Joint inversion of Love-wave dispersion curves and SH-wave first arrivals for near-surface SH-wave velocity

Xiaohan Hao[1], Jianghai Xia[1,2]
[1]Subsurface Imaging and Sensing Laboratory, Institute of Geophysics and Geomatics,
China University of Geosciences (Wuhan)
[2]Hubei Subsurface Multi-scale Imaging Key Laboratory, Institute of Geophysics and Geomatics,
China University of Geosciences (Wuhan)

***Abstract*—Tomography and surface-wave analysis are two established methods in seismic surveys and used successfully in near surface. Data acquisition, processing, and inversion of those two methods have been studied widely. Those two methods possess some different particular advantages respectively. We implemented the joint inversion of Love-wave dispersion curves and SH-wave first arrivals using a damped least-squares algorithm to obtain subsurface SH-wave velocity models. The joint inversion method is tested with two synthetic seismic records. We compared results of those two individual inversion methods with those of the joint inversion method. The results indicate that we could use the joint inversion method to achieve a better real SH-wave velocity model because the joint inversion uses more data for SH-wave velocities in inversion. The results also indicate SH-wave velocities of low velocity layers could also be determined using the joint inversion method.**

Keywords: joint inversion;Love wave; SH-wave

I. INTRODUCTION

Surface-wave analysis methods and body-wave tomography are quite common techniques for near surface applications in areas like geotechnical engineering, seismic hazard and environmental problems. Surface-wave methods have high detecting resolution, and are very robust versus model complexity and data quality [10]. People have widely used body-wave tomography in estimating depth and morphology of the bedrock, and defining geologic boundaries and property distribution in sedimentary environments .

Surface-wave analysis methods have disadvantages, and they are ill-posed, mix determined, strongly non-linear and suffer from severe solution non-uniqueness [7]. Besides, the vertical resolution of them decreases with depth rapidly and it is not sufficient in some areas.

Body-wave tomography method suffers problems too. On the one hand, the experiment data like first arrival traveltimes are hard to pick up precisely, particularly in some complex areas[10], and this will lead to big errors among results of inversion model and true model. On the other hand, the presence of low velocity layers can produce “hidden layers” and significant errors in the final velocity profile[10]. Furthermore, Strong non-uniqueness of solution affects its results as well.

Joint inversion of two kinds of different geophysical data traveltimes can reduce the ambiguities inversion of one in those two kinds of data, and make an improvement in reliability and accuracy of inverse result. Several joint inversion methods have been proposed in near-surface geophysics. Herig et al.[4] gave an iteratively reweighted least-squares algorithm to perform a joint inversion of surface-wave and electrical[4]. Dal Moro[1] adopted a joint inversion of refraction-wave traveltimes and surface-wave dispersion curves using a biobjective evolutionary algorithm. He showed that intrinsic ambiguities related to hidden layers could be mitigated with this approach[1]. Schuler made an inversion of dispersion curves of Love and Rayleigh waves and P-and S-wave first arrivals to calculate parameters of a layered earth model with increasing velocity with depth. Piatti et al. adopted a damped least square algorithm in inversion of surface-wave phase velocities and P-wave refraction traveltimes[9].

In order to verify the effect of joint inversion, we present here a joint inversion of Love-wave dispersion curves and SH-wave first arrival traveltimes using a damped least-squares algorithm. We apply the inversion to two synthetic data sets. The inverse results are compared with inverse model of Love-wave dispersion curves or SH-wave first arrival traveltimes.

II. JOINT INVERSION ALGORITHM

The Love-wave dispersion curves and SH-wave first arrival traveltimes are inverted using damped least-squares algorithm. We assume the underground is a 1D lateral homogenous Earth model lateral homogenous Earth made up of $n+1$ elastic layers (including the half-space) in the inversion. Each layer is parameterized with SH-wave velocity, density and thickness. The parameters are totally $3n+2$.

The objective function can be expressed as:

$$\Phi = \left\| \mathbf{J}\Delta\mathbf{X} - \Delta\mathbf{d} \right\|_2 \mathbf{W} \left\| \mathbf{J}\Delta\mathbf{X} - \Delta\mathbf{d} \right\|_2 + \alpha \left\| \Delta\mathbf{X} \right\|_2^2 \tag{1}$$

In Eq.(1), $\Delta\mathbf{d}$ represents the difference between model forward response and the measured data $\mathbf{d}_{obs}$, and $\Delta\mathbf{X}$ represents the correction of the model.

$$\mathbf{d}_{obs} = [V_{L1}, V_{L2}, \cdots, V_{Lm_1}, t_1, t_2, \cdots, t_{m_2}] \quad (2)$$

$$\mathbf{g}(\mathbf{X}) = [\mathbf{g}_{LW}(\mathbf{X}) \quad , \mathbf{g}_{ST}(\mathbf{X})] \quad (3)$$

$$\mathbf{X} = [V_{SH1}, V_{SH2}, \cdots, V_{SHn}, h_1, h_2, \cdots, h_{n-1}, \rho_1, \rho_2, \cdots, \rho_n] \quad (4)$$

As specified in Eqs.(2–4): $\mathbf{d}_{obs}$ is the experimental data set, which includes both dispersion curves (m_1 points) and SH-wave first arrival traveltimes (m_2 points); V_L and V_{SH} denote Love-wave phase velocities and the SH-wave velocities respectively; t represents the SH-wave first arrival traveltimes; h and ρ are thickness and density of layers.

As reported in Eq.(3), the forward responses $\mathbf{g}(\mathbf{X})$ includes both Love waves $\mathbf{g}_{LW}(\mathbf{X})$ and SH-wave $\mathbf{g}_{ST}(\mathbf{X})$. The terms $\mathbf{g}_{LW}(\mathbf{X})$ and $\mathbf{g}_{ST}(\mathbf{X})$ represent Love-wave dispersion curves and SH-wave first arrival forward algorithms respectively.

The model being updated at the kth iteration can be expressed as follows:

$$\mathbf{X}^{k+1} = \mathbf{X}^k + (\mathbf{A}^T\mathbf{A} + \alpha\mathbf{I})^{-1}\mathbf{A}^T\Delta\mathbf{d} \quad (5)$$

where **A** is the Jacobian or sensitivity matrix, and α is the Marquart damping factor [6].

III. Synthetic Examples

The synthetic datasets for the two models (Table 1 and Table 2) are acquired using the same forward responses in the inversion, then added overlaid with a white Gaussian noise. The dispersion curve is calculated in the frequency band from 1Hz to 100 Hz, and the first arrival traveltimes are computed using 100 receivers. The geophone interval and the nearest offset are both 1m.

The individual inversions are performed with using same damped least-squares approach which is also adopted by the joint inversion. We also use the same initial model which was defined according to surface-wave phase velocities[5]. Mass densities are fixed as priori information because dispersion curves are poorly sensitive to density variations[3] and SH-wave traveltimes do not depend on those parameters.

The first synthetic model [5],Model 1, is a six -layered model (TABLE I).

TABLE I. Parameters of a Six-layered Earth Model

Layers	Thickness(m)	SH-wave velocity (m/s)	Density (g/cm^3)
1	2.0	194	1.82
2	2.3	270	1.86
3	2.5	367	1.94
4	2.8	485	1.96
5	3.2	603	2.02
Half space	∞	740	2.09

The model, Model 2 (TABLE II)[2] is a 15m-thickness sand deposit overlying a homogeneous half space. This model belongs to a hidden layer case, which leads to erroneous velocity inversion and wrong interpretation on refraction traveltimes.

TABLE II. Parameters of a Three-layered Earth Model

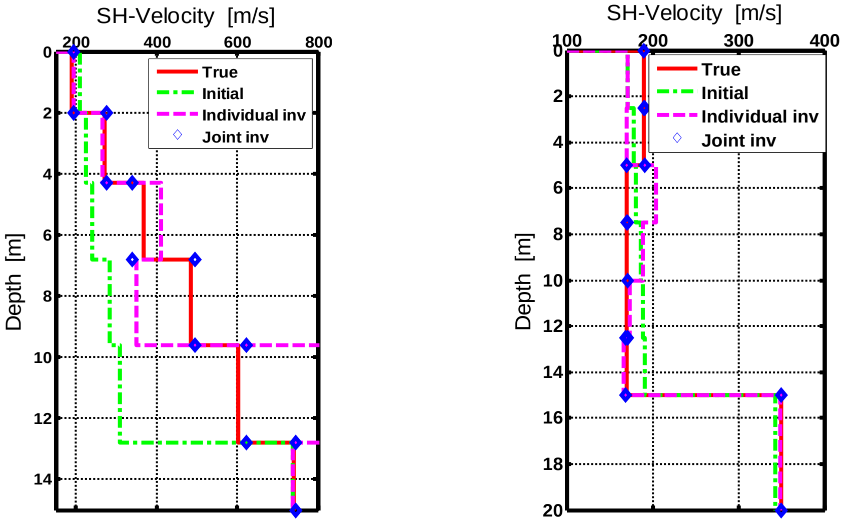
Layers	Thickness(m)	SH-wave velocity (m/s)	Density (g/cm3)
1	5.0	190	2.65
2	10.0	170	2.65
Half space	∞	350	2.65

The joint inversion method for Model 1 (Fig. 1.(a) and Fig.2) shows a quite good performance on inversion of V_{SH}. The V_{SH} profile obtained by the joint inversion method is globally more accurate in V_{SH} than the profile obtained by the individual inversion method. The final models by the two individual inversion methods possess a big deviation as far as the fourth and fifth layers' velocity comparing to the true model, but joint inversion method could generate a more accurate velocity model. Fig.2 indicates that, final models' forward responses are all in agreement with the synthetic data.

The V_{SH} profiles obtained through the joint and individual inversion methods for synthetic Model 2 are shown in Fig.1.(b). The fitting between the data and the forward responses for all the final models are shown in Fig.3. Those responses are all in good fit with the data. The final models obtained by the individual inversion methods, however are not consistent with the true model, and the joint inversion method provides a better final model. The velocity values of joint inversion method are more accurate than individual inversion methods. The thicknesses of the layers and the interface positions are better identified by the joint inversion method. The V_{SH} profile obtained by the joint inversion suggests the presence of a hidden layer in the true model.

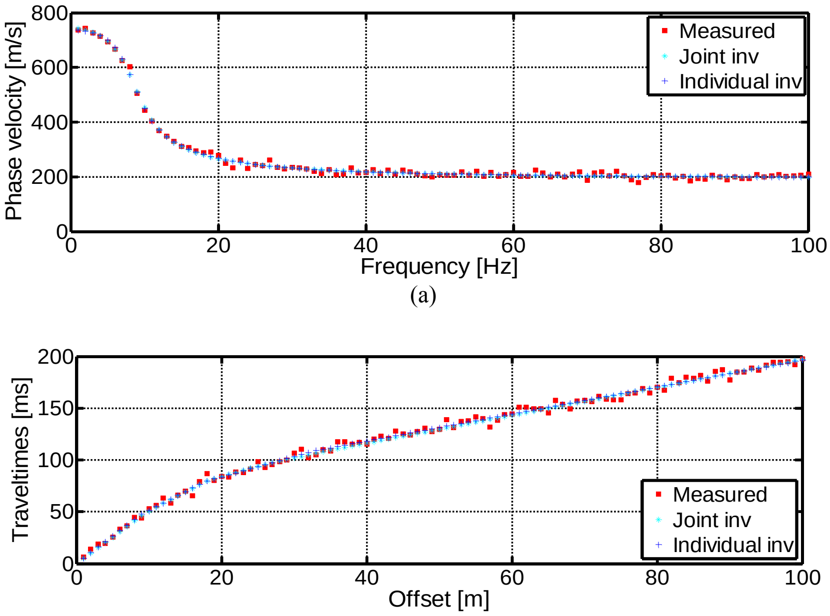
IV. Conclusions

It is demonstrated that the proposed joint inversion of Love-wave dispersion curves and SH-wave first arrival can improve the accuracy of the inversion results: better estimation of thickness and velocity values of layers, more precise distinction of layers' interfaces, and accurate inversion of hidden layers' position. Comparing with other joint inversion methods, this method is simpler, because its datasets can be obtained from the same seismic data and only calculates one same parameter, V_{SH} velocity. The algorithm has been applied to a simple 1D synthetic model, and the extension to a field data or 2D case will be implemented in the future.



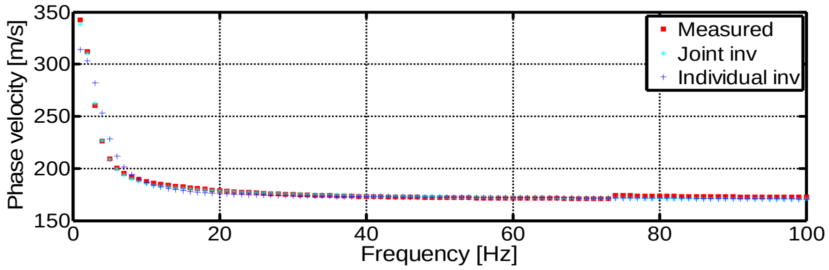
(a) (b)

Fig. 1. VSH final profiles comparison for joint and individual inversions with initial and true profiles. (a) Final models for Model 1 of Table 1; (b) final models for Model 2 of Table 2.

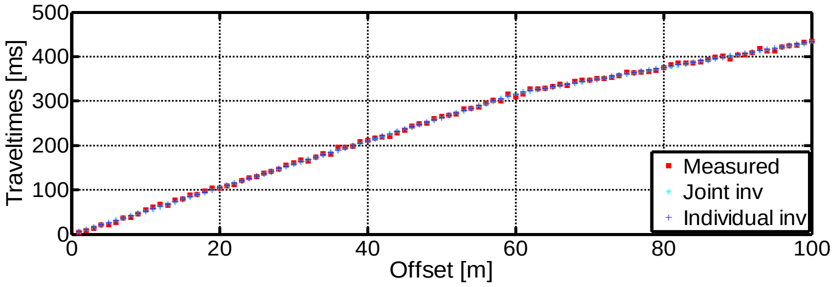


(b)

Fig. 2. Fitting for Model 1 of Table 1 between the data and the forward responses of the final model of individual inversion methods. (a) Comparison among experimental dispersion curves and final models responses, (b) comparison among experimental SH-wave first arrival traveltimes and final model forward responses.



(a)



(b)

Fig. 3.Fitting for Model 2 of Table 2 between the data and the forward responses of the final model of individual inversion methods. (a) Comparison among experimental dispersion curves and final models responses; (b) comparison among experimental SH-wave first arrival traveltimes and final model forward responses.

V. Reference

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