Seismic attenuation of saturated cracked glass: Numerical analyses of experimental creep tests

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***Abstract*—We examine laboratory data from creep tests performed on a thermally cracked water-saturated glass sample and estimate Young's modulus and extensional-mode attenuation as functions of frequency. An SEM image of a vertical cross-section through the upper half of the cylindrical sample has been digitized and provides key geometric information regarding the crack network. Together with the well-known properties of the glass matrix, this allows for deriving a first-order 2D poroelastic model. Comparing the results of numerical compressibility tests with the corresponding laboratory data indicates that the former are capable of capturing some important characteristics of the latter. This is notably the case for the overall magnitude and frequency dependence of the attenuation and Young's modulus, which corroborates the importance of WIFF in interconnected cracks as the dominant dissipation mechanism.**

Keywords—sesmic; attenuation; numerical analyses;creep tests; cracked glass

I. Introduction

The geophysical and hydraulic characterization of fractured rocks is widely regarded as an ultimate objective for a number of important applications, such as the sustainable use of groundwater, the optimized production of hydrocarbons and geothermal energy, and the safe storage of nuclear waste. There is increasing evidence to suggest that a number of seismic attributes are not only sensitive to the presence of fractures *per se*, but also to certain characteristics of fracture networks. Notably the interconnectivity of fractures seems to have a strong effect on the attenuation [1]. To date, the available evidence is, however, entirely based on numerical simulations and hence the importance of this dissipation mechanism remains uncertain.

The attenuation of cracked materials is commonly measured through creep tests, which are based on recording the time-dependent strain in response to a step-function-type axial stress [2]. The lower limit of the covered frequency range is linked to the total observation time and can be of the order of 10^{-5} Hz for a 24~hours creep test. Conversely, the upper limit of the frequency range is linked to the accuracy of the experiment. If the stress is applied quasi-instantaneously, and the resulting strain is recorded every 0.1 s, it can easily be of the order of 10 Hz.

Extracting the frequency-dependent attenuation and Young's modulus dispersion from creep tests is, however, challenging as many of these experiments were not performed with the intention of estimating these characteristics. Correspondingly, high frequencies tend to be poorly defined due to insufficiently dense sampling of the strain response at early times. Moreover, the high stress and strain magnitudes typical of creep tests tend to result in non-linear effects, which in turn may lead to an overestimation of the seismic attenuation[3].

Laboratory studies exploring cracked materials often use glass samples. The primary reasons for this are that glass (i) is essentially homogeneous, (ii) has a negligible porosity, and (iii) can be cracked in a controlled manner. In this study, we analyze creep tests performed on a thermally cracked, water-saturated glass sample in terms of the frequency-dependent attenuation and Young's modulus dispersion. The primary objective of this work is to explore whether and to what extent experimental data from classical creep tests can be interpreted in terms of the seismic attenuation and Young's modulus dispersion.

II. Laboratory Measurements

The sample considered in this study consists of a borosilicate glass cylinder with a diameter of 4 cm and a length of 8 cm produced under ideal conditions of slow cooling to prevent the formation of pores and cracks. The elastic properties of the intact sample were measured. The porosity and permeability of the glass matrix are considered to be essentially negligible. It is, however, known that at the atomic scale, that is, at scales of the order of 10^{-10} m, there is a connected porosity, which in turn corresponds to a permeability of the order of 10^{-20} m^2. The sample underwent thermal cracking by slowly heating it to 300 ºC, keeping it at this temperature during ~12 h, and subsequently quenching it. The sharp decrease in temperature along the sample's surface induces tensile stresses resulting in the nucleation of cracks, which then propagate throughout the interior as the cooling progresses.

The water-saturated cracked sample was submitted to a creep test using a triaxial press[4]. The axial loading was increased and then kept constant during 24 h, while the confining pressure and the pore pressure were kept constant. Please note that, even though this experimental procedure does not correspond to that of a pure uniaxial test, it still allows for an adequate estimation of Young's modulus as the confining pressure is essentially negligible compared to the axial loading. The original objective of this creep test was to explore fracture propagation, which was monitored by recording the acoustic emission activity. The full experiment consists of a succession of creep steps, each with a duration of 24 h. Here, we only consider the first creep step at the lowest level of loading, which was performed under particularly well-controlled conditions. During this first creep step, no acoustic emissions were recorded, which indicates that the original, thermally induced crack network remained unaltered.

To infer the frequency-dependent attenuation and Young's modulus from these experimental data, the time derivatives of the stress and strain histories are evaluated. The resulting stress and strain rates are transformed into the Fourier domain, which then allows for evaluating Young's modulus the extensional-mode attenuation, as quantified by the inverse quality factor. The real part of Young's modulus and the extensional-mode attenuation are shown in Fig.1 as functions of frequency. The attenuation exhibits pronounced frequency dependence and is highest between $3 \cdot 10^{-4}$ and $4 \cdot 10^{-3}$ Hz, which roughly coincides with the steepest gradient of Young's modulus. The latter shows a classical dispersion characteristic with a gradual increase between the nearly constant values at the lower and upper limits of the considered frequency spectrum.

Due to the set-up of the experiment, the characteristics of the attenuation and dispersion curve in the low- and high-frequency regimes, as denoted by grey zones in Fig.1, are likely to contain artifacts. For practical reasons, the axial loading cannot be applied instantaneously, which inherently reduces the accuracy of the estimated attenuation and Young's modulus at the upper end of the prevailing frequency range. This becomes apparent for the inferred attenuation curve at frequencies higher than 10^{-2} Hz. The asymptote in this frequency regime is proportional to $f^{-1.2}$, as denoted by the dashed line in Figure 1, and thus far too steep to be caused by WIFF[4]. In the low-frequency regime, the total time of the creep test might be too short to capture the mechanical behavior of the sample appropriately. Furthermore, we cannot assure that the measured axial strain reached its maximum level over the measured time and thus Young's modulus in the low-frequency limit might be slightly smaller than the inferred value of 59.5 GPa.

Despite these uncertainties in the low- and high-frequency regimes, the characteristic frequency dependence of the attenuation and Young's modulus indicates that WIFF within the interconnected crack network might be the dominant cause of seismic energy dissipation. In the following, we therefore seek to verify this hypothesis through a numerical modeling study.

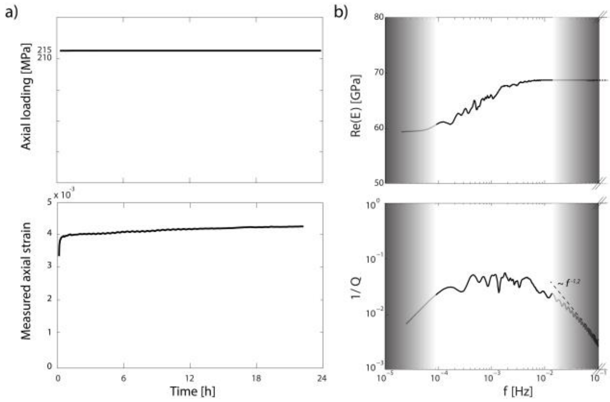

Fig.1 a) Applied stress and measured strain for a creep test performed on a thermally cracked water-saturated glass sample. b) Real part of Young's modulus and attenuation extracted from the creep test.

III. NUMERICAL MODELING

To analyze our laboratory measurements, we perform 2D numerical oscillatory tests on models of the cracked glass sample. These oscillatory tests, which can be regarded as the frequency-domain version of corresponding relaxing tests in the time domain, are based on a finite element solution of the quasi-static poroelastic equations. The numerical simulations yield the complex-valued and frequency-dependent stress and strain values, from which the attenuation and Young's modulus can be inferred. We use an unstructured triangular mesh in such a way that first-order discontinuities in the material properties coincide with element boundaries. This allows for an efficient discretization of cracks having large aspect ratios by correspondingly varying the dimensions of the triangular elements inside the cracks as well as in their immediate vicinity. In this study, the smallest elements have a side length of $2 \cdot 10^{-6}$ m and are ~2000 times smaller than the largest elements used for the intact glass matrix. The boundary conditions consist of a fixed lower boundary, that is, the vertical displacement is set equal to zero at the bottom of the model, free lateral boundaries, and a time-harmonic vertical stress oscillation imposed along the upper boundary.

For defining the crack network for our 2D model, we consider an SEM image of a vertical cross-section through the upper half of the cracked glass sample (Fig.2). Mirror symmetry was assumed in order to reconstruct the crack network in the lower half of the sample. The thermal treatment of the sample causes the cracks to nucleate on the surface of the cylindrical sample and to propagate towards the core. Correspondingly, the assumption of mirror symmetry is an adequate first-order assumption. After image enhancement, the cracks were identified and their lengths and apertures were measured. The thus inferred representative crack aspect ratio is $1.5 \cdot 10^{-3}$. Cracks are represented as highly compliant features of very high porosity and permeability, are fully saturated with water. The porosity in the cracks is fixed at 99%, while their permeability is constrained by the cubic law. We bin the crack apertures into four dominant groups, which results in permeabilities of 100, 550, 2000, and 6000 mD.

According to effective medium theory, the bulk modulus of fractures and cracks is controlled by the product of the aspect ratio and the corresponding bulk modulus of the solid grains. On this basis, we estimate the bulk modulus of the cracks to be 0.09 GPa. Considering the uncertainty associated with the estimation of crack aspect ratios, the bulk modulus of the cracks is expected to be in the range of 0.06-0.15 GPa. Following experimental evidence, assume the bulk modulus of the cracks to be equal to the shear modulus[5].

Radial cross-sections through the sample were also available and, together with the vertical section, allowed for estimating the spatial distribution of the crack density (Fig.2). Working with a 2D numerical model inherently assumes that the 3D effects in the laboratory data are small. The radial sections show that the crack density is constant in the radial sections regardless of the crack orientation, while in the vertical section there is a predominance of the vertical cracks. This indicates that the 3D crack network exhibits an approximate vertical transverse isotropic (VTI) symmetry, which is indeed consistent with the approximate rotational symmetry of our 2D model.

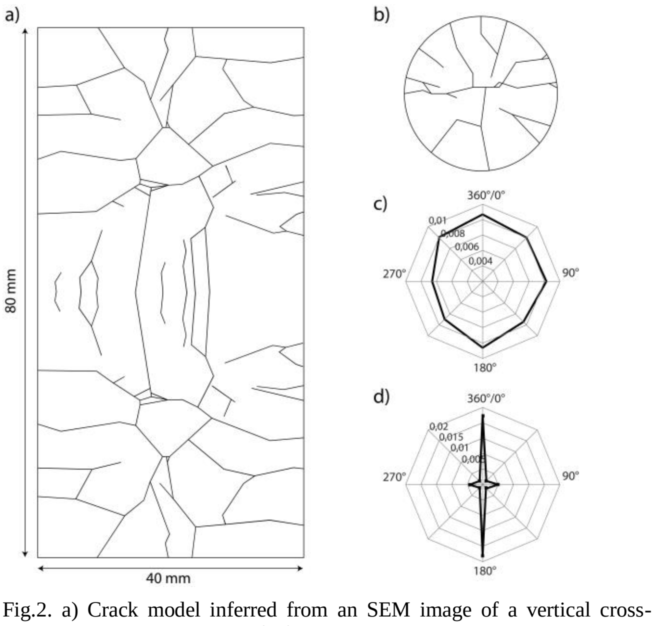

Fig.2. a) Crack model inferred from an SEM image of a vertical cross-section through the upper half of the sample. b) Crack network in a radial plane and c) corresponding crack density distribution with respect to the crack orientation. d) Crack density distribution with respect to orientation in the vertical plane.

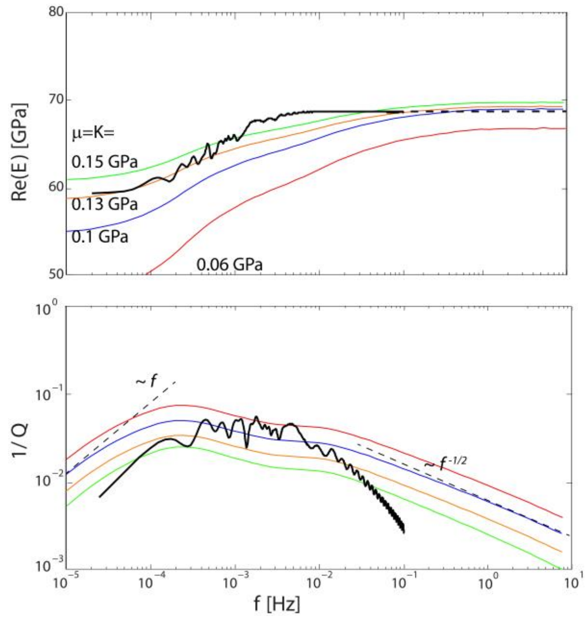

Fig.3 Comparison of experimentally inferred Young's modulus and attenuation with numerical simulations for crack compressibilities ranging from 0.06 to 0.15 GPa. Dashed lines denote the asymptotes of the numerical results at low and high frequencies.

IV. Results

The results of the numerical simulations for varying bulk and shear moduli of the cracks are illustrated in Fig.3. In all cases, we observe a high level of attenuation over a relatively broad frequency range (~10^{-4}-10^{-2} Hz) as well as the presence of two peaks. For WIFF between connected cracks, the peak attenuation is reached when the diffusion length of the material within the cracks is of a similar size as the characteristic crack length[1]. The part of the attenuation curve characterized by a broad region of high values is caused by two dominant populations of cracks with average lengths of ~5 mm and ~0.5 mm. These average crack lengths do indeed correspond approximately to the characteristic frequencies at which the two attenuation peaks are observed in the numerical results.

The overall characteristics of the numerically inferred attenuation curves are in reasonable agreement to that of the experimental data, notably with regard to the relatively broad region of high attenuation. However, the two peaks present in the numerically inferred attenuation curves are not observed in the experimental data. The overall slope of the numerical attenuation curves matches the experimental data reasonably well in the range of 10^{-5}-10^{-2} Hz, while there are pronounced discrepancies at higher frequencies. The same holds true for Young's modulus, where an adequate fit to the experimental data can be achieved for frequencies between 10^{-5} and 10^{-3} Hz, while the discrepancies are again more pronounced at higher frequencies. These discrepancies between the numerical simulations and the experimental data at higher frequencies are, as outlined above, most likely due to the difficulty of recording the very beginning of the creep step with sufficient accuracy.

The magnitude of the attenuation and Young's modulus increases with increasing compressibility contrast, while the overall frequency dependence remains similar. Crack compressibilities of 0.1-0.13 GPa provide a decent match of the attenuation magnitude and the high-frequency limit of Young's modulus observed in the experimental data. Conversely, at lower frequencies, a value of 0.13 GPa produces a better fit with regard to Young's modulus. However, as already discussed above, the experimentally obtained Young's modulus might be overestimated at these frequencies due to the lack of sufficiently long measurement times.

We cannot exclude that part of the observed attenuation is associated with the high strain levels applied to the sample. The corresponding non-linear component of the attenuation is commonly explained by friction between the solid walls of the cracks. This is expected to result in the addition of a quasi-frequency-independent base level attenuation and hence not significantly affect the frequency dependence of the dissipation caused by WIFF[3]. Furthermore, the roughness of the crack walls, which is considered to be the main cause for this non-linear frictional attenuation, is expected to be relatively small in cracked glass. This is consistent with the fact that our numerical models match the peak attenuation frequency of the data with permeability values obtained from the cubic law, which inherently relies on the assumption of planar crack walls.

V. CONCLUSION

The results of this study indicate that frequency-dependent attenuation and Young's modulus dispersion can be extracted from creep tests performed on thermally cracked water-saturated glass samples. One of the experimental uncertainties concerns the shape of the attenuation curve at high frequencies and is due to the fact that the axial loading cannot be applied instantaneously. Another, and probably more important, uncertainty is associated with the commonly insufficient duration of standard creep tests, which may lead to an overestimation of Young's modulus at low frequencies. Finally, the high strain levels used in such experiments may result in non-linear energy dissipation due to solid-to-solid friction along the crack walls, which is expected to simply add a frequency-independent base level to the attenuation associated with WIFF.

Our numerical results indicate that the overall magnitude of the attenuation is quite sensitive to the fracture compressibility. With a uniform crack compressibility based on effective medium theory, the experimental amount of attenuation and the high-frequency value of Young's modulus are reasonably well reproduced by the numerical simulations. Meanwhile, crack permeability controls the frequency at which the attenuation is highest. Using four crack permeability families inferred from the cubic law, that is, assuming perfectly smooth crack walls, provides a reasonable match of the overall frequency dependence of the attenuation inferred from the experimental data. This in turn indicates that the non-linear effects on attenuation due to solid-to-solid friction along the crack walls are likely to be negligible and hence that WIFF effects in interconnected cracks are the predominant mechanism responsible for the observed frequency-dependent attenuation and Young's dispersion.

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