An physical force-driven packing optimization design method in strip packing problems

Xiaoping Liao[1,a], Shuaiyin Guo[2,b] and Chengyi Ou[2,c]

[1] Mechanical manufacturing &automation, science and technology department,

Guangxi University, Nanning, Guangxi, China

[2] Mechanical manufacturing &automation, college of mechanical & engineering,

Guangxi University, Nanning, Guangxi, China

[a] xpfeng@gxu.edu.cn , [b] 1398321860@qq.com, [c] 317676290@qq.com

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Abstract. This thesis put forward a physical force-driven packing optimization design method for solving the Strip Packing Problems (SPP). This thesis researched on the following aspects: The mathematical optimization model of SPP is proposed firstly. Based on the convex hull plus rubber band compact layout method, the method of physical analysis of the layout process and the time-based layout simulation process are presented definitely. An enhanced version of the mate algorithm of surplus rectangle for rectangle packing is proposed. This method use the minimal rectangle to replacing polygon objects and choose the next packing object by a series of score decision rules. Different forces are applied to the packing objects in corresponding stages which would drive them to move compactly and the optimal packing result can be obtained in the end. The comparison of computational experiment results shows that the proposed packing method have a better performance.

Introduction

As the development of society, how to realize the resource utilization rate maximization has become one of the problems to be solved. There are packing problem in sheet metal, shipbuilding, furniture plates and other industries, the packing is a technique that array production in the formulary master-batch by using some way to achieve the master-batch utilization maximization.

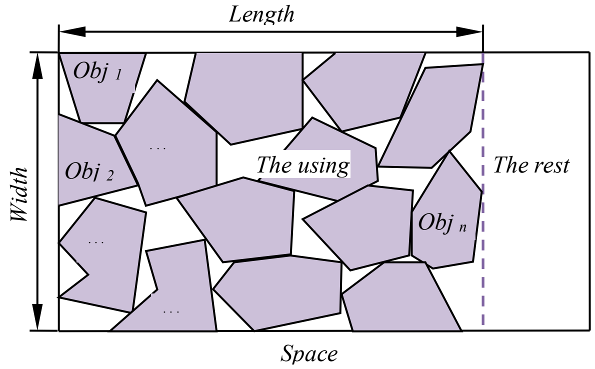
How to lay out all the components in the limited space and meet the assembly performance of the product, is a key problem of precise design of complex products and a challenge. In order to solve the key problem and improve production efficiency, quite one part modern enterprise utilized the computer-aided software for packing design, and the software is often through a packing optimization algorithm, thus the packing can obtain better results in the acceptable calculation time by combining computer and optimization algorithm than the manual.

The research of packing optimization algorithm is a recognized difficult problem in the field of international packing optimization [1]. In this paper, we are based on the deficiencies of packing optimization algorithms at this stage, mainly research correlative theory method and engineering application about the 2D compact layout design, a feasible physical force-driven packing optimization method is proposed in order to optimize the layout of 2D irregular polygons. the characteristics of the method are that change the traditional "one by one" sequence packing, all the packing objects are arranged together, and objects can be rotated at any angle on the plate.

Strip packing problems

SSP can be described that the objects $\{Obj_1, Obj_2 \dots Obj_n\}$ are arrayed in the proper position and angle in the plate *Space.* All the objects must be in the plate and can not overlap with each other, and the area of *The using* is minimum in the end(refer with: Fig. 1).

According to the conditions, the mathematical optimization model of strip packing problems can be proposed as Eq. 1.



$$
\begin{aligned}
&Find \;\; X^{*} = [X_1, X_2, ..., X_i, ..., X_n] \in R^{n} \\
&\min \;\; Length \\
&s.t. \;\; Obj_i \subset Space \\
&\qquad Obj_i \cap Obj_j = \Phi \\
&\qquad i, j = 1,2...n, \text{and} \, i \neq j
\end{aligned} \tag{1}
$$

Fig. 1 Diagram of strip packing problems

The *Width* is a constant and the number of objects is *n*. The position and angle of each object can be expressed as $X_i = [x_i, y_i, \theta_i]^T$,among $i = 1,2 \dots n$, x_i and y_i are the abscissa and ordinate of Obj_i barycenter, thus $X = [X_1, X_2, \dots X_i, \dots, X_n]^T \in R^n$ indicates that all the objects. That all the objects must be in the plate *Space* and can not overlap with each other are two constraints, they can respectively be expressed as $Obj_i \subset Space$ and $Obj_i \cap Obj_j = \Phi$, among $i = 1,2 \dots n, i \neq j$.

The quality evaluation of a packing scheme is usually Packing Density (PD)[2], PD is area utilization rate that is the ratio of the total area of objects and the area of *The using*, higher number indicates that packing scheme is better, maximum is 100%. The number of PD only is related with *Length*, when the *Length* is minimum the PD can get the maximum.

The force and motion analysis of convex hull plus rubber band compact layout process

In a certain sense, the packing optimization design of 2D parts is the process of obtaining the best compact layout by using the compact layout method. The convex hull plus rubber band compact layout method originate from a common physical phenomenon, a detailed description is:

(1) Firstly, a series of layout objects are converted to the " convex hull "and are free placed in the 2D layout plane, all layout objects are not mutually overlap, and then a virtual rubber band will bind the all the layout objects from the periphery(refer with: Fig. 2).

(2) Since the rubber band is stretched and needs to restore the original length, the contiguous layout objects are affect by the rubber band force, layout objects mutually move in the role of rubber band force. During the movement, the layout objects may collide with each other, and produce translation, rotation, sliding and other complex movements under the affect of combined force, and ultimately all objects draw close together ,a compact layout form.

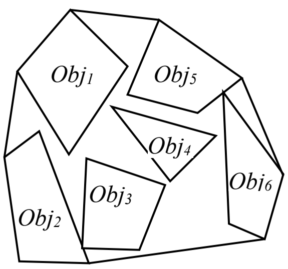


Fig. 2 Layout objects

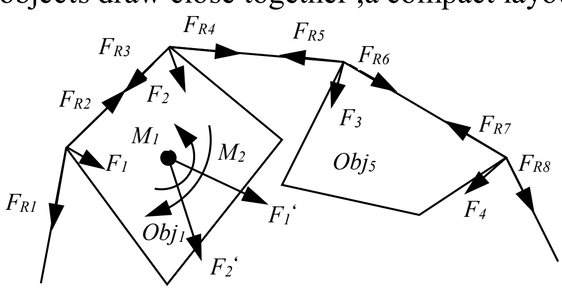


Fig. 3 Force analysis

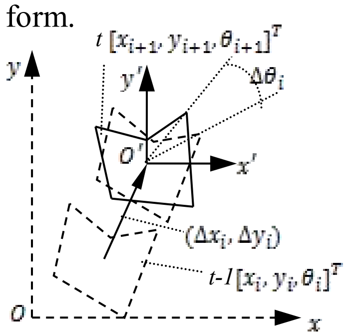


Fig. 4 Motion analysis

During the movement, there are three sorts of objects that act upon by forces(refer with: Fig. 3). First, the objects only act upon by rubber band force as the Obj_1. Second, the objects only act upon by interaction forces between the objects as the Obj_3. Three, the objects act upon by rubber band force and interaction forces as the Obj_2, Obj_5 and Obj_6. The Obj_4 doesn't get force.

The Obj_1 acts upon by $\boldsymbol{F_{R1}}$, $\boldsymbol{F_{R2}}$, $\boldsymbol{F_{R3}}$ and $\boldsymbol{F_{R4}}$, Rubber elastic force is in pairs, so the direction of the composite force is the angle bisector of each pair of rubber elastic force as $\boldsymbol{F_1}, \boldsymbol{F_2}$. The targets can be moved to the barycenter as $\boldsymbol{F'_1}, \boldsymbol{F'_2}$, but the couple $\boldsymbol{M}$ must attach as $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$, its torque is equal to the torque that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ act on barycenter. The composite force and couple of layout object Obj_1 are shown in Eq. 2.

$$\vec{F}_{total} = \sum \vec{F_i'} + \sum \overrightarrow{F_{obj}}$$
$$M_{total} = \sum M_i + \sum M_{obj} \quad (2)$$

Based on the force analysis, the layout objects have the motion analysis and can update the motion state. The objects' motion can be divided into two sections(refer with: Fig. 4):

(1) The translation motion in the xOy coordinates can be divided into the horizontal displacement Δx and vertical displacement Δy.

(2) The rotation motion is that object rotates $\Delta\theta_i$ around the origin of $x'O'y'$(barycenter).

Initial packing optimization design

The initial packing design method must take into account the balance of time efficiency and the packing results. The geometric complexity of the rectangle is much lower than that of the polygon, and the layout design of the rectangle is more simple, thus the initial packing design is performed by using the rectangle instead of the complex polygon. The packing process have two parts:

(1) Firstly, the smallest envelope rectangle of the objects can be solved, and replace the objects.

(2) Then initial packing design utilizes the envelope rectangle of the packing object by using mathematical programming method.

The rectangular is placed on the minimum residual rectangle region in each layout that utilizes the mate algorithm of surplus rectangle [3], but the packing process does not consider that placing the rectangular is or not optimal. Referring the game theory, the "Situation" has great changes in the packing process after putting rectangular into the region every time. So it is difficult to obtain the optimal packing results that follow the fixed packing sequence. Diagram of the initial packing design is shown in Fig.5.

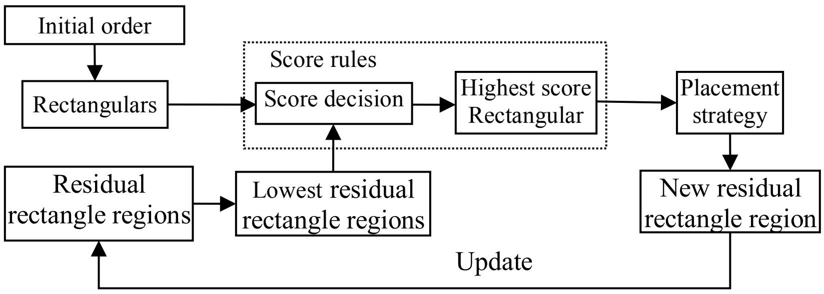


Fig. 5 Diagram of initial packing design.

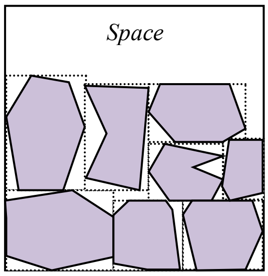


Fig. 6 Initial packing design result

The diagram shows, it is reasonable that choose the next packing object by a series of score decision rules, according to the current "Situation", timely to adjust the subsequent packing order is necessary. The initial packing design result refer with Fig. 6.

Physical force-driven packing optimization design

Packing objects only dispersedly put into the *Space* according to the prescribed manner in the initial packing design, there is a large gap between the objects, and the layout is not compact. Based on the initial packing design and compact layout method of physical analysis and simulation, the physical force-driven packing optimization design method is proposed, which is used to optimize the packing result and improve the utilization ratio of space.

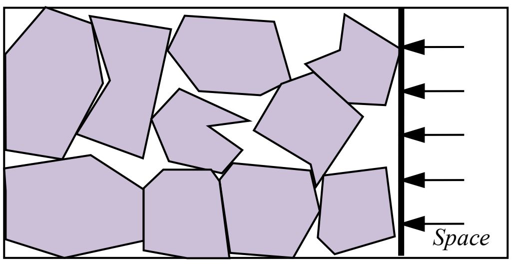


Fig. 7 Uniform force-driven stage

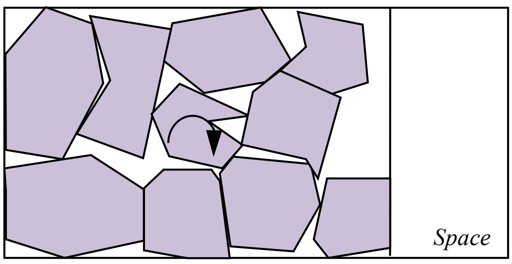


Fig. 8 Packing adjustment stage

The packing design process have four stages: Initial packing stage, Rubber band force-driven stage, Uniform force-driven stage, Packing adjustment stage.

(1) Firstly, the packing objects put into the space by using the initial packing method.

(2) Based on the initial packing, the rubber band force is applied to objects by building the rubber band model, the force can drive objects to the bottom of space along the length direction.

(3) After packing objects have balance in the rubber band force, the border is uneven on the other side of direction motion, the highest level of packing objects have decline through give the objects a uniform force. (refer with: Fig. 7)

(4) It will prevent packing process and thus fall into local solution that the objects may occur "stuck" in the physical force- driven method, the packing process can continue and further optimize the results by rotation or translation object. (refer with: Fig. 8)

Experiments

The computational experiments are necessary to verify the feasible and effective of the packing methods.

It can be directly seen from the Fig. 9, after finishing the initial layout design, most of the cases have achieved relatively good results with high time efficiency.PD1 is area utilization rate of using the initial packing method, Refer[5] indicates the area utilization rate after finishing the initial packing stage in Reference[5].

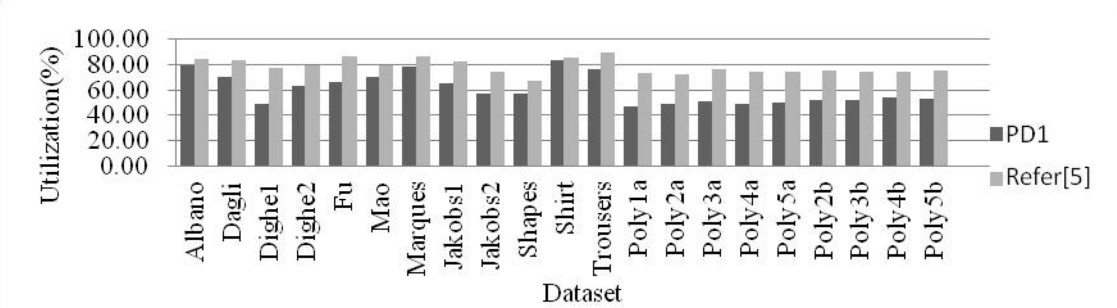


Fig. 9 The comparison of initial packing results.

In order to verify the physical force-driven packing design method, the computer experiments and the results obtained in the Reference[5] were compared(refer with: Table 1).

Table 1 The result comparison of different packing design methods

Dataset	PD2(%)	Refer[5](%)	T1(s)	T2(s)
Albano	85.00	84.55	16.834	93.30
Dagli	85.25	83.70	14.328	188.80
Dighe1	81.34	77.40	23.752	8.80
Dighe2	84.04	79.40	8.680	7.10
Fu	83.74	86.93	11.887	20.70
Mao	81.92	79.44	9.995	29.70
Marques	84.72	86.50	6.029	4.80
Jakobs1	76.18	82.60	8.170	43.40
Jakobs2	73.49	74.80	14.582	81.40
Shapes	68.97	67.68	36.62	31.40
Shirt	86.71	85.69	36.964	806.50
Trousers	85.51	89.65	76.761	3612.00
Poly1a	74.31	73.20	19.679	12.50

In the Table 1, PD2 is area utilization rate of using the proposed packing method, T1 is the total time of packing process, Refer[5] indicates the area utilization rate and T2 is computation time in Reference[5].

According to the physical force-driven packing design method, auto-layout system was developed by using C++ and Box2D.The system main interface is shown in Fig. 10.

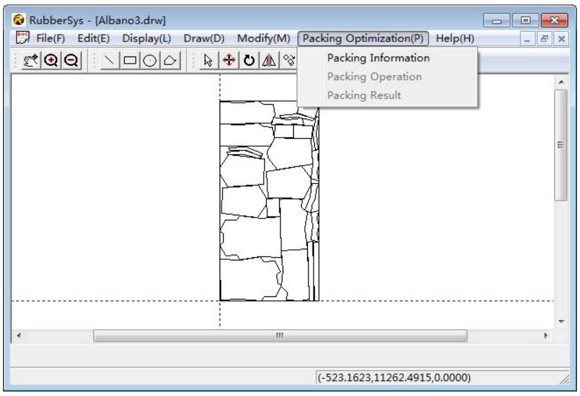


Fig. 10 System main interface

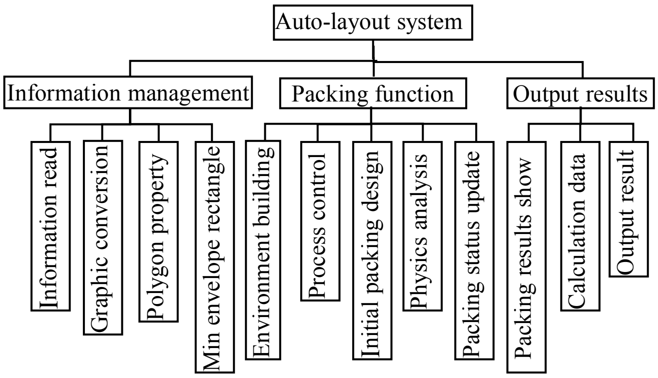


Fig. 11 Overall framework of the auto-packing system

Overall framework of the auto-packing system is shown in Fig. 11. The system packing process can be realized by the three sections: information management, packing function, output results.

Conclusions

In this paper, aiming at the strip packing problem of 2D irregular polygons, the initial packing design got optimized, the force and motion of packing objects got analysis, so a physical force-driven packing optimization design method was proposed, and the feasibility of the method was verified by computational experiments, utilizing the method had achieved relatively good results with high time efficiency, according to the method, auto-layout system was developed.

Acknowledgements

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