-1),i} - \hat{Z}_{k|k-1}\right] \cdot \left[\gamma_{(k|k-1),i} - \hat{Z}_{k|k-1}\right]^T + R \quad (23)$$

$$P_{XZ} = \sigma_k \cdot \sum_{i=0}^{2n} \omega_i^c \cdot \left[\chi_{(k|k-1),i} - \hat{X}_{k|k-1}\right] \cdot \left[\gamma_{(k|k-1),i} - \hat{Z}_{k|k-1}\right]^T \quad (24)$$

$$K_k = P_{XZ} P_{ZZ}^{-1} \quad (25)$$

$$\hat{X}_k = \hat{X}_{k|k-1} + K_k(Z_k - \hat{Z}_{k|k-1}) \quad (26)$$

$$P_k = P_{k|k-1} - K_k P_{ZZ} K_k^T \qquad (27)$$

V. SIMULATION VERIFICATION AND ANALYSIS

In order to verify the effectiveness of the Geomagnetic/Celestial autonomous navigation algorithm based on adaptive UKF proposed in this paper, the simulation experiment is carried out under the condition of supposing the aircrafts maneuver flight at the same height.

The data used in this simulation is set as follow:

1) Initial state vector of aircraft:

$$X_0 = [40^o \quad 116^o \quad 20000m \quad 1000m/s \quad 200m/s \quad 0m/s]^T$$

2) Initial covariance matrix:

$$P_0 = diag\{(0.5rad)^2 \quad (0.5rad)^2 \quad (50m)^2 \quad (50m/s)^2 \quad (50m/s)^2 \quad (50m/s)^2\}$$

3) Initial process noise covariance matrix:

$$Q_0 = diag\{0 \quad 0 \quad 0 \quad (0.01m/s)^2 \quad (0.01m/s)^2 \quad (0.01m/s)^2\}$$

4) geomagnetic noise: The accuracy of star sensor is very high, the measurement noise is mainly from the geomagnetic model error. According to the literature [10] The average error of IGRF model is about $149nT$, so in order to verify the influence of the model error to the filtering algorithm, the geomagnetic noise is set to $150nT$ in this simulation.

5) simulation period: T=1s; *simulation time:* 3000s.

Based on the above simulation conditions, in order to verify the sensibility to the selection of initial value of the two filtering algorithms, the general UKF and adaptive UKF algorithms are used to filtering respectively under the conditions that the smaller offset and larger offset between the initial value of the filter and the initial value of the state variable .

A. Simulation with Small Initial Offset

Firstly, simulation is carried out in the case of smaller offset between the initial value of the filter and the initial value of the state vector. Suppose the initial value of the filter is set as follow:

$$\acute{X}_0 = [40.2^o \quad 116.2^o \quad 20000m \quad 1000m/s \quad 200m/s \quad 0m/s]^T$$

Figure 2 and figure 3 represent two different filtering methods to obtain the error of position and velocity of Geomagnetic/Celestial autonomous navigation in a simulation respectively. Table 1 shows the mean square error of the corresponding navigation accuracy. (RMSE).
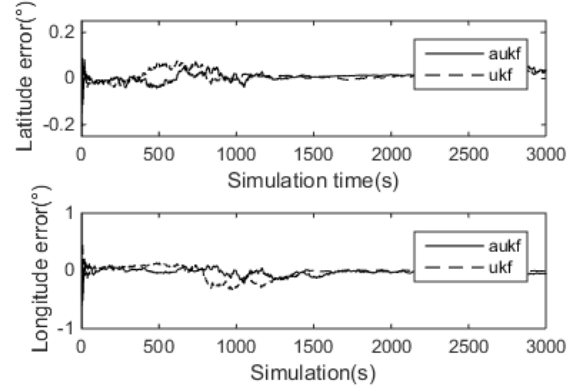

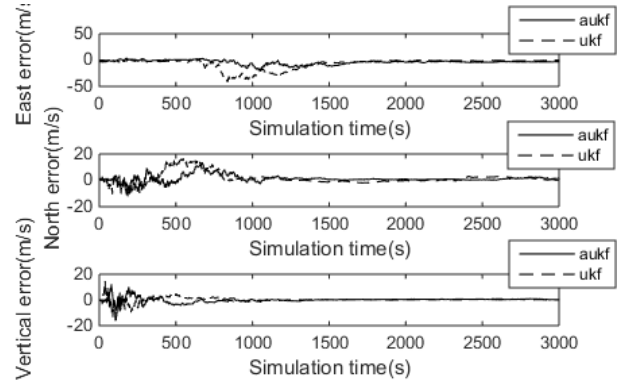FIGURE II. ESTIMATED POSITION ERRORS WITH SMALL INITIAL OFFSET

FIGURE III. ESTIMATED VELOCITY ERRORS WITH SMALL INITIAL OFFSET

B. Simulation with Large Initial Offset

Subsequently, Simulation is carried out in the case of larger offset between the initial value of the filter and the initial value of the state vector. Suppose the initial value of the filter is set as follow:

$$\acute{X}_0 = [35^o \quad 111^o \quad 20010m \quad 996m/s \quad 195m/s \quad 4m/s]^T$$

Figure 4 and figure 5 represent two different filtering methods to obtain the error of position and velocity of Geomagnetic/Celestial autonomous navigation in a simulation respectively. Table 1 shows the mean square error of the corresponding navigation accuracy. (RMSE).
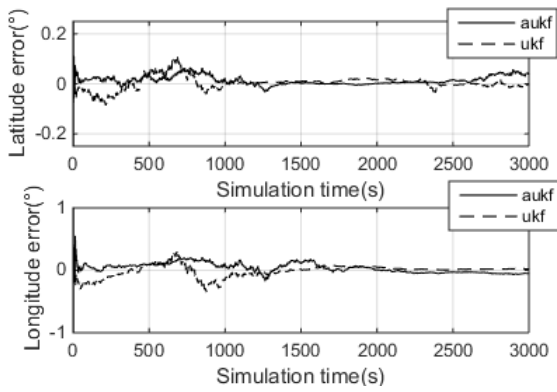
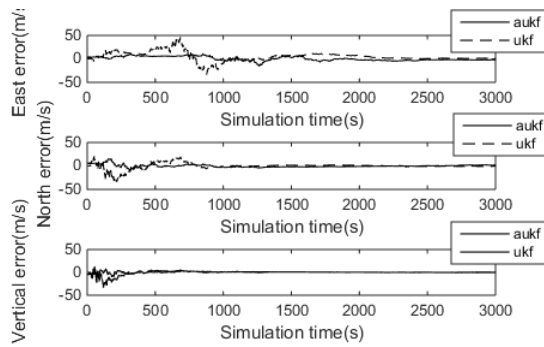FIGURE IV. ESTIMATED POSITION ERRORS WITH LARGE INITIAL OFFSET

FIGURE V. ESTIMATED VELOCITY ERRORS WITH LARGE INITIAL OFFSET

TABLE I. ROOT MEAN SQUARE ERRORS OF GNS/CNS

Error	Condition	Small initial offset		Large initial offset	
		UKF	AUKF	UKF	AUKF
Latitude error (°)		0.042	0.040	0.069	0.045
Longitude error(°)		0.269	0.227	0.323	0.300
Three axis velocity error (m/s)	North	8.62	6.97	14.10	6.70
	East	5.79	5.46	6.08	5.13
	Vertical	5.58	2.37	4.66	4.60
	Synthesis	11.79	9.16	16.03	9.61

Simulation results show:

1) The Geomagnetic/Celestial autonomous navigation system based on the adaptive unscented Kalman filtering(UKF) method has a better filtering stability and convergence, whose filtering error does not accumulate over time and navigation accuracy is higher, so that the autonomous navigation of near ground aircraft can be achieved.

2) Under the condition of smaller initial offset, The filtering effect of the two kinds of filtering algorithm is roughly the same, The adaptive UKF algorithm is superior to the general UKF algorithm in terms of convergence speed, fluctuation range and navigation accuracy. But the difference is small.

3) Under the condition of larger initial offset, before the filtering reaches stability, the error fluctuation of Adaptive

UKF algorithm is slow, and its convergence rate is faster. When the filtering is stable. The steady state value is more closer to the true value, the root mean square error of navigation does not increase substantially with the increase of initial offset. According to the data in Table 1, compared with the general UKF, The adaptive unscented Kalman filtering(UKF) method improves the navigation accuracy of Geomagnetic/Celestial Integrated Navigation System. The positioning accuracy is increased by about 8%, the speed accuracy is increased more obviously , which is promoted by about 40% .

VI. CONCLUSION

An adaptive unscented Kalman filtering(UKF) method for geomagnetic/celestial autonomous navigation is presented to solve the problem that the conventional UKF is sensitive to the initial value and declines in accuracy and further diverges due to the system model inaccuracy. Firstly, the residual sum of squares of observations is taken as the criteria to determine the trend of filter divergence. When the residual sum of squares of observations is more than theoretical prediction, an adaptive factor is chosen to adjust the con-variance matrixes of state vectors and observed vectors adaptively in order to reduce the impact on filtering caused by initial value and inaccuracy of the system model. Simulation results show that adaptive UKF can restrain filter divergence efficiently, ensure convergence and stability of the filter and improve the precision of the autonomous navigation method using geomagnetism/celestial information under the condition of large initial errors or system model inaccuracy.

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