A 3D finite volume method solver for scattering from lossy objects in layered media

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Abstract—**A 3D frequency-domain finite volume method (FVM) solver is developed for scattering from lossy objects embedded in layered media. Based on the Yee staggered grids, the formulation derivation for FVM are presented in this paper. Two numerical examples with a 3D cuboid embedded in a half space and in a three-layer background are designed to demonstrate the accuracy and efficiency of the method. The comparison with the finite different method shows that the FVM is capable of solving the electromagnetic scattering problems for lossy objects embedded in a layered medium.**

Keywords—finite volume method; scattering modeling; lossy object; layered medium.

I. INTRODUCTION

As a high resolution numerical method, the finite volume method (FVM) was originally developed for shock capturing applications in nonlinear problems, such as gas dynamics based on Euler equations. However, it is also well suited for solving the problems of acoustic, elastic, and electromagnetic waves [1,2] in heterogeneous media [3,4]. Recently the FVM algorithms is described in Reference[5] and implemented in marine controlled source electromagnetic successfully. The same approach is also applicable to other hyperbolic systems with rapidly varying coefficients. Scattered fields from lossy objects embedded in a layered medium is one important research topic because of its wide application in areas such as geophysical exploration, remote sensing , biomedical imaging.

In this paper, a new FVM algorithm based on the Yee staggered grid for scattering from homogeneous objects embedded in layered media is presented. The principle of the finite volume method is briefly derived for lossy media firstly. The accuracy and efficiency of the method are studied via two numerical examples.

II. THEORY

In frequency domain, partial differential equation in electromagnetic wave with complex permittivity $\tilde{\varepsilon} = j\omega\varepsilon + \sigma$, and without considering permeability ($\mu = \mu_0$) is governed

by the following equation

$$\nabla \times \nabla \times \boldsymbol{E} + j\omega\tilde{\varepsilon}\mu_0\boldsymbol{E} = -j\omega\mu_0\boldsymbol{J}_s \qquad (1)$$

where $\boldsymbol{E}$ is the electrical field and ω is the angular frequency, $\mu_0 = 4\pi \times 10^{-7} H/m$ is the permeability of the vacuum, $\boldsymbol{J}_s$ is the impressed electric current density.

From (1), one can find it is singular at the source point, in order to solve this problem, we introduce into a reference model $\tilde{\varepsilon}_0$ (for example, $\tilde{\varepsilon}_0 = j\omega\varepsilon_0 + \sigma_0$, where $\varepsilon_0 \approx 8.8541 \times 10^{-12} F/m$ is the permittivity of the vacuum and σ_0 is a conductivity constant). The primary fields $\boldsymbol{E}_0$ in the reference model can be found easily by analytical resolution.

We also have

$$\nabla \times \nabla \times \boldsymbol{E}_0 + j\omega\tilde{\varepsilon}_0\mu_0\boldsymbol{E}_0 = -j\omega\mu_0\boldsymbol{J}_s \qquad (2)$$

from (1) and (2), we have

$$\nabla \times \nabla \times \boldsymbol{E}^s + j\omega\mu_0\tilde{\varepsilon}\boldsymbol{E}^s = -j\omega\mu_0(\tilde{\varepsilon} - \tilde{\varepsilon}_0)\boldsymbol{E}_0 \qquad (3)$$

where $\boldsymbol{E}^s = \boldsymbol{E} - \boldsymbol{E}_0$ is defined as the second field. Formulation (3) is the PDE in electromagnetic wave, one can find the second field by solving the partial differential equation.In order to use FVM to discretize the electromagnetic field, we need to make up a finite volume around the calculated point, as illustrated in Fig. 1.Integrating in a control finite volume Ω to (3), we obtain

$$\int_\Omega \nabla \times \nabla \times \boldsymbol{E}^s d\Omega + j\omega\mu_0\int_\Omega \tilde{\varepsilon}\boldsymbol{E}^s d\Omega$$
$$= -j\omega\mu_0\int_\Omega (\tilde{\varepsilon} - \tilde{\varepsilon}_0)\boldsymbol{E}_0 d\Omega \qquad (4)$$

Using the Gauss theorem $\int_\Gamma \tilde{n} \times \boldsymbol{A} d\Gamma = \int_\Omega \nabla \times \boldsymbol{A} d\Omega$, where the normal direction $\tilde{n}$ of Γ is outward.Then we have

$$\int_\Gamma \tilde{n} \times (\nabla \times \boldsymbol{E}^s) d\Gamma + j\omega\mu_0\int_\Omega \tilde{\varepsilon}\boldsymbol{E}^s d\Omega$$
$$= -j\omega\mu_0\int_\Omega (\tilde{\varepsilon} - \tilde{\varepsilon}_0)\boldsymbol{E}_0 d\Omega \qquad (5)$$

From (5), one can note that the second term in left hand side and the term in right hand side consider the variation of the permittivity and primary field inside the control volume, which are different from FDM. To solve (5), we first discretize the electromagnetic field at Yee grid system, in which the electric fields are located at the center of each edge of cubic cell and the magnetic fields are located at the center of the surface of cubic cell, as shown in Fig. 2.
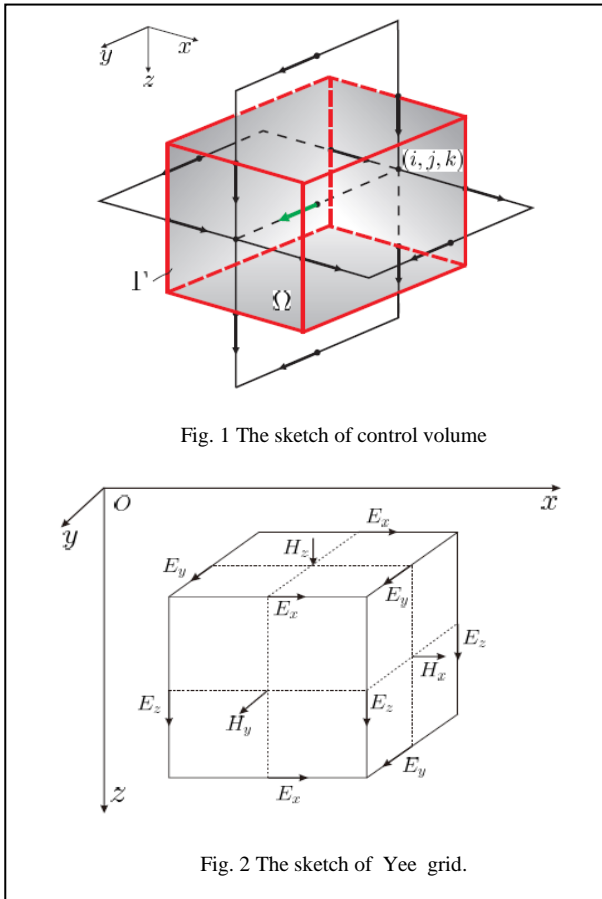

Fig. 1 The sketch of control volume

Fig. 2 The sketch of Yee grid.

III. NUMERICAL RESULTS

The incident electric field for the model is generated by a z-directed dipole. The operating frequency is 100 MHz. To validate the FVM code, the results are compared to those of an independent finite difference method[5]. Fig. 3a shows the geometry of a homogeneous cuboid located at the bottom layer of a half space. We used $60 \times 60 \times 80$ cells for discretization.

Fig. 3b shows the normalized total field as a function of x-direction. The normalization is performed with respect to the maximum total field at the receiver array. The total electric field from the FVM agrees very well with those of the FDM. The relative residual error is less than 0.02%. Fig. 4 shows the geometry of a homogeneous cuboid located at the bottom layer of a three-layer background. We used $60 \times 60 \times 100$ cells for discretization. Fig. 5 shows the normalized total field as a function of x-direction. Again , the total electric field from the

FVM agrees very well with those of the FDM. The relative residual error is less than 0.006%.
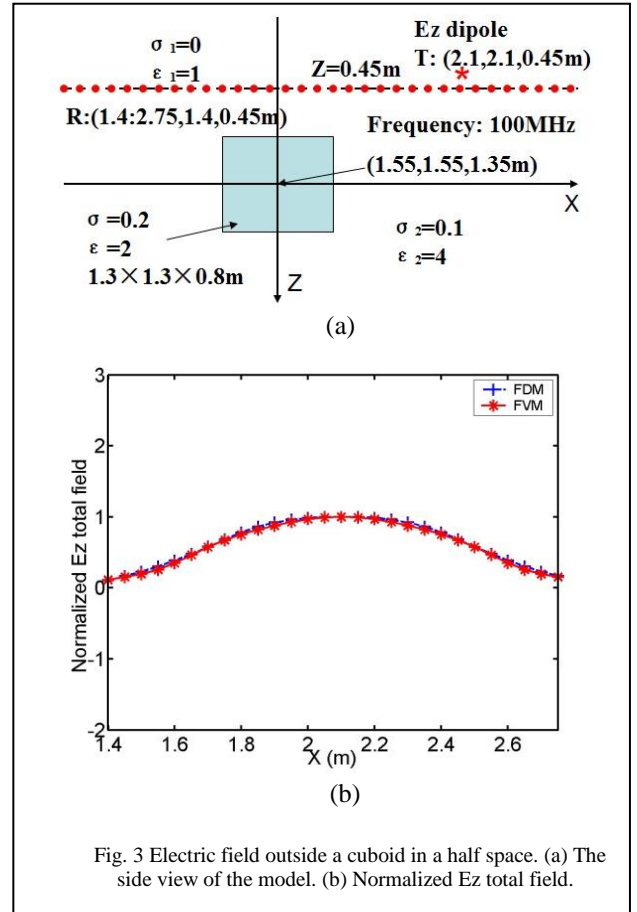

(a)

(b)

Fig. 3 Electric field outside a cuboid in a half space. (a) The side view of the model. (b) Normalized Ez total field.
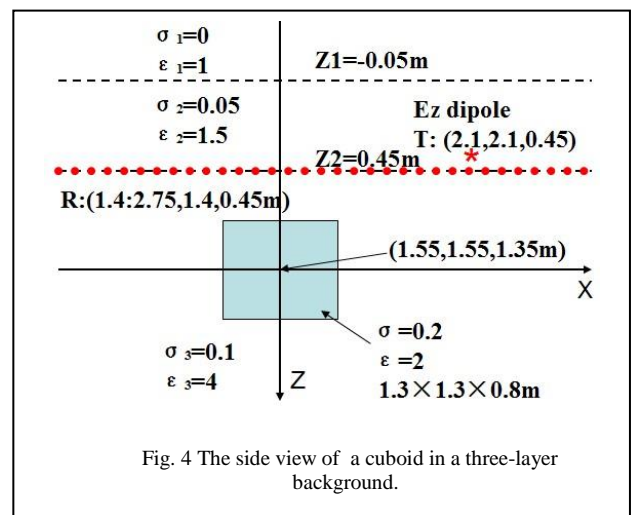

Fig. 4 The side view of a cuboid in a three-layer background.

IV. CONCLUSIONS

A 3D finite volume method based on Yee staggered grids has been developed and applied to calculate scattering field from homogeneous objects embedded in layered media. The accuracy and efficiency of the method are demonstrated by two numerical examples. Compared with the results of finite different method, the FVM presented in this paper is well

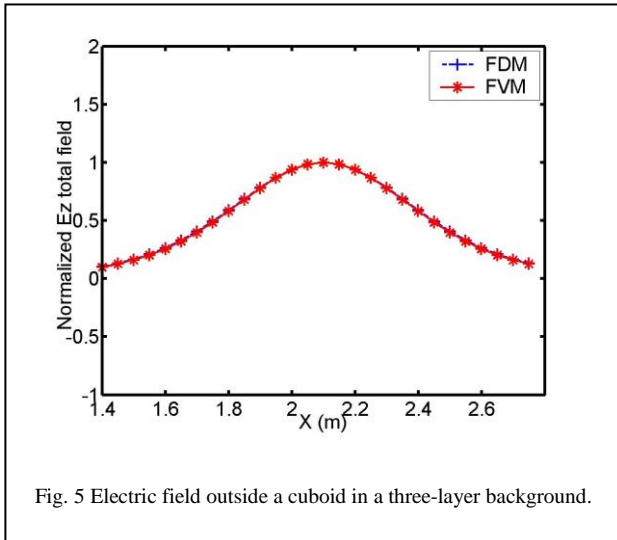suited to calculating the scattering field from lossy objects in layered media.

Fig. 5 Electric field outside a cuboid in a three-layer background.

REFERENCES

[1] T. R. Fogarty, and R. J. LeVeque,"High-resolution finite-volume methods for acoustic waves in periodic and random media," J. Acoust. Soc. Am., vol. 106, no. 1, pp. 17-28, 1999.

[2] R. J. LeVeque,"Finite-volume methods for non-linear elasticity in heterogeneous media," Int. J. Numer. Meth. Fluids., vol. 40, pp. 93-104, 2002.

[3] W. Z. Shen, J. A. Michelsen, and J. N. Sorensen, "A collocated grid finite volume method for aeroacoustic computations of low-speed flows,"Journal of Computational Physics., vol. 196, pp. 348-366, 2004.

[4] N. Botta, R. Klein, S. Langenberg, and S. Lutzenkirchen, "Well balanced finite volume methods for nearly hydrostatic flows," Journal of Computational Physics., vol.196, pp. 539-565, 2004.

[5] B. Yang, Y. X. Xu, Z. X. He, and W.-B. Sun, "3D frequency-domain modeling of morine controlled source electromagnetic responses with topography using finite volume method,"Chinese J. Geophys., vol. 55, no. 4, pp. 1390-1399, 2012.