Dry Type Brake Billiard plate Structure Thermoelastic Coupling Vibration Analysis

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Abstract. This paper analyses the influence of the billiard plate stiffness by heat from the billiard plate dynamics equations and the brake temperature field distribution. Put forward the additional stiffness matrix calculation method through analysing the influence of elastic modulus by temperature and the influence of stiffness by thermal stress. Obtain the thermal vibration analysis results of billiard plate structure, under the influence of temperature and the pre-stressed. Calculation has been completed through the finite element and numerical iterative. This paper provides a meaningful reference for brake design.

Introduction

Working process is often accompanied by vibration for dry type brake. In the process of actual use, vehicle brakes often due to the high speed vehicle, braking frequently makes heating temperature rising sharply, which brings us difficulties to study the brake vibration characteristics. The main effect of temperature on the structure vibration characteristics embodied in two aspects: On the one hand reduces the elastic modulus of structure material; on the other hand, the uneven heat stress, eventually changes the structure stiffness characteristics. Billiard-plate Structure is loading device on dry type brake. The structure is shown in Fig. 1.

Figure 1 Structure of Billiard plate

The device through the ball and the ball groove convert rotary motion to linear motion, which can greatly increase the load pressure. Billiard-plate stability directly affects the stability of load pressure. Due to the high temperature, the thermal vibration effect needs further analysis. The brake structure diagram is shown in Fig. 2.
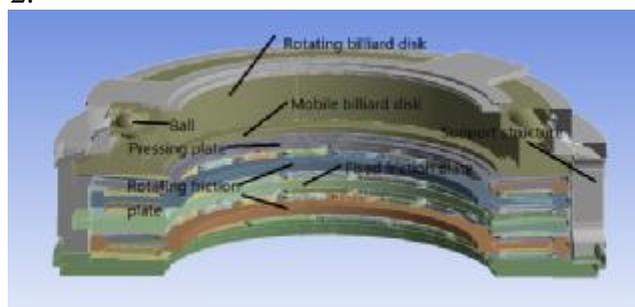
Figure 2 Brake structure

The thermal vibration is mostly focused on the rectangular plate achievements, for example, circular plate and elliptical plate on the establishment and solving the basic equations of thermal vibration. Equation established from the perspective of energy, using Hamilton principle or directly

from the force balance to build. The researchers often adopt finite element method and finite difference method. Noor [1] and Reedy [2] has made a detailed review on static and linear thermoelastic analysis. Chang [3, 4] made a lot of work in thermoelastic coupling vibration of rectangular plates. Huang and Tauchert [5] analyzed on the angle of plates layer heated geometric nonlinear problem, but limited to the static. Chan [6] analyzed on the nonlinear static and dynamic response of the heated laminated plates, but based on the theory of mode 1. Kant [7] established thermal vibration model based on the theory of high mode. Spain, Soistmann and Linville [8] calculated two different materials of the shuttle's whole machine model under the condition of flight Mach number 6, aerodynamic heating effect on 5 modes natural frequency.

The influence of temperature on the vibration characteristics are implemented through influencing structural stiffness. Heating can weaken the structure stiffness from two aspects, one is reducing of the material elastic modulus, and the second is the stiffness change caused by the temperature gradient. Usually we adopt linear elastic theory analysis. Thermal stress as surface structure internal force will cause additional stiffness matrix (called the geometric stiffness matrix) change. When the thermal stress change is big, the additional stiffness matrix significantly changed the whole stiffness characteristics.

The establishment of the billiard plate dynamics equation

As shown in Fig. 3 is the billiard plate structure force analysis. The boll's radius is R. Rotating billiard plate subjected to force (F_1). Mobile billiard plate's circumferential direction is constrained. The rolling friction are f_1 and f_2. Groove angle is a.
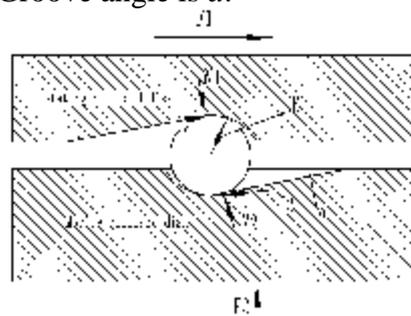
Figure 3 Billiard plate structure force analysis

Assuming that bolls do pure rolling motion billiard plate rotates. General dynamics relationship of billiard plate structure is shown in Fig. 4.
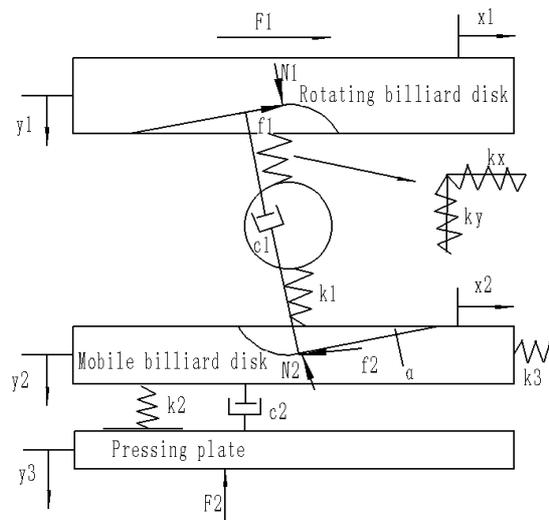
Figure 4 Billiard-plate dynamic model

The five degrees of freedom model is established. In fact y_1 freedom degree we don't focus on, if the support stiffness is sufficient. So actually four degrees of freedom system is established. According to the second class Lagrange's equation, the general form of the equation is:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}+\frac{\partial V}{\partial q_j}=Q_j^*$$

(1)

Where T is the kinetic energy of the system, V is the potential energy of the system, q_j is generalized coordinates of the system, $\dot{q}_j$ is generalized velocity, Q_j^* is generalized force of the system.

Ignoring the quality and the moment of inertia of the balls, kinetic energy and potential energy expression are as follows:

$$T=\frac{1}{2}m_1\dot{x}_1^2+\frac{1}{2}m_1\dot{y}_1^2+\frac{1}{2}(m_2+m_3)\dot{x}_2^2+\frac{1}{2}m_2\dot{y}_2^2+\frac{1}{2}m_3\dot{y}_3^2$$

(2)

$$V=\frac{1}{2}k_1(x_1-x_2)^2\left(\frac{\tan\alpha}{\cos\alpha}\right)^2-\frac{1}{2}k_1\frac{(y_2-y_1)^2}{\cos^2\alpha}+\frac{1}{2}k_2(y_2-y_3)^2+\frac{1}{2}k_3x_2^2-gm_1y_1-gm_2y_2-gm_3y_3$$

(3)

Due to the effect of constraint of the structure, it is assumed that stiffness of the supporting body enough. So the y_1 don't change. The generalized coordinates are:

$$\begin{cases} \frac{\partial V}{\partial x_1}=k_1(x_1-x_2)\left(\frac{\tan\alpha}{\cos\alpha}\right)^2 \\ \frac{\partial V}{\partial x_2}=k_1(x_2-x_1)\left(\frac{\tan\alpha}{\cos\alpha}\right)^2+k_3x_2 \\ \frac{\partial V}{\partial y_2}=\frac{k_1y_2}{\cos^2\alpha}+k_2y_2-k_2y_3-m_2g \\ \frac{\partial V}{\partial y_3}=k_2y_2+k_2y_3-m_3g \end{cases}$$

(4)

Generalized forces which corresponding to the generalized coordinates are:

$$\begin{cases} Q_{x_1}^*=F_1-c_1(\dot{x}_1-\dot{x}_2)\frac{\tan\alpha}{\cos\alpha} \\ Q_{x_2}^*=c_1(\dot{x}_1-\dot{x}_2)\frac{\tan\alpha}{\cos\alpha}-F_3 \\ Q_{y_2}^*=c_1\dot{x}_1\left(\frac{1}{\tan\alpha}-\tan\alpha\right)-c_2(\dot{y}_2-\dot{y}_3) \\ Q_{y_3}^*=c_2(\dot{y}_2-\dot{y}_3)-F_2 \end{cases}$$

(5)

Then the matrix form of the dynamic differential equation is:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2+m_3 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix}$$

$$+\begin{bmatrix} c_1\frac{\tan\alpha}{\cos\alpha} & c_1\frac{\tan\alpha}{\cos\alpha} & 0 & 0 \\ -c_1\frac{\tan\alpha}{\cos\alpha} & c\frac{\tan\alpha}{\cos\alpha} & 0 & 0 \\ -c_1\left(\frac{1}{\tan\alpha}-\tan\alpha\right) & 0 & c_2 & -c_2 \\ 0 & 0 & -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix}$$

$$+\begin{bmatrix} k_1\left(\frac{\tan\alpha}{\cos\alpha}\right)^2 & -k\left(\frac{\tan\alpha}{\cos\alpha}\right)^2 & 0 & 0 \\ -k_1\left(\frac{\tan\alpha}{\cos\alpha}\right)^2 & k_1\left(\frac{\tan\alpha}{\cos\alpha}\right)^2+k_3 & 0 & 0 \\ 0 & 0 & \frac{k_1}{\cos^2\alpha}+k_2 & -k \\ 0 & 0 & k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ y_2 \\ y_3 \end{bmatrix}$$

(6)

Above model is based on the traditional dynamic analysis, which does not consider the heat effect on the stiffness. In addition the brake vibration process is usually under the compression state, so the influence of pre-stress on the system stiffness must be considered. Additional stiffness matrix calculation process is shown in Fig 5.
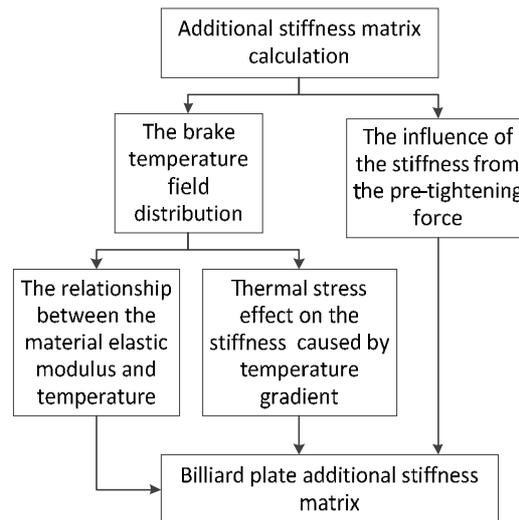
Figure 5 Additional stiffness matrix Calculation

Both temperature gradient generated by thermal stress and pre-tightening force generated by the pre-stress, there is no difference in nature. So in the paper we put the two aspects of influence together for discussion.

The brake temperature field distribution and the impact analysis

The brake thermal field is calculated with finite element method. Brake three-dimensional finite element model is established in this paper.

Table 1 Basic parameters

Vehicle Quality(T)	Number of brake	Number of friction pair	The friction pair area (m^2)
50	2	4	4.48×10^{-2}

Through the analysis of vehicle operation condition complete the calculation of heat flux. Define the convective heat transfer coefficient to determine the heat flux distribution. We calculate the results of the brake temperature distribution under thermal fluid-solid coupling. The distribution of heat flux between the friction pair associates with convective heat transfer on the surface of the friction pair. Heat flow distribution coefficient not only related to the physical and chemical properties of materials, but also related to contact surface temperature gradient ratio. The table below is the material properties of friction materials.

Table 2 Material properties of Friction materials

structure	Density	Specific heat capacity	Coefficient of thermal conductivity
Rotating friction disk	6700	452	8
Fixed friction disk	7850	470	48

According to the working condition of the vehicle, the research got six different working condition, temperature distribution under the different conditions are quite different, so the influence of different working condition for billiard plate thermal vibration situation is different.

Table 3 Simulation Conditions

Working Condition	1	2	3	4	5	6
Braking acceleration(m/s^2)	-5	-4	-2.5	-4	-2.5	-5
Braking time(s)	3	2.7	4.4	2.0	3.3	1.0
Thermal power (W$\times 10^5$)	4.2	2.5	1.5	1.9	1.2	1.8
Heat flow density (w/m$^2 \times 10^5$)	12	7.0	4.2	5.2	3.3	5.0

According to the working condition of the above six through ANSYS-Workbench separately carried on the brake temperature field simulation analysis, the result is shown below.
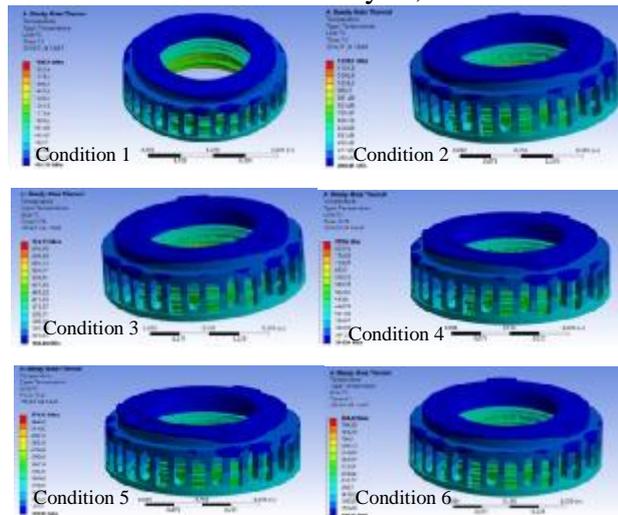
Figure 6 Calculation results of various kinds of working condition

Table 4 Maximum Thermal Stress of Billiard plate structure

Working Condition	1	2	3	4	5	6
Max thermal stress(MPa)	21.78	13.34	8.16	10.49	6.35	1057

From the point of data of any of the above condition of simulation results, billiard plate structure temperature difference is big; billiard plate due to the structural constraint function at the same time, the thermal stress effect is obvious.
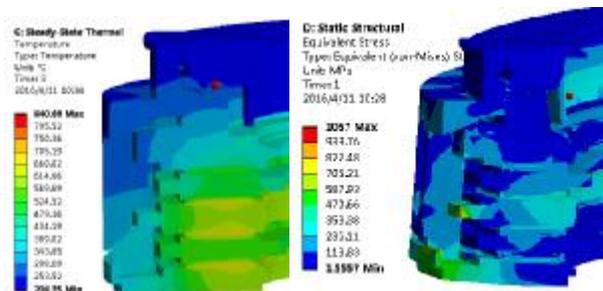
Figure 7 Temperature distribution and Structural Deformation

Table 5 Maximum and Minimum temperature data

structure	Condition 1		Condition 2		Condition 3	
	Max	Min	Max	Min	Max	Min
Rotating Friction disc	1390	1185	960	745	530	442
Fixed friction disc	1390	1275	960	835	530	482
Pedestal	1390	794	960	490	530	430
Body Support	794	416	490	282	350	208
Billiard plate	1390	416	960	282	530	208

Continue Table 5

structure	Condition 4		Condition 5		Condition 6	
	Max	Min	Max	Min	Max	Min
Rotating Friction disc	650	530	430	302	650	542
Fixed friction disc	650	570	430	332	650	582
Pedestal	650	420	430	280	650	430
Body Support	430	214	280	157	430	208
Billiard plate	650	214	430	157	650	208

The relationship between the material elastic modulus and temperature

Through the elastic modulus of metal materials with the temperature change of phenomenological explanation, can obtain the general relation between elastic modulus and temperature.

$$\begin{cases} \alpha = \dfrac{1}{r}\dfrac{dr}{dT} \\ \eta = -\dfrac{1}{r}\dfrac{dE}{dT} \end{cases}$$
(7)

Assume that heated expands atomic spacing follow the following pattern: $r = r_0(1+\alpha T)$.

Where r_0 is the atomic spacing when absolute temperature $T_0 = 0$, α is linear expansion coefficient of material, η is Temperature coefficient of modulus of elasticity E.

The formula shows that elastic modulus decreases with the rise of temperature in general situation of metal material. But whether in a straight line down it depends on the temperature coefficient of elasticity modulus η. The η of metal material is hard to find, but the linear expansion coefficient a experimental data is more fully. A number of studies have demonstrated that many ratios of linear expansion coefficient and temperature coefficient (α/η) of metal and alloy material are close to a constant value.

So the general empirical formula of the elastic modulus with temperature changes is obtained.

$$E = E_0(1 - 25\alpha T) \quad (73.15 < T < 1073.15)$$
(8)

Where E_0 metal elastic modulus under absolute temperature, T s thermodynamic temperature scale. The formula are in good agreement under the $200^{\circ}C \sim 800^{\circ}C$ by experiment proof. It will meet the linear relationship basically. But deviation is larger when the environment temperature is higher than $800\,^{\circ}C$. The relationship between the elastic modulus and temperature of billiard plate materials is shown in the following table.

Table 6 Relationship of _E_ and _T_

			$E*10^{10}$Pa				
	_T_(K)	288	773	823	873	923	973
$4Cr_{14}$	test	21.6	17.2	16.9	16.3	16.0	15.5
$Ni_{14}W_2$ Mo	calculate	21.6	16.6	16.1	15.5	15.0	14.5
$38CrSi$	test	21.0	17.8	17.5	17.3	17.1	16.9
	calculate	21.0	17.7	17.3	17.0	16.7	16.3

Analysis of system Stiffness and Stress relations

The internal stress referred to in this paper includes thermal stress and pre-stress which is generated by pre-tightening force. Studies have shown that natural frequency has close relationship with internal stress. As shown in Fig.8 is the before four mode of system (Refers to the structured modal, does not contain the unstructured modal). Natural frequency produced obvious change, by adding different pre-stress. So the stiffness matrix produced a change in the system.
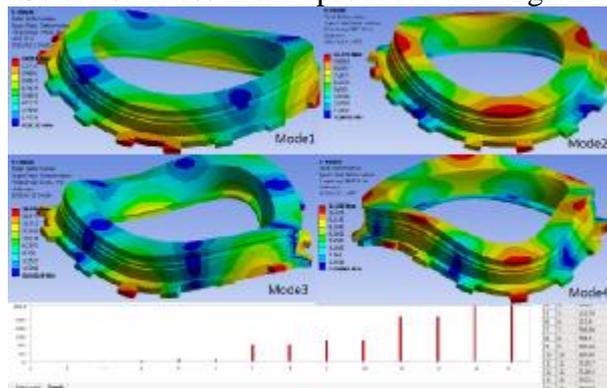
Figure 8 System Natural Frequency

The additional stiffness matrix under the influence of thermal stress and pre-stress is obtained by solving the geometric stiffness matrix. Considering the billiard plate structure characteristics, the vibration strain is mainly composed of two parts: The first is strain because of billiard plate displacement change; the second is additional strain which produced by the deflection in the billiard plate because of the surface internal force. So suppose billiard plate material is isotropic, so additional strain energy is:

$$\Delta U = \tfrac{1}{2}[\delta]^T[K_g][\delta] = -[\delta][Q]_T - [\delta][Q]_F + C$$

(9)

Where $[Q]_T = \int [B]^T[E][\varepsilon]_T dV$ is the equivalent load produced by the thermal stress of the system, $[B]=[\varepsilon]/[\delta]$, $[Q]_F = \int [B]^T[E][\varepsilon]_F dV$ is equivalent load produced by pre-stress, $C = \tfrac{1}{2}\int [\varepsilon]_F^T[E]_T[\varepsilon]_F dV$ is only related to temperature $[E],[\delta],[\varepsilon]_T,[\varepsilon]_F,[K_g]$ are elastic modulus matrix, Unit displacement matrix, thermal strain matrix, pre-strain matrix and additional stiffness matrix respectively.

Results Analysis of the Thermal Vibration

Billiard plate dynamics equation is nonlinear dynamic equation. Use the classic 4 stairs variable step Runge - Kutta numerical integration method to solve. Calculation parameters are shown in table 7.

Table 7 Calculation parameters of Dynamic systems

Parameters	Numerical Value
Rotating billiard plate m_1	40 kg
Mobile billiard plate m_2	40 kg
Pressing plate m_3	60 kg
Groove Angle	20°
Circumferential force F_1	10000 N
Axial force F_2	30000 N
Stiffness coefficient K_1	$8*10^6$ N/m
Stiffness coefficient K_2	$5*10^6$ N/m
Stiffness coefficient K_3	$5*10^6$ N/m
damping ratio ζ_1	0.016
damping ratio ζ_2	0.016

Through calculation, the displacement and velocity curve of components are obtained, under 800^oC, 50000N pre-tightening force. The calculation results are shown in Fig.9. It is shown the additional stiffness matrix's influence of the displacement response of the X direction and Y direction. Its force is exerted at linear way. Linear function time is 0.1 s. After of that, the force maintains a constant.
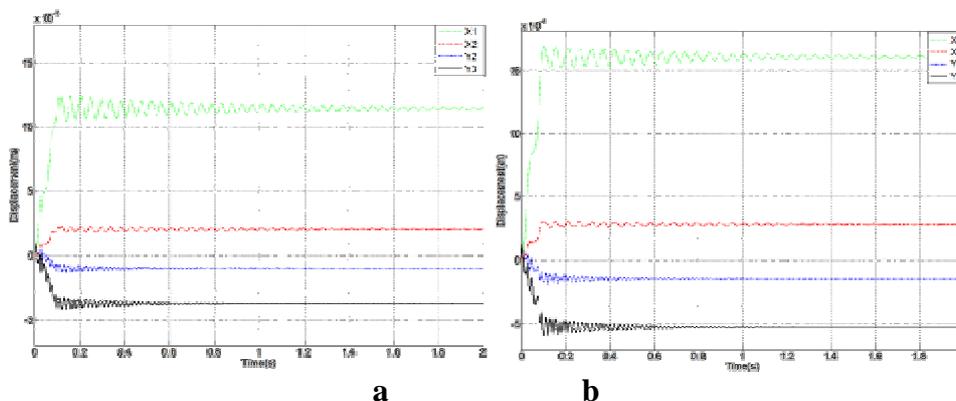
a   b
Figure 9 Different directions Displacement curve
the normal temperature condition for(a) and (b) 800℃ high temperature state

After adding the additional stiffness matrix, Displacements of mobile billiard plate and rotating billiard plate are obvious changed. As the temperature increase stiffness decreases and the displacement increases. But there is no obvious change on amplitude. Stiffness weakening makes the

speed increases obviously in loading process; and makes the velocity attenuation speed up. Stiffness significantly affect the vibration cycle. Smaller stiffness makes smaller vibration frequency.

a   b
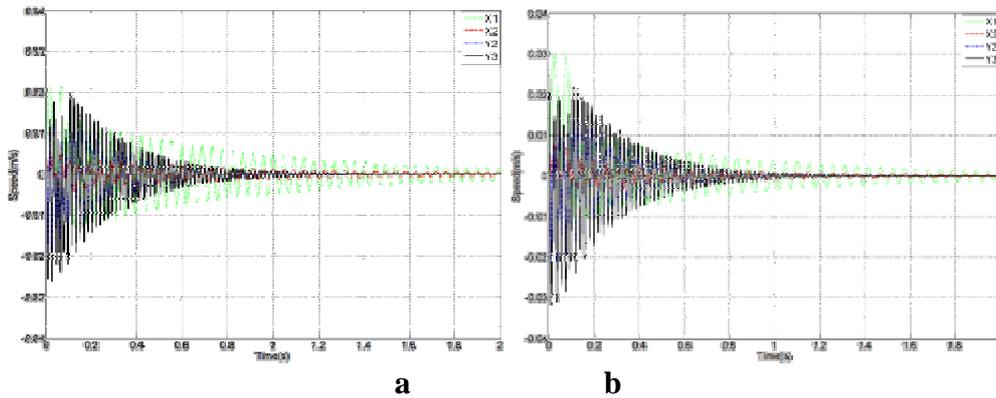
Figure 10 Different directions Speed curve
the normal temperature condition for(a) and (b) 800℃ high temperature state

Through the system natural frequency solved, obtain the change of the natural frequency of the billiard plate under different working conditions. They are natural frequency curves of the billiard plate under different temperatures and the pre-tightening forces drawn in Fig 11. Natural frequency decreases with temperature increasing. And high modes are more sensitive to this effect.
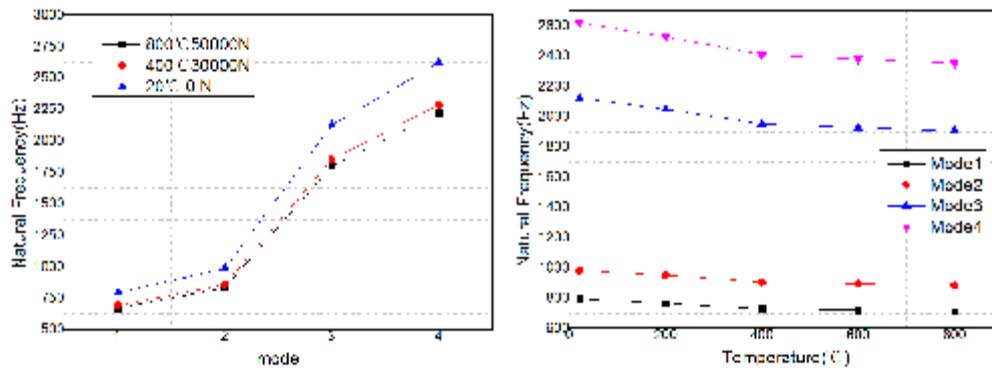
Figure 11 Natural Frequency curves

The influence degree of temperature is different for different modes seen in Fig.11. The effect of friction heat on system natural frequency has reached 19.1%. The higher the temperature, the stronger the impact.
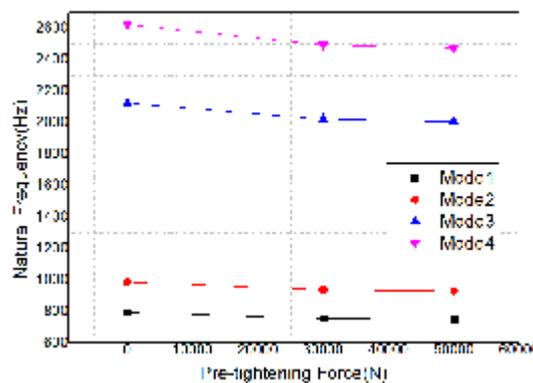
Figure 12 Effect of the Pre-stress

We contrast the maximum thermal stress with the stress produced by pre-tightening force, the temperature effect transform into force effect thereby. Fig.12 shows the effect of the pre-stress. It is found that with load increasing, the system natural frequency decreases. Natural frequency of the system change is not a linear change. For high pre-stress, the system frequency increment gradually smaller.

Fig.13 shows natural frequency change under the first two modes. Different modes show the same change rule. The effect of temperature on vibration is integrity. The effect is applicable for all the natural frequencies of the system.
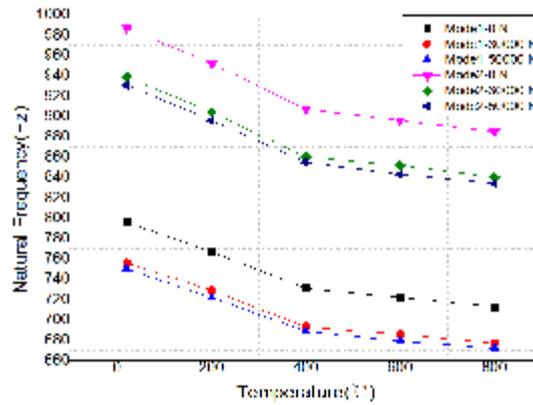
Figure 13 Influence of the first two modes

The effect of pre-stress on system natural frequency has reached 11.2%. The stress produced by friction heat and the stress produced by force are no difference in essence.

Conclusion

In the research we mainly analyze the loading structure of dry type brake (Billiard plate) vibration characteristics under the influence of high temperature, and obtained some conclusions:

1. The main factors of structure stiffness change by high temperature are: the elastic modulus and internal stress.

2. Displacements of system are obvious changed. As the temperature increasing, stiffness decreases and the displacement increases. It makes the speed increases obviously in loading process; and makes the velocity attenuation speed up. Vibration frequency becomes smaller.

3. Temperature effects on system natural frequency are: as the temperature increasing, natural frequency decreases with the nonlinear changes.

Acknowledgment

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