Group Consensus of Second-Order Multi-agent Networks with Multiple Time Delays

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***Abstract*—In this paper, the problem of group consensus of second-order multi-agent networks with multiple time delays is investigated. Based on the theory of frequency-domain, some algebraic criteria are proposed analytically which can guarantee the multi-agent networks to achieve group consensus. Results show that input time delays of the agents, coupling weights and coupling strengths between the agents play a key role in reaching group consensus. Meanwhile, communication time delays can only affect the convergence rate of the systems. Finally, the validity and correctness of our theoretical results are verified by several numerical simulated examples.**

Keywords-time delays; group consensus; multi-agent network; complex networks

I. INTRODUCTION

The consensus problem aims to design control strategies and protocols such that the whole network can achieve stability. Up to now, it has attracted increasing attention of researchers in different fields. So far, many related research works have been successively reported, which can be seen in [1-6] and the references therein.

In cooperative control, under the influence of environments, situations, cooperative tasks or even time, the consensus states of the system will be likely to change. This phenomenon can be described as group consensus of multi-agent networks, which means that all agents within the same cluster can reach a consistent state, while different clusters achieve distinct states. Up to date, lots of research work about group consensus has been reported. In [7], the group consensus problems of second-order multi-agent systems with time delays were investigated. In[8], a novel hybrid protocol was designed, and the average group consensus problem with undirected topologies is studied. In [9], couple-group consensus problems of second-order networks with fixed and stochastic switching topologies were discussed, and the authors provided the sufficient and necessary condition for mean-square couple-group consensus. In [10, 11], Yu studied group consensus of networks with undirected graphs and strongly connected and balanced topology. Furthermore, by applying double-tree-form transformation, they extended group consensus with communication delays and switching topologies[12, 13]. In [14], group consensus of networks with communication delays was investigated and the topology of the system was strongly connected and balanced. In [15], with and without time delays, group consensus of first-order networks with connected bipartite topology were discussed, respectively. And the bound of time delay was obtained analytically. Meanwhile, weighted group consensus of agent with time delays was investigated and an upper bound of maximum time delay was obtained in [16]. Considering the coexistence of input and communication time delays, group consensus for multi-agent networks with undirected and connected bipartite topology were studied, respectively. And some sufficient conditions were proposed in [17].

It is worth noting that there are two delays existed in networks, input and communication time delays. From the relevant works mentioned above, it is not difficult to find out there are some more conservative assumptions in them. First, some related research works only consider communication delays, or only analyze the same communication and input delays, such in [10-13, 15-17]. As mentioned above, the two delays between agents should be different and coexisting. Second, most of the works only focus on the networks with symmetrical topology, such as in [8, 10, 11, 14-17].

Motivated by the related work, we will discuss the group consensus of second-order multi-agent networks with multiple time delays. And the topology of the system is more general which neither needs to be undirected nor strongly connected digraph.

The rest of this paper is organized as follows as In Section 2, some preliminaries are briefly outlined. The main results are addressed in Section 3. In Section 4, some examples are illustrated to verify our theoretical results and conclusions are drawn in Section 5.

II. PRELIMINARIES

In a multi-agent network, we can represent each agent and the information exchange among them as a node and an edge of a weighted directed graph, respectively. For convenience, let $G = (V, E, A)$ denotes a weighted directed graph, where $V = \{v_1, v_2, ..., v_N\}$ is the node set, the set of neighbors of v_i is described by $N_i = \left\{v_j \in V : \left(v_i, v_j\right) \in E\right\}$, ¥ $= \{1, 2, ..., N\}$ denotes the node index set, $E \subseteq V \times V$ is the edge set and $A = (a_{ij}) \in$ ¡ $^{N \times N}$ is the weighted adjacency matrix. If the edge $e_{ij} \in E$, $a_{ij} > 0$, which means node v_i

can receive information from node v_j. Otherwise, $a_{ij}=0$. In this paper, we assumed that $a_{ii}=0$ for all $i \in \mathbb{N}$.

For second-order multi-agent networks, the dynamics are listed as (1),

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases}, \tag{1}$$

where $x_i(t), v_i(t), u_i(t) \in \mathbb{R}^n$ denotes the position, velocity state, control input of the agent i. For convenience, we assume $n=1$. We only consider $x_i(t), v_i(t), u_i(t) \in \mathbb{R}$. The results we proposed can be easily expanded for $x_i(t), u_i(t) \in \mathbb{R}^n$ by using the Kronecker product $\otimes$.

Next, we will list some related definitions and lemma.

A. *Definition 1*

Consensus in second-order system (1) is said to be reached asymptotically if and only if $\lim_{t\to\infty}\left\| x_i(t) - x_j(t) \right\| = 0$, $\lim_{t\to\infty}\left\| v_i(t) - v_j(t) \right\| = 0$

B. *Definition 2*

If there exists a path in G from node v_i to node v_j, then v_j is said to be reachable from v_i. If a node is reachable from every other node in G, then it is treated as a globally reachable node.

Assumes that the network contains $n+m(n,m>1)$ agents, its topology G consists of two sub-graphs, $G_1 = (V_1, E_1, A_1)$ and $G_2 = (V_2, E_2, A_2)$, where $V_1 = \{v_1, v_2, ... v_n\}$, $V_2 = \{v_{n+1}, v_{n+2}, ..., v_{n+m}\}$. The set of finite index is $L_1 = \{1, 2, ..., n\}$, $L_2 = \{n+1, n+2, ..., n+m\}$. $N_{1i} = \{v_j \in V_1 : (v_i, v_j) \in E\}$, $N_{2i} = \{v_j \in V_2 : (v_i, v_j) \in E\}$ represent the neighbor set of v_i in the two sub-graphs, respectively. If the first n agents converge to a consistent state, and the other m agents converge to a differently consistent state, then we can say that the network realizes group consensus.

C. *Lemma 1* [18]

If the graph G exists a globally reachable node, its Laplacian matrix will obtain a simple eigenvalue 0.

D. *Lemma 2* [19]

For $\forall \gamma \in [0,1)$, when $\omega \in \mathbb{R}$, convex hull $\gamma Co(0 \cup \{E_i(j\omega), i \in \quad\})$ does not contain $(-1, j0)$, where $E_i(j\omega) = (\pi / 2T) \times (e^{-j\omega T} / j\omega)$, and T denotes the system delay.

E. *Lemma 3* [20]

The set $\bigcup_{i\in \mathbb{N}} G_i$ is included in the convex hull $\gamma Co(0 \cup \{E_i(j\omega), i \in \quad\})$ for $\omega \in \mathbb{R}$.

III. GROUP CONSENSUS OF MULTI-AGENT NETWORKS WITH MULTIPLE DELAYS

In this section we will discuss group consensus of multi-agent networks with multiple time delays.

In [10-13], based on the in-degree balance conditions(A1) and (A2), average group consensus problems of multi-agent networks with undirected, strongly connected and balanced topologies were investigated, respectively. The input protocol is as follows:

$$u_i(t) = \begin{cases} \sum_{V_j \in N_{1i}} a_{ij}(x_j(t) - x_i(t)) + \sum_{V_j \in N_{2i}} a_{ij} x_j(t), \forall i \in L_1 \\ \sum_{V_j \in N_{2i}} a_{ij}(x_j(t) - x_i(t)) + \sum_{V_j \in N_{1i}} a_{ij} x_j(t), \forall i \in L_2 \end{cases} \tag{2}$$

In (2), for $\forall i, j \in L_1, a_{ij} \geq 0$; $\forall i, j \in L_2, a_{ij} \geq 0$; $\forall i, j \in \phi = \{(i,j) : i \in L_1, j \in L_2\} \cup \{(i,j) : j \in L_1, i \in L_2\}$, $a_{ij} \in \mathbb{R}$, and the in-degree balance assumptions are

$$\text{(A1): } \sum_{j=n+1}^{n+m} a_{ij} = 0, \forall i \in L_1; \text{(A2): } \sum_{j=1}^{n} a_{ij} = 0, \forall i \in L_2.$$

Based on (3), Ren et al.[18] analyzed the group consensus of second-order multi-agent systems with time delays.

$$\begin{aligned} u_i(t) = \alpha \sum_{V_j \in N_i} a_{ij}(x_j(t - T_{ij}) - x_i(t - T)) \\ + \beta \sum_{V_j \in N_i} a_{ij}(v_j(t - T_{ij}) - v_i(t - T)) \end{aligned}. \tag{3}$$

where T_{ij}, T denote communication delays and input delays, , and $\alpha > 0$, $\beta > 0$ are the coupling strengths of systems.

Motivated by the related work, we will investigate the group consensus problems of second-order multi-agent networks with multiple delays. Consider the following control protocol (4),

$$u_i(t) = \begin{cases} \alpha \sum_{V_j \in N_{1i}} a_{ij}(x_j(t - T_{ij}) - x_i(t - T_i)) + \beta \sum_{V_j \in N_{1i}} a_{ij}(v_j(t - T_{ij}) - v_i(t - T_i)) \\ + \alpha \sum_{V_j \in N_{2i}} a_{ij} x_j(t - T_{ij}) + \beta \sum_{V_j \in N_{2i}} a_{ij} v_j(t - T_{ij}), \forall i \in L_1 \\ \alpha \sum_{V_j \in N_{2i}} a_{ij}(x_j(t - T_{ij}) - x_i(t - T_i)) + \beta \sum_{V_j \in N_{2i}} a_{ij}(v_j(t - T_{ij}) - v_i(t - T_i)) \\ + \alpha \sum_{V_j \in N_{1i}} a_{ij} x_j(t - T_{ij}) + \beta \sum_{V_j \in N_{1i}} a_{ij} v_j(t - T_{ij}), \forall i \in L_2 \end{cases} \tag{4}$$

where T_i, T_{ij} denotes the input delay and the communication delay of a node v_i, respectively.

A. *Theorem 1*

Suppose the multi-agent networks (1) with control protocol (4) contain $n+m(n,m>1)$ agents, and its topology includes a globally reachable node, if $\tilde{d}_i(\alpha\cos\omega_{i0}T_i+\beta w_{io}\sin\omega_{i0}T_i)<\frac{1}{2}\omega_{i0}^2$ holds, then the system can achieve group consensus asymptotically. Where $\tilde{d}_i=\sum_{k=1,k\neq i}^{m+n}a_{ik}$, ω_{i0} is the intersection between the Nyquist curve of $G_{i0}(j\omega)$ and the negative real axis of the complex plane, $G_{i0}(j\omega)=\tilde{d}_i\frac{e^{-j\omega T_i}}{j\omega}\frac{\alpha+\beta j\omega}{j\omega}$ and $\tan(\omega_{io}T_i)=\frac{\beta}{\alpha}\omega_{io}$.

B. *Proof:*

Applying the Laplace transform to (1) with (4), it is easy to get its characteristic equation, which is $\det\left(s^2I+\alpha L(s)+\beta L(s)s\right)=0$, where I is identity matrix

$$L(s)=\left(l_{ij}(s)\right)=\begin{cases}-a_{ij}e^{-sT_{ij}}, & j\neq i\\ \sum_{k=1,k\neq i}^{m+n}a_{ik}e^{-sT_i}, & j=i\end{cases}. \quad (5)$$

For convenience, let $F(s)=\det\left(s^2I+\alpha L(s)+\beta L(s)s\right)$. Based on the theory of frequency-domain, we will prove that all zeros of $F(s)$ have negative real parts or $F(s)$ has a simple zero at $s=0$.

1) Let $s=0$, $F(0)=\det(\alpha L)$. Based on Lemma 1, we know that $F(s)$ indeed exists a simple zero at $s=0$.

2) Let $s\neq 0$ and $P(s)=\det(I+G(s))$, where $G(s)=(\alpha L(s)+\beta L(s)s)/s^2$.

Based on the general Nyquist stability criterion, if the track of the $\lambda(G(j\omega))$ does not enclose the point $(-1,j0)$, all zeros of $P(s)$ have negative real parts. Let $s=j\omega$. According to the Gerschgorin disk criterion, the eigenvalue of $G(j\omega)$ satisfies $\lambda(G(j\omega))\in\bigcup_{i\in N}G_i$, where

$$G_i=\left\{\zeta:\zeta\in C\left|\zeta-\sum_{k=1,k\neq i}^{m+n}a_{ik}\frac{\alpha+\beta j\omega}{j\omega}\frac{e^{-j\omega T_i}}{j\omega}\right|\leq\left|\sum_{k=1,k\neq i}^{m+n}a_{ik}\frac{\alpha+\beta j\omega}{j\omega}\frac{e^{-j\omega T_i}}{j\omega}\right|\right\}, \quad (6)$$

and C denotes complex number set, $\tilde{d}_i=\sum_{k=1,k\neq i}^{m+n}a_{ik}$.

From (6), by Lemma 2 and 3, it is clear that if $(-x,j0)$ with $x\geq 1$ is not in G_i, the following inequality can hold

$$\left|-x-\tilde{d}_i\frac{\alpha+\beta j\omega}{j\omega}\frac{e^{-j\omega T_i}}{j\omega}\right|>\left|\tilde{d}_i\frac{\alpha+\beta j\omega}{j\omega}\frac{e^{-j\omega T_i}}{j\omega}\right| \quad (7)$$

By calculations, we can obtain the inequality easily ,

$$x-(2\tilde{d}_i/\omega^2)\times(\alpha\cos\omega T_i+\beta\omega\sin\omega T_i)>0 \quad (8)$$

When $x\geq 1$, the following inequality holds $(2\tilde{d}_i/\omega^2)\times(\alpha\cos\omega T_i+\beta\omega\sin\omega T_i)<1$. At the moment, we obtain that the track of $\lambda(G(j\omega))$ does not enclose the point $(-1,j0)$, so all zeros of $P(s)$ have negative real parts.

From (6), we can get the center of the disk is $G_{i0}(j\omega)=\tilde{d}_i\frac{e^{-j\omega T_i}}{j\omega}\frac{\alpha+\beta j\omega}{j\omega}$.Suppose ω_{i0} is the intersection point between the Nyquist curve of $G_{i0}(j\omega)$ and the negative real axis of the complex plane, thus we get $\tan(\omega_{io}T_i)=\frac{\beta}{\alpha}\omega_{io}$.

Theorem 1is proved.

C. *Remark 1*

From Theorem1, we can know that group consensus of second-order systems is related to input delays and coupling strengths of agents or systems, and is independent of communication delays between the agents.

IV. SIMULATION EXAMPLES

According to Theorem 1, some simulation examples are given to verify the effectiveness and the correctness of the criteria established in section 3.

A. *Experiment 1*

Consider a dynamical network (1) with 5 nodes, the topology and connection weights between nodes illustrated in Fig.1. Set agents 1,2,3in a group and agents 4,5 in another group. Meanwhile, the topology owns a globally reachable node and (A1) and (A2) are also satisfied.

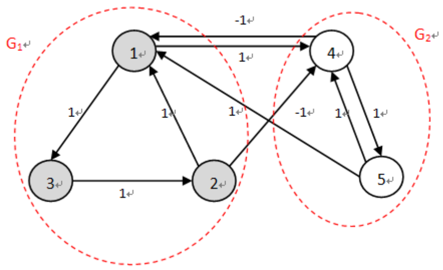
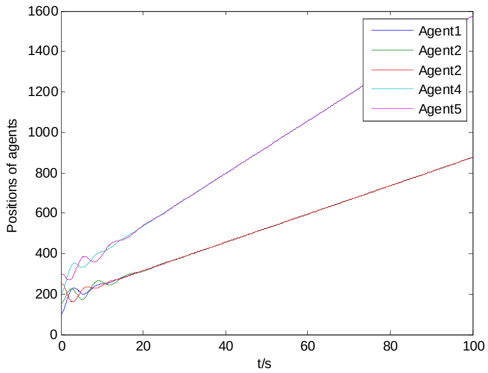
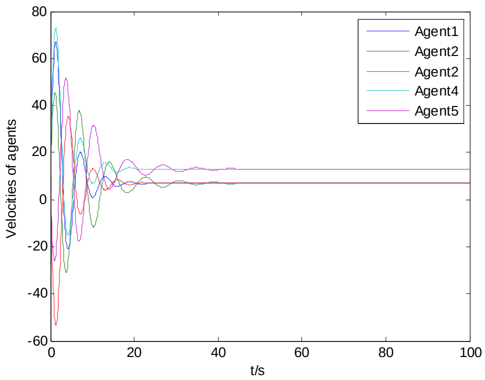


FIGURE I. INTERCONNECTION GRAPH OF MULTI-AGENT SYSTEM (1)

Set $\alpha=0.6, \beta=0.3$, respectively and randomly generate the initial states of the agents. Meanwhile, the input delay are 0.04s, 0.02s, 0.01s, 0.02s, 0.02s, respectively. It is not difficult to verify that the condition of group consensus in Theorem 1 can be satisfied. For simplicity, assume all the communication delays between the nodes are equal to 0.02s. The trajectories of the positions and the velocities in system (1) are shown in Figure 2. From the results, we can easily find that the group consensus of the system is achieved .



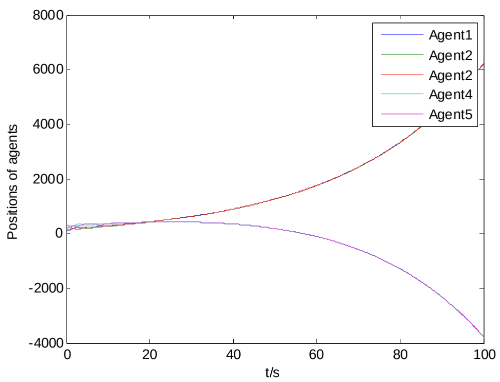
(a) Position state



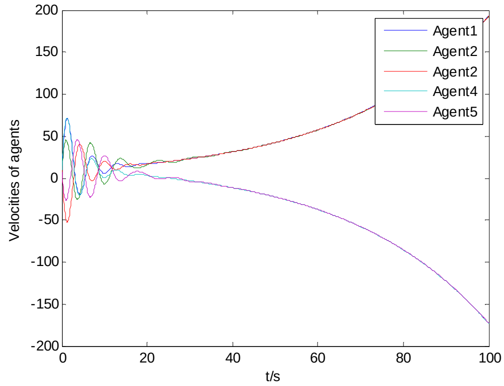
(b) Velocity state

FIGURE II. TRAJECTORIES OF ALL NODES IN THE SYSTEM (1), WHERE $T_1 = 0.04$S

Next, reset T_1=0.1s and keep all the other parameters not changed, it is easy to found that the condition cannot be satisfied in Theorem 1. In this case, the trajectories of the agents in system (1) are illustrated in Figure 3.



(a) Position state



(b) Velocity state

FIGURE III. TRAJECTORIES OF ALL NODES IN THE SYSTEM (1), WHERE $T_1 = 0.1$S

V. CONCLUSION

For second-order systems, this technical note is aimed at exploring the issue of group consensus of multi-agent networks with diverse communication delays and input delays. By applying the theory of frequency-domain, some algebraic criteria of the group consensus are derived. It can be shown that the group consensus of systems is determined by input delays, coupling strengths and connection strengths between agents, independent of communication delays.

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