An improved method of elastic impedance based on the relationships between P- and S- wave velocities

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***Abstract*—In the elastic impedance (EI) equation proposed by Connolly (1999), the relationship between the P-wave velocity α and S-wave velocity β, $K=\beta^2/\alpha^2$, is assumed to be constant, which is not consistent with the statistical pattern between P- and S-wave velocities in real rocks, thus affecting the accuracy of elastic impedance analysis. Considering that P- and S- wave velocities satisfy a linear relationship, a new approach for calculating elastic impedance was derived based on a simplified form of Zoeppritz equation proposed by Aki and Richards (1980). This novel approach is formed by multiplication of two terms. The first term represents the similar form for Connolly's EI equation. As for the second term, it is interpreted as the correction of the linear relationship between the P-wave velocity and S-wave. Numerical simulation experiments are used to verify performance of the new EI equation in reproducing the amplitude-versus-offset (AVO) response of artificial sandstone strata generated by two different general empirical linear equations for P- and S- wave velocities. As shown by the results, compared with Connolly's elastic impedance, the reflection coefficient obtained using the new elastic impedance calculation method was more accurate and less affected by the error of S-wave velocity. In addition, the variation tendency of the new elastic impedance with the incidence angle is closely associated with the coefficient of the linear relationship, and proper velocity relationships need to be applied according to the actual conditions for elastic impedance analysis.**

The general empirical equations between P- and S- wave velocities are suitable for describing the relationships in water saturated sandstone. Through Gassmann's saturated fluid substitution analysis, it was found that the reflection coefficients calculated using the new elastic impedance equation under saturated oil and saturated oil and gas conditions were still more accurate than that calculated using Connolly's equation. At the same time, the difference in empirical equations also has considerable influence on the variation of new elastic impedance at different incidence angles.

Keywords—elastic impedance, fluid substitution, zoeppritz equation

I. INTRODUCTION

Amplitude variation is the most important seismic data interpretation method used to study the lithology and hydrocarbon identification. With the advent of AVO technology, the relationship between amplitude variations and lithologic and physical property parameters with respect to different incidence angles can be analyzed, moreover, the interpretation for seismic amplitude has developed rapidly[1-3]. In 1999, Connolly proposed the Elastic Impedance (EI)[4], and introduced the concept of impedance to nonzero-incidence seismic data, and also employed angle stacks data to invert the impedance values with different incidence angles. EI inversion may reflect the characteristics of the amplitude-versus-offset (AVO), and overcome some deficiencies of AVO technology on near and far offsets analyses, it is gradually applied in fluid identification and predictions for reservoir physical properties[3][5].

Connolly integrated and derived EI from the simplified forms of Zoeppritz Equations that were proposed by Aki and Richards[15]. One important assumption in Zoeppritz Equations is that $K=\beta^2/\alpha^2$ is constant (β is S-wave velocity, α is P-wave velocity). And in the studies of improving EI provided by Connolly, the value of K is also a constant[6], but the relationship of P- and S-wave is not in considerations of the assumed K, not being consistent with actual geological conditions. For real stratums, P- and S-wave velocities of rock not only are intrinsically related, but apparent statistical regularity exists. Castagna et al.[9][11] presented the linear relationship between P-wave velocity and S-wave velocity for mudrocks according to logging data, called linear equations for mudrocks. Pickett[10], Han et al.[16], Smith et al.[17], Greenberg and Castagna[13], Gan et al. [20]and Han et al.[14] established various empirical equations of P- and S-wave velocities based on laboratory data. Li [13] concluded a quadratic fitting relation for P- and S-wave velocities on the basis of previous studies. Therefore, when the P- and S-wave of rocks satisfy a specific empirical equation, K=β2/α2 is not a constant, but an explicit function.

In the present work, a novel approach of calculating elastic impedance was proposed, which was based on the linear relationship between P- and S-wave velocities, and also referred to the process that Connolly deduced EI. When computing EI by this approach, K was not regarded as a `constant any more, it was a fitting function satisfying a specific relation of P- and S-wave velocities. Hence, the developed EI formula was more in line with actual geological conditions, also with a higher accuracy.

II. THEORY AND METHOD

Connolly Elastic Impedance

The simplified forms of Zoeppritz equations proposed by Aki and Richards (1980):

$$R(\theta) = A + B\sin^2\theta + C(\tan^2\theta\sin^2\theta)$$
$$A = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right) \tag{1}$$
$$B = \frac{\Delta\alpha}{2\alpha} - 4K\frac{\Delta\beta}{2\beta} - 2K\frac{\Delta\rho}{\rho}$$
$$C = \frac{\Delta\alpha}{\alpha} \qquad K = \frac{\beta^2}{\alpha^2}$$

Where $R(\theta)$ is the reflection coefficient of P-wave, α, β, ρ are an average of P-wave velocity, S-wave velocity, *and* density of the upper and lower media, θ is incidence angle; $\Delta\alpha$, $\Delta\beta$ and $\Delta\rho$ are differences of P-wave velocity, S-wave velocity, and density between upper and lower stratums in reflection interfaces.

Introducing the concept of EI to nonzero incidence angles, thus the EI in *n-1* and *n* layer is related to the reflection coefficient in this interface as:

$$R(\theta) = \frac{EI_n(\theta) - EI_{n-1}(\theta)}{EI_n(\theta) + EI_{n-1}(\theta)} \approx \frac{\Delta EI(\theta)}{2EI(\theta)} \approx \frac{1}{2}\Delta\ln EI(\theta) \tag{2}$$

Substituting Eq. (2) into Eq. (1), one can get:

$$\Delta\ln(EI) = (1+\tan^2\theta)\Delta\ln(\alpha) - 8K\sin^2\theta\cdot\Delta\ln(\beta) + (1-4K\sin^2\theta)\Delta\ln(\rho) \tag{3}$$

Connolly hypothesized K is a constant, and obtained EI from:

$$EI_{\text{con}} = \alpha^{1+\tan^2\theta}\beta^{-8K\sin^2\theta}\rho^{1-4K\sin^2\theta} \tag{4}$$

The new Elastic Impedance

In accordance with the actual geological circumstances, it is assumed that P-wave velocity is linearly related to S-wave velocity, denoted as $\alpha = w(\beta - b)$, where w and b are constants, generally $w>0$ and $b<0$.

$$k = \frac{(\beta-b)^2}{\alpha^2} = \frac{1}{w^2} \tag{5}$$

Taking Eq. (5) into Eq. (3), the following equation can be got:

$$\Delta\ln(EI) = (1+\tan^2\theta)\Delta\ln(\alpha) - 8\frac{(\beta-b+b)^2}{\alpha^2}\sin^2\theta\cdot\Delta\ln(\beta) + (1-4\frac{(\beta-b+b)^2}{\alpha^2}\sin^2\theta)\Delta\ln(\rho)$$

$$\Rightarrow \Delta\ln(EI) = (1+\tan^2\theta)\Delta\ln(\alpha) - 8k\sin^2\theta\cdot\Delta\ln(\beta) + (1-4k\sin^2\theta)\Delta\ln(\rho) - 8\frac{2b\beta-b^2}{\alpha^2}\sin^2\theta\cdot\Delta\ln(\beta) - 4\frac{2b\beta-b^2}{\alpha^2}\sin^2\theta\cdot\Delta\ln(\rho)$$

Integrating the above equation, obtain:

$$\int d\ln(EI) = \int[(1+\tan^2\theta)d\ln(\alpha) - 8k\sin^2\theta\cdot d\ln(\beta) + (1-4k\sin^2\theta)d\ln(\rho)] - \int[8\frac{2b\beta-b^2}{\alpha^2}\sin^2\theta\cdot d\ln(\beta) + 4\frac{2b\beta-b^2}{\alpha^2}\sin^2\theta\cdot d\ln(\rho)]$$

Substituting the above equation, obtain:

$$EI_{\text{new}} = \alpha^{1+\tan^2\theta}\beta^{-8k\sin^2\theta}\rho^{1-4k\sin^2\theta}\cdot e^{-\chi} \tag{6}$$

Where,

$$\chi = \int\left(8\frac{2b\beta-b^2}{\alpha^2}\sin^2\theta\, d\ln(\beta) + 4\frac{2b\beta-b^2}{\alpha^2}\sin^2\theta\, d\ln(\rho)\right)$$

From Gardner equation[16], density and P-wave meet the relation of $\rho=m\alpha^n$, where m and n is coefficient and index, respectively. The integral can be written as:

$$\chi = \left[8k\ln(\frac{\beta-b}{\beta}) - 8k\frac{b}{\beta-b} - \frac{2nb^2}{\alpha^2} - \frac{8nb}{w\alpha}\right]\sin^2\theta$$

The new EI defined in Eq. (6) is similar to that is defined by Connolly (Eq. (4)), but it is derived on the basis of the linear relationship between P-wave and S-wave. When b=0, they are equal.

III. Numerical Simulation Analysis

Sandstone reservoir is important in oil and gas exploration. In actual geological conditions, sandstones distribute complicatedly, and a large amount of relevant studies have been presented in previous works[9]. Thus, the classical empirical equations of relationship between P- and S-wave velocity are used to conduct the numerical simulation experiments.

Accuracy analysis

The relationship between P- and S-wave velocities for sandstones is fitted according to the data of water saturated sandstones, and the velocity of sandstones varies with the cement and consolidation of rocks from loose to dense, moreover, the correlations of P- and S-wave velocity differ and hence the corresponding linear empirical equations change. Li[13] discussed that when P-wave velocity was less than 3km, Castagna's [9] formula fitted better, nevertheless the formulae proposed by Smith et al.[12] were suitable for analyzing the sandstones deeply embedded. Smith's empirical formula $\alpha = 1.42\beta+0.79$ and Castagna's empirical formula $\alpha = 1.16\beta + 1.36$ were selected to construct the rock models, as shown in Tables 1 and 2, where density was obtained from Gardner's equation (1974) $\rho=1.74\alpha^{0.25}$.

Employing the simplified forms of Zoeppritz equations (Eq. (1)) given by Aki and Richards to calculate the accurate variations of reflection coefficient in reflection interfaces with respect to the incidence angles in rock models. In addition, substituting EI equation (Eq. (4)) defined by Connolly into Eq. (2) to calculate the reflection coefficient (denoted as Rcon, and supposed K=0.21), and applying Eq. (6) in Eq. (2) to compute the reflection coefficient (denoted as Rnew), as shown in Fig.1.

TABLE I. MODEL PARAMETERS FOR CALCULATING REFLECTION COEFFICIENTS USING SMITH'S EMPIRICAL FORMULA

Rock No.	Elastic Parameters Of Model			
	α (km/s)	β(km/s)	ρ (g/cm^3)	β^2/α^2
1	2.21	1.00	2.00	0.20
2	2.50	1.20	2.06	0.23
3	3.21	1.70	2.21	0.28
4	3.35	1.80	2.24	0.29
5	4.12	2.33	2.38	0.32
6	5.49	3.30	2.57	0.36

TABLE II. MODEL PARAMETERS FOR CALCULATING REFLECTION COEFFICIENTS USING CASTAGNA'S EMPIRICAL FORMULA

Rock No.	Elastic Parameters Of Model			
	α (km/s)	β(km/s)	ρ (g/cm^3)	β^2/α^2
1	2.21	0.73	2	0.11
2	2.5	0.98	2.06	0.15
3	3.21	1.59	2.21	0.25
4	3.35	1.72	2.24	0.26
5	4.12	2.38	2.38	0.33
6	5.49	3.56	2.57	0.42

In Fig.1, Rock 1 and2, Rock 3 and 4, and Rock 5 and 6 (see in Tables 1 and 2) among rock models were selected, respectively, to calculate the reflection coefficients with the variations of incidence angles. Seismic waves propagated from low impedance medium to high impedance medium. From Fig.1, it can be seen that when coping with the rocks in line with specific relationships between P- and S-wave, the theoretical coefficient curves obtained from Eq. (6) and simplified forms of Zoeppritz equations are basically coincide. The accuracy of Eq. (6) is clearly higher than that of Connolly's formula (see Eq. (4)).

It also can be seen from Figs. 1A-C that the relationship between P- and S-wave velocities satisfied Smith's empirical equation, and the reflection coefficient Rcon derived from Connolly's formula (Eq. (4)) was increasingly deflected with the increase of incidence angles. Meanwhile, with the augment of average values in upper and lower reflection interfaces, the errors of Rcon were getting larger, even for small incidence angles, it gradually deflected away, which was mainly related to the value of constant K. When the discrepancy between and K was significant, the error of reflection coefficient obtained from Eq. (4) was correspondingly larger. The same results could be concluded when the relationship between P- and S-wave velocities satisfied Castagna's empirical formula, as shown in Figs.1D-F.

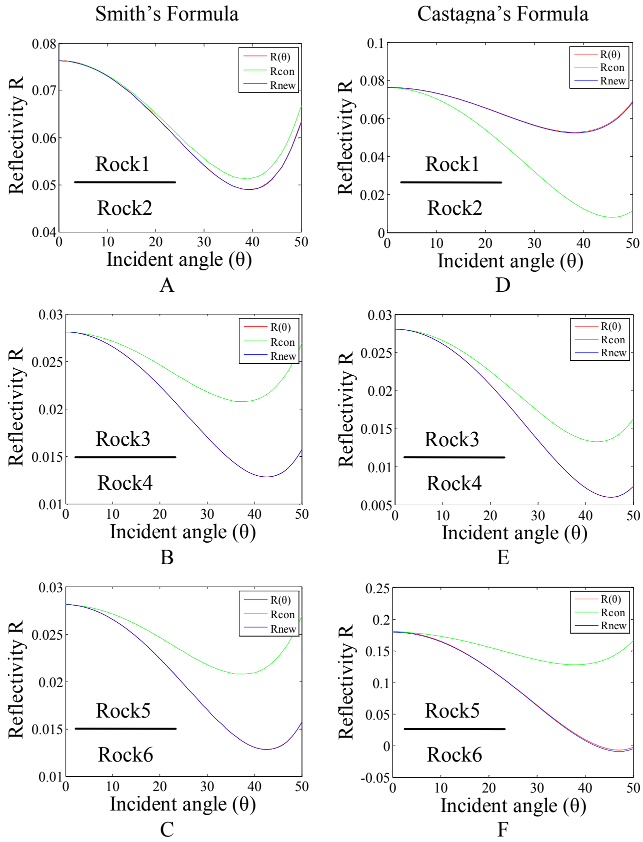

Fig. 1. *Reflectivity versus incidence angle for different rock parameters*

Therefore, when the correlation between P- and S-wave velocities meets a definite linear relation, the discrepancies between reflection coefficients derived from Connolly's formula (Eq. (4)) and theoretical values are influenced by K significantly, yet the approach presented in this work could ensure the calculation precision.

Analysis of Fluid Substitution

Fluid substitution was generally carried out using Gassmann's formul[17] (Eq. (7)), analyzing the effects of different fluid compositions and volume percentages on the seismic response to detect the hydrocarbon.

$$\rho\alpha^2 = K_{dry} + 4/3\mu_{dry} + \frac{\left(1-K_{dry}/K_{ma}\right)^2}{\left(1-\phi-K_{dry}/K_{ma}\right)K_{ma}+\phi/K_{fl}} \qquad (7)$$

$$\rho\beta^2 = \mu_{dry}$$

Where Φ is porosity, is the volume modulus of dry rocks, is the shear modulus of dry rocks, is the volume modulus of matrix, and represents the volume modulus of fluid in pores.

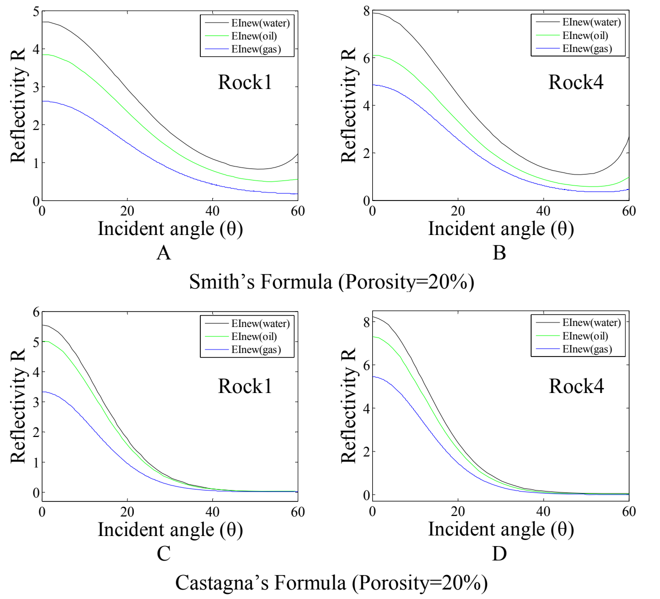

Fig. 2. *New ealstic impedance calculated using different fluid-bearing sands*

The single water-bearing rocks in Tables 1 and 2 were replaced by oil-bearing and gas-bearing rocks using fluid substitution, to calculate the new elastic impedance *EI*new for different fluid-bearing conditions, as shown in Fig. 2. Some discoveries can be concluded: First, *EI*new exhibited different discrepancies with varied filled fluid, and with the augment of incidence angles, the discrepancies caused by low-velocity Rock 1 and high-velocity Rock 4 were reducing. Second, differences between different empirical equations affected the variations of new elastic impedances versus incidence angles under varied fluid-bearing conditions. For Smith's empirical formula, *EI*new displayed obvious distinctions with respect to the large incidence angles (or far-offset) when subjected to fluid-bearing conditions, however, for Castagna's empirical formula, *EI*new tended to zero impedance for large incidence angles, thus its differences could not be exhibited. These further indicated that when the new elastic impedance is utilized in practical application, people need to choose an appropriate relationship between P- and S-wave velocities, as well as a proper incidence angle to analyze and evaluate the reservoir.

IV. CONCLUSIONS

Connolly's and the improved elastic impedances were deduced based on the hypothesis that $K=\beta^2/\alpha^2$ is a constant, which affected the accuracy of EI analysis. Hence a novel approach was proposed to calculate EI considering that P- and S-wave velocities satisfied the linear fitting relationship. The velocity model of P- and S-wave was constructed using Smith's and Castagna's empirical formula, and it was compared with Connolly's elastic impedance. When P- and S-wave meeted a specific linear relationship, the following conclusions could be obtained:

(1) Compared with the elastic impedance proposed by Connolly, the new elastic impedance had a higher accuracy to calculate the reflection coefficient, and was also less influenced by S-wave velocity errors.

(2) The tendency of new elastic impedance in regard to the variations of incidence angles was closely related to the coefficient of the linear relationship between P- and S-wave. And for large incidence angles, new EI approached zero impedance. Therefore, in practical applications, appropriate relationship between P- and S-wave, and proper incidence angles need to be chosen to analyze AVO phenomenon.

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