Anisotropic CSAMT responses of 2D model stimulated by infinite line transmitter

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***Abstract*—Based on Maxwell equations with source terms, EM responses for infinite line source of 2-D symmetrical anisotropic formation in frequency domain are computed. The partial differential equation solutions based on a finite element method of electric field parallel to line source for TE mode and magnetic field parallel to line source for TM mode are derived and calculated. The result shows that the TM mode can present the geoelectricl variations of earth model properly, while TE mode is less reliable compare with TM mode for situation of infinite line source. The value of TE mode is much larger than TM mode to at least two decades. By calculating the 2-D anisotropic anomalous body in either homogeneous half-space or one-dimensional layered media, the isotropic background and anisotropic anomalous body can be clearly figured out. By changing the anisotropic coefficient and rotation angle from main electrical axis of anisotropic model, it is obvious different from anisotropic case both for apparent resistivity and impedance phase curves.**

Keywords—anisotropic, CSAMT, responses, stimulation, infinite line transmitter

I. Introduction

Since the underground electric structures within a certain range of scales always show anisotropic, if we still carry out the process according to the isotropic assumption, we will sometimes get wrong interpretation results[1]. So the study of electrical anisotropy has an important theoretical and application value for CSAMT method.

For the present electrical anisotropy in electromagnetic exploration data, it has been the focus of attention of scholars at domestic and overseas. Pek et al. (1997) introduced the finite difference method to solve the problem of two-dimension electrical anisotropy of MT[2]. Xiong (1989) showed the results of horizontal dipole source excitation in on vertical anisotropic medium [3]. Li et al. (1991) discussed the forward and inversion problems of layered formation of CSAMT with azimuthal anisotropy[4]. Løseth et al. (2007) proposed the theory of numerical calculation of layered anisotropic medium in wave number domain by horizontal electric dipole excitation[5], the algorithm is based on propagator matrix to calculate the electromagnetic field. Puzyrev et al. (2013) used the finite element method to achieve the parallel computing of three-dimension anisotropy marine electromagnetic model data[6]. However, for the calculation of CSAMT electromagnetic field response on electrical anisotropic formations by infinite source excitation has not yet been reported. The paper, based on the predecessors calculation, applied the two-dimension finite element method to calculate frequency domain electromagnetic responses on anisotropic medium excited by infinite line source and compared the results with the isotropic situation, and it has an important guiding significance to improve CSAMT exploration effect.

II. Forward theory of CSAMT of two-dimensional electr-ical anisotropy and the solution of the Finite Element Method

In the Cartesian coordinate system, it is assumed that infinite line source and two-dimensional anisotropic electrical anomalous body is in the direction parallel to the axis x , the axis z is perpendicular down to the plane xy . Let surrounding rock background model be isotropic medium, two-dimensional anisotropy parameters is set to be the spindle electrical conductivity $\sigma_x = \sigma_z > \sigma_y$,the rotation angle between measurement axis x and the electrical spindle axis x' is θ (assume xy measuring plane and electrically principal plane are parallel to the horizontal plane), so the conductivity tensor can be expressed as:

$$\sigma = \begin{pmatrix} \sigma_x\cos^2(\theta)+\sigma_y\sin^2(\theta) & \sin(\theta)\cos(\theta)(\sigma_y-\sigma_x) & 0 \\ \sin(\theta)\cos(\theta)(\sigma_y-\sigma_x) & \sigma_x\sin^2(\theta)+\sigma_y\cos^2(\theta) & 0 \\ 0 & 0 & \sigma_z \end{pmatrix} \quad (1)$$

From (1), Maxwell's equations can be written as (taking into account the fact that the structural trends parallel to the axis, so $\partial/\partial x = 0$):

$$\begin{cases} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = i\omega\mu H_x \\ \dfrac{\partial E_x}{\partial z} = i\omega\mu H_y \\ -\dfrac{\partial E_x}{\partial y} = i\omega\mu H_z \end{cases} \quad (2)$$

Where, I is unit matrix, E_x and E_y are horizontal and vertical component of electric field strength ,respectively (unit V/m), j is current density (unit A/m^2), ω is angular frequency, μ is magnetic permeability, and ε is the dielectric constant.

$$\begin{cases} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma_{11}E_x + \sigma_{12}E_y + J_S \\ \frac{\partial H_x}{\partial z} = \sigma_{21}E_x + \sigma_{22}E_y \\ -\frac{\partial H_x}{\partial y} = \sigma_{33}E_z \end{cases} \quad (3)$$

Where H_x and H_y are horizontal component and the vertical components of magnetic field intensity (unit A/m), respectively,

$$\sigma_{11} = \sigma_x \cos^2(\theta) + \sigma_y \sin^2(\theta) - i\omega\varepsilon$$
$$\sigma_{22} = \sigma_y \cos^2(\theta) + \sigma_x \sin^2(\theta) - i\omega\varepsilon$$
$$\sigma_{12} = \sigma_{21} = \sin(\theta)\cos(\theta)(\sigma_y - \sigma_x)$$

Formula (2) and (3) can be simplified to:

$$\frac{1}{i\omega\mu}\left(\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) + \left(\sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}}\right)E_x + \frac{\sigma_{12}}{\sigma_{22}}\frac{\partial H_x}{\partial z} + J_S = 0 \quad (4)$$

$$\frac{1}{\sigma_{33}}\frac{\partial^2 H_x}{\partial y^2} + \frac{1}{\sigma_{22}}\frac{\partial^2 H_x}{\partial z^2} + i\omega\mu H_x - \frac{\sigma_{12}}{\sigma_{22}}\frac{\partial E_x}{\partial z} = 0 \quad (5)$$

Use the finite element method to solve the equation. The entire calculation area will be divided into a number of small rectangular area units, and then conduct bilinear interpolation in each grid:

$$\begin{aligned} &\iint_\Omega \frac{1}{i\omega\mu}\left(\frac{\partial N}{\partial y}\frac{\partial E_x}{\partial y} + \frac{\partial N}{\partial z}\frac{\partial E_x}{\partial z}\right)dydz - \iint_\Omega N\left(\sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}}\right)E_x dydz \\ &-\iint_\Omega N\frac{\sigma_{12}}{\sigma_{22}}\frac{\partial H_x}{\partial z}dydz - \iint_\Omega NJ_S dydz = 0 \end{aligned} \quad (6)$$

$$\begin{aligned} &\iint_\Omega \left(\frac{1}{\sigma_{33}}\frac{\partial N}{\partial y}\frac{\partial H_x}{\partial y} + \frac{1}{\sigma_{22}}\frac{\partial N}{\partial z}\frac{\partial H_x}{\partial z}\right)dydz \\ &-\iint_\Omega i\omega\mu N H_x dydz + \iint_\Omega \frac{\sigma_{12}}{\sigma_{22}} N\frac{\partial E_x}{\partial z}dydz = 0 \end{aligned} \quad (7)$$

Two formulas above are final calculation formulas for CSAMT to process two-dimension anisotropic conditions, Ω is unit area, N presents the interpolation function matrix and $N = (N_1, N_2, N_3, N_4)$.

In order to using finite element method to solve formula (6) and (7), we will do unit analysis and obtain the solved linear equations:

$$\begin{pmatrix} S_{nm} & T_{nm} \\ -T_{nm} & W_{nm} \end{pmatrix}\begin{pmatrix} E_{xn} \\ H_{xn} \end{pmatrix} = \begin{pmatrix} P \\ Q \end{pmatrix} \quad (8)$$

Where,

$$S_{nm} = \iint_\Omega \left(\frac{1}{i\omega\mu}\left(\frac{\partial N_n}{\partial y}\frac{\partial N_m}{\partial y} + \frac{\partial N_n}{\partial z}\frac{\partial N_m}{\partial z}\right) - N_n\left(\sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}}\right)\right)dydz$$

$$W_{nm} = \iint_\Omega \left(\frac{1}{\sigma_{33}}\frac{\partial N_n}{\partial y}\frac{\partial N_m}{\partial y} + \frac{1}{\sigma_{22}}\frac{\partial N_n}{\partial z}\frac{\partial N_m}{\partial z} - i\omega\mu N_n\right)dydz$$

$$T_{nm} = -\frac{\sigma_{12}}{\sigma_{22}}\iint_\Omega N_n\frac{\partial N_m}{\partial z}dydz$$

Q is all zero column vector, P is column vector containing source.

Since the near field source is singular, the paper loaded source by pseudo function δ , the expression of infinite line source is as follow:

$$J_S = I\delta_y(y)\delta_z(z) \quad (9)$$

Where I is for the current strength, $\delta_i(i)(i = y, z)$ is the pseudo function of δ .

By solving linear equations, and we can obtain the node value of E_x and H_x ,then substitute them into the formula (10) and (11) and the formula to solve the node values of E_y and H_y

$$H_y = \frac{1}{i\omega\mu}\frac{\partial E_x}{\partial z} \quad (10)$$

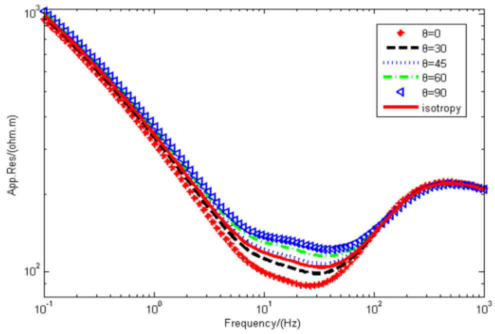
$$E_y = \frac{1}{\sigma_y - i\omega\varepsilon}\frac{\partial H_x}{\partial z} \quad (11)$$

For a two-dimensional infinite line source, the calculated results of TE mode is unreliable, so the following only discusses TM mode, which is equivalent to the result of the scalar CSAMT way.

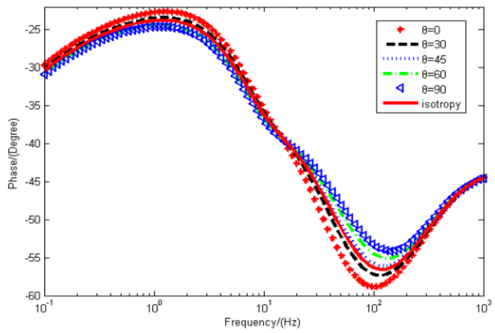
III. MODEL CALCULATION

A. *Results of Anisotropic Two-dimension Anomalies of Different Depths in Homogeneous Half Space*

Model set: embed a two-dimensional rectangular cylinder in a homogeneous half-space, its top surface depth is 400m, the scale in x and z directions is $500\,(m) \times 200\,(m)$, the vertical direction is unlimited extension along the y axis, the resistivity value is $\rho_x = \rho_z = 10\Omega \cdot m$, $\rho_y = 20\Omega \cdot m$, take the angle of rotation $\theta = 0、30^\circ、45^\circ、60^\circ、90^\circ$. Uniform half-space resistivity is $200\,\Omega \cdot m$. Fig. 1 shows apparent resistivity and impedance phase curves of two-dimensional anisotropic media model in the depth of 400 meters with the same anisotropy parameters and different rotation angles. Abscissa is frequency (unit HZ), vertical axis is apparent resistivity or impedance phase (unit degree). Meanwhile the figure also compared with the results under conditions of equivalent isotropic.



(a)



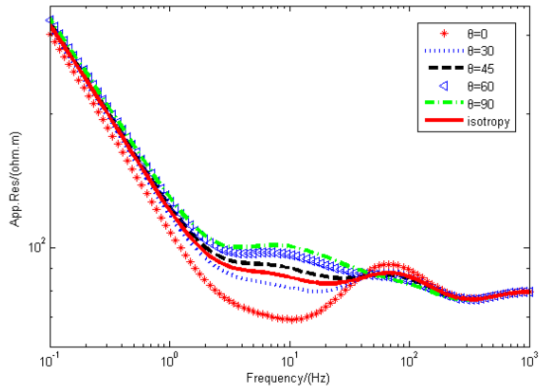
(b)

Fig. 1. Apparent resistivity and phase curves of 2D anisotropic model berried 400m beneath the surface under the same anisotropy coefficient and different rotation angles

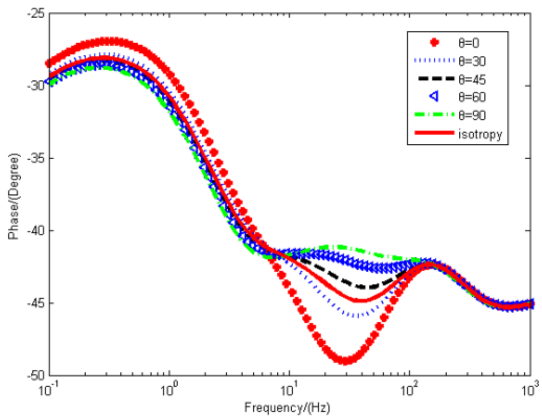
As can be seen from Figure 1, the effect of anisotropy is large. The value of high frequency in the apparent resistivity curve (Fig. 1a) is close to 200 $\Omega \cdot m$, with frequency decreasing, it gradually deviates from the true value, at last, the curve is straight up with the angle of 45° and eventually tend to overlap, which is equivalent to entering the near area of CSAMT. In the band of detecting abnormal body, when the angle is equal to 0 degrees, the resistance value is lowest, and as the angle increases, the resistivity value also increases. From the phase curve (Fig. 1b), the value of each curve in the high frequency band are close to -45°, and as the frequency decreases the curves gradually separate, and as the angle increases, the phase separated greater, curves intersect at around 15Hz, over the intersection of the curve, the curves slightly apart with the lower frequencies, the smaller angle is, the larger the phase separates, and finally each curve tends to overlap. Results of equivalent isotropic is between the angle 30° and 45°.

B. Calculation of Two-Dimension Anisotropic Layered Media Abnormal Body

Embed a square cylinder in a three-layered medium (K type), a first layer's thickness of 400 m, its resistivity of $80\,\Omega \cdot m$, the second layer's thickness of 600m, its resistivity of 600 $\Omega \cdot m$, the third layer's resistivity of $80\,\Omega \cdot m$, the depth of the top of 2D body is 600m, the scale of x and z directions is , resistivity value is $500(m) \times 200(m)$, resistivity value is $\rho_x = \rho_z = 5\Omega \cdot m$, $\rho_y = 15\Omega \cdot m$ the value of the rotation angles are $\theta = 0^\circ$、30°、45°、60°、90° , respectively. The apparent resistivity and impedance phase curves of the model are shown in Figure 3.



(a)



(b)

Fig. 2. Apparent resistivity and phase curves of 2D anisotropic model embedded in layered strata berried 600m beneath the surface under the same anisotropy coefficient and different rotation angles

As can be found From Fig. 2 that apparent resistivity curve shape is consistent with the set K models feature. After changing the angle of rotation, each curve separates at the site corresponding to the anomaly body, and as the angle increases, the resistivity value becomes larger, which reflects that the sensitivity of the model on anisotropic is more obvious, and the curve substantially coincides in high frequency band whose value is close to 80 $\Omega \cdot m$, which is consistent with the results of surface resistivity of theoretical model, after entering the near zone, it straights up with close 45 degree angle. Both phase curve (Fig. 3b) and apparent resistivity curve can reflect the changing feature of model forms, but the performance is more obvious than the apparent resistivity curve.

IV. CONCLUSION

This paper designed a series of geological model, studied electromagnetic response on two-dimensional anisotropic formation with different anisotropy angles and different anisotropy coefficients, and made a comparison with its corresponding isotropic model, which can go to the following points: the calculation results of the model indicate that in the case of the infinite long source, the results of TM mode is reliable and high precision, however, the results of TE mode are unreliable. This is equivalent to a scalar field exploration of CSAMT way. For two-dimension electrical anisotropy model, when the anomaly body shows anisotropy, their differences in regardless of the apparent resistivity or impedance phase compared with the isotropic case are both existed. The calculation results show that, in the case of anisotropic media, even if the anisotropy coefficient is small, it can not be approximated by isotropic case, the degree of anisotropy influence can not be ignored, in order to improve the interpretation accuracy and application effect of controlled source electromagnetic method, it is essential to use anisotropic theory for data processing and interpretation

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