# Owe Period Fast North-Seeking Method Based On Interpolation 

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#### Abstract

In order to make rationally use of the original data and improve the data fitting precision of the sine waveform, the owe period fast North-Seeking method of the polynomial interpolation is based on the least square method. According to the character of the large deflection north point data, the polynomial interpolation subdivision is carried out based the eighth data. fitting a quarter data. To owe period fitting of the quarter data of sinusoidal parameter. Use a certain type gyro as prototype. Through software simulation algorithm,the experimental results show that the algorithm reasonable use of the data, shortened the time for the north, improve the fitting accuracy.


## Introduction

North seeking technology has been widely used in aerospace, mining, undersea tunnel, geographic mapping, energy exploration and military fields. The development direction of the current gyro theodolite is high precision, fast and full automation, but high precision and fast is often a pair of contradictory requirements. GYROMAT series gyro -theodolite of German DMT company representative the international advanced level [1-4] , Can provide high accuracy and moderate precision and fast search three measurement modes.

Domestic research on the precision of the north seeking algorithm is more mature, but the research on the rough north seeking is backward, relevant literature is relatively small
[5,6] .Literature [7] proposed a full azimuth fast forward method based on swing speed detection, Time and accuracy are greatly improved, the problem is that there is no theory and documentation to show whether the follow-up of the shell can be tracked on the gyro swing. Literature[8]proposed use of one-fourth-cycles to find out rough north, use the method of integral precision complex north of north.

In this paper, the author studied mathematical models of gyro trail and gyro sensitivity of the trail, proposed a method is used to find the middle and low precision, based on increase the data quantity by polynomial interpolation, and then tuse the less - period fitting method.

## Motion equation of gyro sensitive part

Established the dynamics model of gyro sensitive parts, as shown in figure 1 [9,10] . The gyro sensitive part is suspended from the O point of the instrument case through a suspension belt. $\mathrm{O}_{1}$ for the sensitivity of the center of gravity, the center of gravity $\mathrm{O}_{1}$ and the distance between the suspension point L is called the heart of high, mg for the sensitivity of the Ministry of gravity, $\mathrm{H}_{\mathrm{G}}$ for the gyro rotor moment of momentum.


Fig. 1 Pendulous gyro dynamics model


Fig. 2 Curve of under period fitting

Define the following parameters: The moment of inertia of the gyro is around Ox, Oy, Oz axis Jx, Jy , Jz ( Jx does not include the moment of inertia of the gyro rotor, $\mathrm{Jy}=\mathrm{Jz}$ ); $\varphi$ said instrument erection latitude; $D_{B}$ said torsion coefficient sling $K=H_{G} \cdot \omega e \cdot \cos \varphi$, Omega phi, said North moment coefficient, $\mathrm{D}=\mathrm{D}_{\mathrm{K}}+\mathrm{D}_{\mathrm{B}}$; Tracking method zero correction coefficient: $\lambda 1=\mathrm{D}_{\mathrm{B}} / \mathrm{D}_{\mathrm{K}}$, non tracking method zero correction coefficient: $\lambda 2=\mathrm{D}_{\mathrm{B}} / \mathrm{D} ; \alpha_{0}$ said zero angle torque range sling.

In a small angle, the small North case, motion equations of oscillation of gyro sensitive parts can be expressed as:

$$
\left\{\begin{array}{l}
J_{x} \frac{d^{2} \gamma}{d t^{2}}+m g l \gamma=0  \tag{1}\\
J_{y} \frac{d^{2} \beta}{d t^{2}}+H_{G} \omega_{e} \sin \varphi+H_{G} \frac{d \alpha}{d t}+m g l \beta=0 \\
J_{z} \frac{d^{2} \alpha}{d t^{2}}+H_{G} \omega_{e} \cos \varphi \bullet \alpha-H_{G} \frac{d \beta}{d t}+D_{B}\left(\alpha-\alpha_{0}\right)=0
\end{array}\right.
$$

In gyro sensitive parts of the precession process, $\beta$ and $\gamma$ to $\alpha$ is relatively small, usually in the study of gyro sensitive movement, only the gyro shaft deviates from the north to the azimuth angle $\alpha$, the gyro sensitive parts of precession model equivalent to general rigid torsion swing.Formula (1) can be simplified into:

$$
\begin{equation*}
\left(J_{Z}+\frac{H_{G}^{2}}{M_{h}}\right) \bullet \frac{d^{2} \alpha}{d t^{2}}+D_{K} \bullet \alpha+D_{B} \bullet\left(\alpha-\alpha_{0}\right)=0 \tag{2}
\end{equation*}
$$

According to the stress situation of the sling, the equations of motion of gyro sensitive parts can be mainly divided into tracking and tracking. In the tracking case, the sling of gyro sensitive parts of torque $D_{B} \cdot\left(\alpha-\alpha_{0}\right)$ to a fixed value, in the case of no trace, the $\alpha_{0}$ is fixed, and the torque applied to the $D_{B} \cdot\left(\alpha-\alpha_{0}\right)$ is changed.

It can be concluded that the swing of the gyro sensitivity:
(1) whether it is tracking or not tracking, the gyro's sensitivity is around a balanced position in accordance with the sine wave.
(2) the equilibrium position of gyro sensitive parts of swing is not necessarily the North direction, the difference between the calculation of the equilibrium position and the position of the tracking is different from that of the zero position correction factor.

## Owe cycle sine curve fitting based on polynomial interpolation

Owe cycle sine curve fitting. The curve fitting of the sine curve needs to establish the motion model of the gyroscope, and the reliability of the model is verified. Using the model to fit the cycle data.

The cursor center on the PSD panel, the amplitude is $\pm 5 \mathrm{~V}$. A nonlinear regression model based on the sine function is used to establish the model of the least square estimation. Can get:

$$
\begin{equation*}
y=A \sin (2 \pi / T t+\theta)+B \tag{3}
\end{equation*}
$$

Type: undetermined coefficients $A$ for the amplitude of the cursor; periodic oscillation is $T$; relative cursor Zhenbei initial phase is $\theta$; cursor center of oscillation and the offset of zero
reference is $B$.
By minimizing the sum of squares of residuals, the estimated coefficients of the nonlinear least squares are estimated, the principle of regression is to minimize the sum of squared residuals. The sum of squared residuals of the unknown coefficient derivative is equal to 0 , after finishing by solving the matrix about unknown coefficient.

Using Matlab to carry out the nonlinear least square fitting. The fitting effect was tested by the coefficient of determination $\left(\mathrm{R}^{2}\right), R^{2}$ the closer to 1 , the higher the goodness of fit. After calculation, $R^{2}=0.9999$, the result shows that the mathematical model is effective.
literature [11] point out the mean period of gyro motion trajectory is approximately 448s. The data acquisition frequency is 100 HZ ,according to the method, the following functions are obtained:

$$
\begin{equation*}
y=5.00 * \sin (2 * \text { pit } * / 150.00)+0.06 \tag{4}
\end{equation*}
$$

After fitting analysis, there is a large deviation between the curve and the original curve, and the accuracy is not high, but the fitting time is shorter, as shown in Figure 2.

Polynomial interpolation. The main advantage of polynomial fitting is that the previous data fitting is more accurate, the principle of polynomial fitting is to set the goal of curve fitting is the minimum variance or the least square method, polynomial equation:

$$
\begin{equation*}
y=p_{1} x^{n}+p_{2} x^{n-1}+p_{3} x^{n-2} \cdots p_{n} x+p_{n+1} \tag{5}
\end{equation*}
$$

To obtain a polynomial, just determine the value of the coefficient. Polynomial fitting coefficients are calculated by ployfit function.

From the point to the distance of the curve, that is, the square of deviation as:

$$
\begin{equation*}
R^{2}=\sum_{i=1}^{n}\left[y_{i}-\left(p_{1} x^{n}+p_{2} x^{n-1}+p_{3} x^{n-2} \cdots p_{n} x+p_{n+1}\right)\right]^{2} \tag{6}
\end{equation*}
$$

partial derivation and simplified to the right side of the type 2 :

$$
\left[\begin{array}{cccc}
1 & x_{1} & \cdots & x_{1}^{n}  \tag{7}\\
1 & x_{2} & \cdots & x_{2}^{n} \\
\vdots & \vdots & \ddots & \vdots \\
1 & x_{3} & \cdots & x_{3}^{n}
\end{array}\right]\left[\begin{array}{c}
p_{n+1} \\
p_{n} \\
\vdots \\
p_{1}
\end{array}\right]=\left[\begin{array}{c}
y_{n} \\
y_{n-1} \\
\vdots \\
y_{1}
\end{array}\right]
$$

Can get coefficient matrix and the fitting curve. Polynomial fitting method is used to fit the standard sine curve, and the fitting order is 10 . The fitting effect is shown in figure 3 . From figure 3, The front of fitting curve is more accurate, the error of behind fitting curve is gradually increased.


Fig. 3 Polynomial fitting curve Fig. 4 fitting curve of improved method
Fig. 5 north seeking data
Improved owe cycle sine curve fitting improve the characteristics of less cycle sine curve fitting method based on polynomial interpolation fitting error smaller and shorter fitting time. Interpolation subdivision data according to the characteristics of the collected data points.

Will the following form into discrete sine curve formula, $1 / 4$ period data were used to fit the $1 / 8$ period data. Will sine curve formula becomes the following discrete forms:

$$
\begin{equation*}
y(i)=A_{0} \cos (\omega i)+B_{0} \sin (\omega i)+D_{0} \tag{8}
\end{equation*}
$$

Sampling frequency of data signals is known. Selection $A_{0}, B_{0}, D_{0}$, making the sum of squares of residuals is minimized.

$$
\begin{equation*}
\varepsilon=\sum_{i=1}^{n}\left[y_{i}-A_{0} \cos (\omega i)-B_{0} \sin (\omega i)-D_{0}\right]^{2} \tag{9}
\end{equation*}
$$

Seeking least square fitting value of $\mathrm{A}_{0}, \mathrm{~B}_{0}, \mathrm{D}_{0}$. Structure matrix:

$$
\beta=\left|\begin{array}{ccc}
\cos (\omega \cdot 1) & \sin (\omega \cdot 1) & 1  \tag{10}\\
\cos (\omega \cdot 2) & \sin (\omega \cdot 2) & 1 \\
\ldots & \ldots & \ldots \\
\cos (\omega \cdot n) & \sin (\omega \cdot n) & 1
\end{array}\right| ; y=\left|\begin{array}{c}
y_{1} \\
y_{2} \\
\ldots \\
y_{n}
\end{array}\right| ; \quad x_{0}=\left|\begin{array}{c}
A_{0} \\
B_{0} \\
D_{0}
\end{array}\right|
$$

Fitting function:

$$
\begin{equation*}
\hat{y}(i)=A_{0} \cos (\omega i)+B_{0} \sin (\omega i)+D_{0} \tag{11}
\end{equation*}
$$

The expression of amplitude and phase is:

$$
\begin{align*}
& A=\sqrt{A_{0}{ }^{2}+B_{0}{ }^{2}}  \tag{12}\\
& \theta=\arctan \left(\frac{-B_{0}}{A_{0}}\right) \tag{13}
\end{align*}
$$

Using the improved method to fit the data, as shown in figure 4.The fitting effect is good, conforms to the movement rule.

The comparison of the fitting error between the cycle fitting method and the improved method is shown in table 1, from table 1, we can see that the amplitude ratio of the improved method is closer to 5 V than that of the direct less period, and the fitting error is also greatly reduced.

Table 1 Comparison of the fitting error between the less period fitting and the improved method

|  |  | Result of owe cycle fitting | Result of improved method |
| :---: | :---: | :---: | :---: |
| Amplitude <br> (A) | Fitted values (V) | 4.9187 | 5.0427 |
|  | Accuracy | 0.1174 | 0.0257 |

## Experimental verification

Experiments Relying on a certain type of prototype, selected range of-30 degrees to the +30 ,transform once in every 10 positions, collection 6 sets of experimental data for each location, sampling frequency is 100 HZ .north seeking data of eastern 10 degree as shown in figure 5 . Analysis of the data, get the results of the analysis as shown in table 2.

From the table 2 , north seeking time is only $1 / 8$ cycles, the improvement method is better than the North standard deviation of the results, there is a good repeatability, the design requirements of high precision and fast north seeking are realized.

## Summary

In this paper, the basic principle of gyro north seeking is set up, and a new fast and low precision north seeking method is proposed based on the characteristics of polynomial fitting and under cycle fitting. By using polynomial interpolation and nonlinear regression analysis, the function information of the whole period can be predicted by the part of the data, and the results can be calculated in the $1 / 8$ period. The method breaks through the traditional measurement methods, effectively solve the fundamental problems of low efficiency of north seeking, and improve the accuracy of the precision of sine curve fitting parameters and local data processing, provides a new idea for fast north seeking.

Table 2 The result between improvement method and $1 / 8$ owe cycle sine curve fitting

| Locati on | Frequency | Result of owe cycle fitting | Accuracy | Result of improved method | Accuracy |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $+30^{\circ}$ | 6 | $329^{\circ} 56^{\prime} 55^{\prime \prime}$ | $1^{0} 44^{\prime} 56{ }^{\prime \prime}$ | $329^{\circ} 58^{\prime} 33^{\prime \prime}$ | $1^{0} 37^{\prime}$ 21" |
| $+20^{0}$ | 6 | $340^{\circ} 39^{\prime} 12^{\prime \prime}$ | $1^{0} 21^{\prime} 42^{\prime \prime}$ | $340^{\circ} 40^{\prime} 37^{\prime \prime}$ | $1^{0} 13^{\prime} 05^{\prime \prime}$ |
| $+10^{0}$ | 6 | $350^{\circ} 26^{\prime} 24^{\prime \prime}$ | $0^{0} 59{ }^{\prime} 33^{\prime \prime}$ | $350^{\circ} 25^{\prime} 12{ }^{\prime \prime}$ | $0^{0} 52^{\prime} 04{ }^{\prime \prime}$ |
| $0^{0}$ | 6 | 00 ${ }^{\circ} 25^{\prime} 38{ }^{\prime \prime}$ | $0^{0} 45^{\prime} 12{ }^{\prime \prime}$ | 000 ${ }^{\circ} 6^{\prime} 49$ " | $0^{0} 38^{\prime} 55^{\prime \prime}$ |
| $-10^{0}$ | 6 | $10^{\circ} 37^{\prime} 22^{\prime \prime}$ | $0^{0} 58^{\prime} 35{ }^{\prime \prime}$ | $10^{\circ} 38^{\prime} 43^{\prime \prime}$ | $0^{0} 52^{\prime} 07{ }^{\prime \prime}$ |
| $-20^{0}$ | 6 | $20^{\circ} 22^{\prime} 41^{\prime \prime}$ | $1^{0} 24^{\prime} 40{ }^{\prime \prime}$ | $20^{\circ} 21^{\prime} 33^{\prime \prime}$ | $1^{0} 15^{\prime} 01^{\prime \prime}$ |
| $-30^{0}$ | 6 | $30^{\circ} 54^{\prime} 11^{\prime \prime}$ | $1^{0} 46^{\prime} 45{ }^{\prime \prime}$ | $30^{\circ} 52^{\prime} 21^{\prime \prime}$ | $1^{0} 40^{\prime} 11^{\prime \prime}$ |

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