An Algorithm of Image Edge Detection Based on Wavelet Transform

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Abstract: Image edge structure is often the most important feature in image processing and pattern recognition, and edge detection is mainly used to measure, detect and locate the gray level change. Wavelet analysis is a kind of multi scale edge detection method which has excellent denoising ability and complete edge detection ability. Using quadratic B-spline strip as operator, using edge detection algorithm based on wavelet analysis and dealing with the characteristics of texture image and heartbeat advantages and wavelet transform for image detail description of organic unifies in together, get the edge image quality is better.

Introduction

The basic feature of the image is the edge, which exists between the target and background, the target and the target, the region and the region, the basic element and the base element. The edge, which refers to the pixel gray levels or changes in the roof of the collection-pixels, is the image of a variety of different points. Such as: oscillation, shadow, bright spot, peak points and texture, etc. It can outline the characteristics of the set outline of the object, transfer the image of a variety of information, is an important feature of the describing image of the object [1]. It provides important characteristic parameters to describe or identify the target image or to interpret the image. It contains most of the energy of image. The image edge detection is an important stage of image processing. It is widely used in image segmentation, image classification, image registration and pattern recognition, etc.

In 1992, Mallat [2] presented using wavelet analysis of singular signal detection method, and the Gaussian function and its derivative respectively as the smoothing function and the mother wavelet function the method was verified, gives good experimental results [3]. The wavelet analysis theory is applied to digital image edge detection. Using wavelet transform of the signal of multiscale analysis is very suitable for local signal, using a different window to detect image edge is multi-scale edge detection, in the extraction of image edge and can effectively suppress noise.

Wavelet modulus maxima edge detection algorithm

Multiscale edge detection is in different scales to polish the original signal (the original image) and then by polished signal of first-order or second-order derivative to detect the changes of the original signal. The changes are the original signal edge, the edge points are connected into a curve form signal (image) edge. If wavelet function would be viewed as a derivative of a smooth function, signal wavelet transform modulus local maxima corresponding to signal the mutation point (or edge). If the wavelet function would be regarded as the second derivative of a smooth function, the zero crossings of the signal wavelet transform modulus, corresponding to the signal point mutation (or edge). Therefore detection of wavelet coefficient of variation method with zero and local extremum can the edge position of the signal detection.

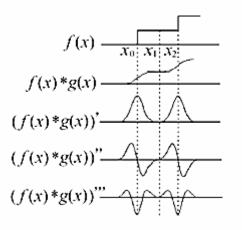


Fig.1 Signal and Gauss smoothing of first order, second order and third order derivative

As shown in Figure 1, f(x) as a step function, g(x) for one dimensional Gauss function, * representation of convolution, the following three figure indicates the first order of Gauss smoothing, second order and third order derivative. From the graph, we can see that zero point is not always the point of the corresponding signal, and the modulus maxima of the wavelet analysis correspond to the point of the signal. It can be seen that the wavelet analysis can be used to detect the edge of the image by using the modulus maximum value.

In the wavelet domain, the edge detection is equivalent to the modulus maxima of the wavelet transform, and the basic principle of the wavelet transform is briefly introduced[4].

When $\theta(x, y)$ is a two dimensional smooth function. It satisfied $\theta(x, y) \ge 0$, $\iint \theta(x, y) dx dy = 1$

 $\lim_{x,y\to\pm\infty}\theta(x,y) = 0$ condition. Scale along the two into the sequence of $s = \{2^j\}_{j\in\mathbb{Z}}$ traversal, the and two-dimensional wavelet transforms are:

$$W_{2^{3}}^{1}f(x, y) = f(x, y) * \psi_{2^{3}}^{1}(x, y)$$
$$W_{2^{3}}^{2}f(x, y) = f(x, y) * \psi_{2^{3}}^{2}(x, y)$$

It can be proved that the model f of the synthetic gradient of the wavelet transform is proportional to the gradient vector of θ_s after polishing, and the proportion coefficient is $s = 2^{j}$. That is proving

$$\begin{pmatrix} W_{2^{j}}^{1}f(x,y) \\ W_{2^{j}}^{2}f(x,y) \end{pmatrix} = 2^{j} \begin{pmatrix} \frac{\partial}{\partial x}(f * \theta_{2^{j}}(x,y)) \\ \frac{\partial}{\partial y}(f * \theta_{2^{j}}(x,y)) \end{pmatrix} = 2^{j} \vec{\nabla}(f * \theta_{2^{j}})(x,y)$$

And ∇ represents the gradient operator. Wavelet transform can also be used to represent the $\vec{\nabla}$ modules and angles:

$$\binom{Mod[W_{2^{j}}f(x,y)]}{Arg[W_{2^{j}}f(x,y)]} = \binom{\sqrt{|W_{2^{j}}^{-1}f(x,y)|^{2} + |W_{2^{j}}^{-2}f(x,y)|^{2}}}{\arctan\left(\frac{W_{2^{j}}^{-2}f(x,y)}{W_{2^{j}}^{-1}f(x,y)}\right)}$$

Wavelet transform modulus of $Mod[W_{2^{j}}f(x, y)]$ is proportional to the modulus of the gradient vector $\vec{\nabla}(f * \theta)(x, y)$, the wavelet transform of the argument $Arg[W_{2^{j}}f(x, y)]$ is the angle between the horizontal direction of the gradient vector $\vec{\nabla}(f * \theta)(x, y)$, which is the direction of the image edge. In different scales, dyadic wavelet transform modulus maxima along the direction perpendicular to the gradient vector are connected into the curve form the image multiscale edge, then calculating a smooth function $(f * \theta_s)(x, y)$ along the direction gradient modulus maxima equivalent in the calculation of wavelet transform modulus maxima, namely through detecting the 2D wavelet modulus maxima edge points can be determined.

simulation experiment

In image edge detection, the selection of wavelet function is directly related to the detection results. Through the analysis of the efficient method based on B-spline edge detection is constructed to minimize the loss of information of a three B-spline image structure in the image processing, the structure to be superior to those commonly used in Gaussian and Laplacian pyramid structure.

Multiscale edge detection of wavelet transform of the image Gatlin, according to establish multi resolution of two scale difference equation shows the wavelet function $\psi(t) = \sum_{n} g_n \sqrt{2}\varphi(2t-n)$,

 $\varphi(t)$ as the scale function, said $\varphi(t) = \sum_{n} h_n \sqrt{2}\varphi(2t - n)$, where h_n is the low pass filter, g_n is

the high pass filter. Wavelet function and scaling function to do the Fourier transform:

$$\begin{cases} \widehat{\varphi}(w) = \sum_{n \in \mathbb{Z}} \frac{h_n}{\sqrt{2}} \widehat{\varphi}(\frac{w}{2}) e^{-iwn/2} = H(\frac{w}{2}) \widehat{\varphi}(\frac{w}{2}) \\ \widehat{\psi}(w) = \sum_{n \in \mathbb{Z}} \frac{g_n}{\sqrt{2}} \widehat{\varphi}(\frac{w}{2}) e^{-iwn/2} = G(\frac{w}{2}) \widehat{\varphi}(\frac{w}{2}) \end{cases}$$

The space of energy conservation: $|\hat{\varphi}(w)|^2 = |\hat{\psi}(w)|^2 + |\hat{\varphi}(w)|^2$. A cardinal spline wavelet is rectangular pulse as n-1 time convolution in the delay, that is, n-1 order B-spline displacement, Fourier transform of the scaling function $\varphi(t)$ is $\hat{\varphi}(w) = e^{-jk\frac{w}{2}} \left[\frac{\sin(w/2)}{w/2}\right]^n$. Select

structure form and Gauss function similar to the cubic spline function as smoothing function and its derivative is quadratic spline function, image shows function and quadratic spline wavelet convolution maxima correspond to the edge points.



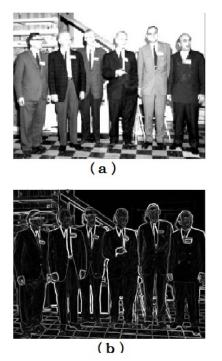


Fig.2 Multiscale edge detection image

As shown in Figure 2, is the multiscale edge detection of wavelet transform of the image Gatlin left is Gatlin original image and the right (a) is the first level wavelet transform edge detection results and (b) is the second wavelet transform edge detection results. The selected wavelet basis is the two-B-spline wavelet, which can be seen that the image edge detection effect is good.

Summary

In this paper, we use the two B spline wavelet edge detection algorithm, the wavelet transform to deal with image heartbeat texture advantages and the characteristics of wavelet transform image details of the description of the combination of [5]. The experimental results show that the description of the algorithm to enhance the clarity of the image edge outline, road closures the details of image edge, the edge of the linear structure has emerged, edge detection result is satisfactory, the edge image quality is better.

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