

## An Overview of Neuronal Network and Neurodynamics

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**Abstract.** The nervous system is composed of a large number of neurons, and the electrical activities of neurons can present multiple modes during the signal transmission between neurons by changing intrinsic bifurcation parameters or under appropriate external forcing. In this review, the dynamics for neuron, neuronal network is introduced, for example, the mode transition in electrical activity, functional role of autapse connection, bifurcation verification in biological experiments, interaction between neuron and astrocyte, noise effect, coherence resonance, pattern formation and selection in network of neurons. Finally, some open problems in this field such as electromagnetic radiation on electrical activities of neuron, energy consumption in neurons are presented.

### Introduction

The Diseases associated with nervous system have brought much pain and anxiety for patients. As a result, clinical diagnosis such as big data analysis based on functional magnetic resonance imaging (fMRI) [1], computational neurodynamics, and cognitive neurodynamics have been paid much attention. It is believed that potential mechanism for abnormality [2] in nervous system could be discerned by detecting dynamic behavior of central nervous system and discovering general theories of cognitive functions in terms of theoretical biophysics. The brain is a high-dimensional nonlinear system and its collective electrical activities are dependent on signal propagation among one hundred billion neurons and also a variety of gliocytes.

Cognitive neurodynamics is considered capable of information processing in the nervous system. Theoretical models have been set up at different levels ranging from microcosmic, mesoscopic, up to macroscopic scale, so that the generation of cognition and potential mechanisms underlying neural information processing could be discerned. Indeed, the deterministic neuronal models could be available for bifurcation analysis, synchronization transition between neurons and networks, networks of networks, and information encoding. Furthermore, some reliable neuronal circuits could be implemented for signal detection as sensors. Some biological experiments and also theoretical neuronal models have shown that single neuron exhibits rich dynamical behaviors such as periodic spiking. Multistability is an important feature of the nervous system. For certain dynamical system with given parameters, it is considered that this system has multistable phenomenon [3] when the final developed state is dependent on the selection of initial values.

Multistability provides certain convenience for functional diversity of the nervous system, and the changes of initial values can alter final states of the system [4]. The application of stability theory could provide sufficient conditions for multi-resting states of the neuronal networks, and coexistence of multi-periodic states could be generated by applying appropriate peripheral external stimulation [5].

The topic about neural coding and decoding in neuroscience is the most challenging problem. Indeed, the nerve system develops its self-adaption that the neural activities in the brain must consume the minimal energy to keep normal activities during the process of signal transmission the brain's information-processing capacity is much dependent on the amount of energy supplied by blood flow in active brain areas. The technology of functional magnetic resonance imaging is available to discern the brain activity [6], but the activity of blood flow cannot be described exactly.

It is believed that neural information can be expressed by energy [7] that the energy can be used to unify the neural models of various levels. Unfortunately, the investigation about energy consumption in the nervous system has been carried on experiments instead of quantitative theoretical analysis due to the brain complexity. Interestingly, most of neuronal models are described by a variety of oscillators and we argued that dimensionless Hamilton energy function based on Helmholtz's theorem could be effective to detect the mode transition in electrical activities.

Neurons are widely accepted to be organized into networks, and neuronal networks exchange information through electrical and chemical synapses. Increasing evidences indicate that astrocytes are also organized into networks [8], and astrocyte networks are interconnected through gap junction channels. The channels are regulated by extra- and intracellular signals that enable the exchange of information. Based on these two networks, a recent review paper suggests the concept of "astroglial networks" [9]. Many recent modeling works focus on the neuronal synchronization in the astroglial network [9-10]. As an example, by integrating Norris-Lecar neuron model and Li-Rinzel calcium model, Amiri et al. constructed a model to study how astrocytes participate in the interplay between the pyramidal cells and interneurons [11]. Furthermore, they extended their three-unit model to a neuronal population model to study the effect of astrocyte on neuronal synchronization. Astrocytes are concluded to be capable of changing the threshold value of transition from synchronous to asynchronous behavior among neurons [12].

## Preliminaries

The information transmission between neurons is studied by using a model that contains two neurons and one astrocyte. First, we identify the parameter region in which the information can be transferred from N1 to N2. The effect of astrocyte does not influence this parameter region [13]. Secondly, in the parameter region for information transmission, we find BLSs in two neurons simultaneously. The parameter values for the occurrence of BLSs are also identified, and the results show that the higher expression level of mGluRs and the existence of astrocyte facilitate the occurrence of BLSs. Meanwhile, the rate for the occurrence of BLSs is calculated, and the rate is not very sensitive to the parameters. Third, time delay in information transmission is studied although glial cells have been widely accepted to serve an important function in synaptic transmission in neuron system, theoretical knowledge on the mechanism of interaction between glial cell and neurons is lacking. The modeling studies in this paper can help us to understand the mechanism by which the astrocytes participate in neuronal information transmission. Based on the WEH effect [13-14], we present a new method to liberate pinned spiral waves on heterogeneities. The effect of autapses on collective behaviors of neuronal network is detected by imposing time-delayed feedback current on membrane of neuron, and thus electric activities of neurons are regulated in close loop. As reported by previous works, the feedback autapse current is dependent on the gain and time delay. To approach realistic model, diversity in time delay (and gain) in autapses are considered. It is found that target waves can be induced and then some target waves can convert into spiral waves and regulate the collective behaviors of network like pacemaker. Under appropriate coupling intensity, spiral wave can coexist with target waves, or perfect spiral waves can grow up and regulate the network synchronously.

The free end of the emitted wave is generated by the REF itself from the rotating hyper-polarization instead of touching the refractory tail of the spiral in EP [15]. Consequently, the time window to apply pulse for successful liberation is much wider in REF than that in EP, which makes the REP more efficient. There are different heterogeneities in the cardiac tissue, such as blood vessels, fatty tissue, boundaries between regions of different fiber alignment directions, and intercellular clefts. Thus, during cardiac arrhythmia, various spiral waves with different

frequencies, phase, and rotation direction may be pinned in various obstacles with different size. That means it is impossible to find an optimal rotating frequency, phase, and rotation direction of REP. A possible treatment of actual application is just letting the EP rotate so that to increase the

efficiency, like that presented obviously. We address one point that the amplitude and duration of the REP is kept as that in EP, which indicates no additional energy is required while its efficiency is increased. We hope this strategy may improve manipulations with pinned spiral waves in related experiments, in cultured cardiac myocytes. Lastly we study the repulsive and attractive forces exerted on a spiral wave by the heterogeneity in an excitable medium. We apply external fields to push the spiral tip into the heterogeneity, or to push the pinned tip out of the heterogeneity. The critical values of the intensity of the external fields  $E_r$  and  $E_a$  are used to quantify the forces. We find that repulsive and attractive forces increase with the enhancement of the level of heterogeneity for both parametric heterogeneity and impermeable inclusions. Notably, the attractive force is much larger than the repulsive force for all parameter values, suggesting that unpinning of the spiral tip is more difficult than pinning. Finally, the comparison shows that, for small-sized heterogeneities, the level and the size of heterogeneity both influence the forces remarkably. However, for large-sized heterogeneities, the forces are independent of the size, but are influenced by the level of the heterogeneity. This work may shed some light on the control or suppression of spiral waves.

### Spectral Properties of the Temporal Evolution of Brain Network Structure

**FMRI Data Preprocessing.** The functional images were preprocessed based on AFNI (<http://afni.nimh.nih.gov/afni/>)<sup>34</sup> and the FSL software Library (<http://www.fmrib.ox.ac.uk/fsl/>). The first four volumes were excluded from analysis due to the initial stabilization of the fMRI signal. For each subject, motion correction was performed through a 3D image realignment with the AFNI program 3dvolreg function, which uses a weighted least-squares rigid-body registration algorithm. Echo planar imaging (EPI) images were motion and slice-time corrected and spatially smoothed using a Gaussian kernel of 6mm full width at half maximum (FWHM). The temporal band-pass filtering ( $0.009 \text{ Hz} < f < 0.1 \text{ Hz}$ ) was performed to reduce the effects of low-frequency drift and high-frequency physiological noise. After eliminating redundant information pertaining to cerebrospinal fluid (CSF) and white matter, fMRI data were further spatially normalized to the Montreal Neurological Institute (MNI) EPI template and resampled to a 3mm cubic voxel.

**Construction of Dynamic Brain Network.** The time series for each ROI was first collected by averaging the voxel time series within the ROI. Then, the dynamic Pearson correlation coefficients between the time series of all pairs of brain ROIs were computed based on a sliding window with a sliding step of one TR. The calculation was carried out as follows:

$$r_{X,Y}^t = \frac{\sum_{i=1}^N (X_i^{t+w} - \overline{X_i^{t+w}})(Y_i^{t+w} - \overline{Y_i^{t+w}})}{\sqrt{\sum_{i=1}^N (X_i^{t+w} - \overline{X_i^{t+w}})^2 \sum_{i=1}^N (Y_i^{t+w} - \overline{Y_i^{t+w}})^2}} \quad (1)$$

Where  $X^{t+w}$  and  $Y^{t+w}$  are the fMRI time series from time  $t$  to time  $t + W$  for different ROIs and  $\overline{X^{t+w}}$  and  $\overline{Y^{t+w}}$  are the average values corresponding to the time series, respectively. The  $W$  is the width of the sliding window,  $N$  is the total number of sampled points in each ROI, and the relation between  $N$  and  $W$  is TR.

**Random Matrix Theory.** RMT investigates the spectral fluctuation of complex systems by separating the intrinsic properties from the random universal part. To obtain more universal properties of spectral fluctuation, all eigenvalues must be placed on the same footing and work at a constant spectral density on average, which is performed by removing the spurious effects caused by variations in spectral density. Thus, it is customary in RMT to unfold eigenvalues  $k$  through transformation

$$\overline{\lambda}_i = \overline{N}(\lambda_i) \quad (2)$$

where  $\overline{N}(\lambda_i) = \int_{\lambda_{\min}}^{\lambda_i} \rho(\lambda') d\lambda'$  measures the probability of an eigenvalue falling within the intervals  $[\lambda_{\min}, \lambda_i]$ . Because the function  $N$  is unknown, the unfolding process is implemented with the help of numerical curve fitting. The NNSD calculated from complex systems can be predicted by the Brody formula

$$p_{\beta}(s_1) = A s_1^{\beta} \exp(-\alpha s_1^{\beta+1}) \quad (3)$$

where  $0 \leq \beta \leq 1$ . when  $\beta = 0$  NNSD follows Poisson statistic and  $p(s_1) = \exp(-s_1)$  which reflects the uncorrelated properties among spectra. In contrast, the spectra are correlated when NNSD obeys the Gaussian orthogonal ensemble(GOE) statistic for  $\beta = 1$  and  $p(s_1)$  satisfies

$$p(s_1) = \frac{\pi}{2} s_1 \exp(-\frac{\pi s_1^2}{4}) \quad (4)$$

## Discussion

Most previous studies on neural network analyzed the spectral properties of time-evolution functional connectivity within and between different brain sub networks at a sub-second resolution with the aim of understanding how to detect the non-stationary statistical behavior of the dynamic brain system. We first found that the shifting of eigenvalue distribution and the decrease in the largest eigenvalue are affected by visual stimulation. Then, using RMT, we not only predicted the universal behavior of the dynamic brain network but also demonstrated the effective role of long-range correlation among spectra in characterizing changes in the dynamic brain network caused by visual stimulation. To further analyze the dynamic brain network with RMT, we explored the long-range correlation among spectra. The greatest difference between short-range correlation and long-range correlation is that long-range correlation counts integrated eigenvalue fluctuations for more brain regions this result strongly indicates that long-range correlation indexes can gather small changes in contribution corresponding to brain regions and can effectively distinguish functional changes in the brain caused by visual stimulation. Further, we observed a more random WBN, which means that the visual stimulation activates the whole brain with a positive effect. The eigenvalues of complex systems can completely shed light on the whole properties and have different meanings in different systems, such as the intrinsic frequency in mechanical systems and energy levels in nuclear physics systems.

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