Wiener State Estimator for Non-regular Descriptor System

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Abstract. Using the modern time-series analysis method in the time domain, based on the autoregressive moving average (ARMA) innovation model and white noise estimator, non-regular descriptor discrete-time stochastic linear systems are researched. Under assumption 1~3, an asymptotically stable reduced-order Wiener state estimator for descriptor systems is given by using projection and block matrix theories. Non-regular descriptor systems include general descriptor systems in them. And the algorithm is reduced-order. It avoids the solution of the Riccati equations and Diophantine equations. So that it reduces the computational burden, and is suitable for real time applications.

Introduction

Descriptor systems are often seen in such fields as circuit, economics and robotics, thus receiving more and more attention. In recent years, a number of very useful research results have been obtained in terms of regular descriptor systems [1~4]. However, research on non-regular descriptor systems is still lacking. In this paper, based on ARMA innovation model and white noise estimators, Wiener state estimators for non-regular descriptor systems are presented by using modern time series analysis method [5] and block matrix theory [6,7]. The presented Wiener state estimators have more universal significance since they can be applied to both non-regular and regular descriptor systems.

Consider the discrete time descriptor stochastic system

$$Mx(t+1) = \Phi x(t) + \Gamma w(t), \tag{1}$$

$$y(t) = Hx(t) + v(t), \tag{2}$$

where the state $x(t) \in R^n$, the measurement $y(t) \in R^m$, $w(t) \in R^q$, $v(t) \in R^m$, constant matrices M 、 Φ 、 Γ and H are respectively $p \times n$, $p \times n$, $p \times q$ and $m \times n$, and $n > m$.

Assumption 1. $w(t)$ and $v(t)$ are correlated white noises.

$$\mathrm{E}\left\{\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(j) & v^{\mathrm{T}}(j) \end{bmatrix}\right\} = \begin{bmatrix} Q_w & S \\ S^{\mathrm{T}} & Q_v \end{bmatrix}\delta_{ij}, \begin{bmatrix} Q_w & S \\ S^{\mathrm{T}} & Q_v \end{bmatrix} > 0, \tag{3}$$

where E is the expectation, T denotes the transpose, $\delta_{tt} = 1$, $\delta_{tj} = 0 \, (t \neq j)$, $S = \mathrm{E}[w(t)v^{\mathrm{T}}(t)]$ is the correlation matrix.

Assumption 2. H can be represented as $H = [I_m \quad O_{m\times(n-m)}]$.

Assumption 3. The system is completely observable, that is, $\forall z \in C$, we have

$$\mathrm{rank}\begin{bmatrix} zM - \Phi \\ H \end{bmatrix} = n, \ \mathrm{rank}\begin{bmatrix} M \\ H \end{bmatrix} = n. \tag{4}$$

M , Φ, $x(t)$ can be divided into the following form

$$M=[\underbrace{M_1}_{m}\quad \underbrace{M_2}_{n-m}],\ \Phi=[\underbrace{\Phi_1}_{m}\quad \underbrace{\Phi_2}_{n-m}],\ x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}\begin{matrix} m \\ n-m \end{matrix}. \tag{5}$$

Substituting it into (1), (2) yields

$$\begin{bmatrix} M_1 & M_2 \end{bmatrix}\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix}=\begin{bmatrix} \Phi_1 & \Phi_2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+\Gamma w(t), \tag{6}$$

$$y(t)=\begin{bmatrix} I_m & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+v(t). \tag{7}$$

That is

$$M_1x_1(t+1)+M_2x_2(t+1)=\Phi_1x_1(t)+\Phi_2x_2(t)+\Gamma w(t), \tag{8}$$

$$y(t)=x_1(t)+v(t). \tag{9}$$

Lemma 1. M_2 is full column rank matrix, that is $\operatorname{rank}(M_2)=n-m$.

Proof. According to Assumption 3, $n=\operatorname{rank}\begin{bmatrix} M \\ H \end{bmatrix}=\operatorname{rank}\begin{bmatrix} M_1 & M_2 \\ I_m & 0 \end{bmatrix}$, then $\operatorname{rank}(M_2)=n-m$.

From Lemma 1 we know, $M_2^+=\left(M_2^{\mathrm T}M_2\right)^{-1}M_2^{\mathrm T}$ is existed and premultiplying (8) by it yields

$$x_2(t+1)=M_2^+\Phi_1x_1(t)+M_2^+\Phi_2x_2(t)+M_2^+\Gamma w(t)-M_2^+M_1x_1(t+1). \tag{10}$$

Formula (9) can be rewritten as $x_1(t)=y(t)-v(t)$, substituting it into (10) yields the state equation of $x_2(t)$

$$x_2(t+1)=M_2^+\Phi_2x_2(t)+M_2^+\Gamma w(t)-M_2^+(M_1-q^{-1}\Phi_1)y(t+1)+M_2^+(M_1-q^{-1}\Phi_1)v(t+1). \tag{11}$$

Substituting (10) into (8) yields

$$M_1x_1(t+1)+M_2[M_2^+\Phi_1x_1(t)+M_2^+\Phi_2x_2(t)+M_2^+\Gamma w(t)-M_2^+M_1x_1(t+1)]$$

$$=\Phi_1x_1(t)+\Phi_2x_2(t)+\Gamma w(t). \tag{12}$$

Let $(I_p-M_2M_2^+)=E$, then (12) can be simplified to

$$E(M_1-q^{-1}\Phi_1)x_1(t+1)=E\Phi_2x_2(t)+E\Gamma w(t). \tag{13}$$

Substituting $x_1(t)=y(t)-v(t)$ into (13) yields the observation equation of $x_2(t)$

$$E(M_1-q^{-1}\Phi_1)y(t+1)=E\Phi_2x_2(t)+E\Gamma w(t)+E(M_1-q^{-1}\Phi_1)v(t+1). \tag{14}$$

Lemma 2. System (11), (14) is completely observable, that is, ($M_2^+\Phi_2$, $E\Phi_2$) is a completely observable pair.

Proof. According to Assumption 3, $\operatorname{rank}\begin{bmatrix} zM-\Phi \\ H \end{bmatrix}=n$, that is $\operatorname{rank}\begin{bmatrix} zM_1-\Phi_1 & zM_2-\Phi_2 \\ I_m & 0 \end{bmatrix}=n$, then $\operatorname{rank}\begin{bmatrix} zM_2-\Phi_2 \end{bmatrix}=n-m$, so

$$\operatorname{rank}\begin{bmatrix} zI_{n-m}-M_2^+\Phi_2 \\ E\Phi_2 \end{bmatrix}=\operatorname{rank}\begin{bmatrix} I_{n-m} & 0 \\ M_2 & -I_p \end{bmatrix}\begin{bmatrix} zI_{n-m}-M_2^+\Phi_2 \\ EA_2 \end{bmatrix}=\operatorname{rank}\begin{bmatrix} zI_{n-m}-M_2^+\Phi_2 \\ zM_2-\Phi_2 \end{bmatrix}$$

$$= \text{rank}\begin{bmatrix} M_2^+ \\ I \end{bmatrix}[zM_2 - \Phi_2] = \text{rank}[zM_2 - \Phi_2] = n - m. \tag{15}$$

ARMA innovation model

Substituting (11) into (14) yields

$$P(q^{-1})(M_1 - \Phi_1 q^{-1})y(t+1) = P(q^{-1})\Gamma w(t) + P(q^{-1})(M_1 - \Phi_1 q^{-1})v(t+1), \tag{16}$$

where q^{-1} is the backward shift operator, and $P(q^{-1}) = E[I_p + \Phi_2(I_{n-m} - q^{-1}M_2^+\Phi_2)^{-1}M_2^+]$.

Formula (16) can also be rewritten as

$$P(q^{-1})(M_1 - \Phi_1 q^{-1})y(t) = P(q^{-1})\Gamma q^{-1}w(t) + P(q^{-1})(M_1 - \Phi_1 q^{-1})v(t), \tag{17}$$

Introducing the left coprime factorization

$$[P(q^{-1})\Gamma q^{-1}, P(q^{-1})(M_1 - \Phi_1 q^{-1})] = \Phi^{-1}(q^{-1})[B(q^{-1})q^{\tau}, C(q^{-1})], \tag{18}$$

where $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are polynomial matrices having the form $X = X(q^{-1}) = X_0 + X_1 q^{-1} + \cdots + X_{n_x} q^{-n_x}$, X_i are the coefficient matrices and n_x is the degree of X . We define $X_i = 0\ (i > n_x)$, and $A_0 = I_p$, $B_0 \neq 0$, $C_0 \neq 0$, τ is an integer.

Substituting (18) into (17) yields the ARMA innovation model

$$C(q^{-1})\mathrm{y}(t) = D(q^{-1})\varepsilon(t), \tag{19}$$

$$D(q^{-1})\varepsilon(t) = q^{\tau}B(q^{-1})w(t) + C(q^{-1})v(t), \tag{20}$$

where $D(q^{-1})$ is a stable polynomial matrix, $D_0 = I_p$, $\varepsilon(t) \in R^p$ is the white noise with zero mean and variance matrix Q_ε . $D(q^{-1})$ and Q_ε can be obtained by using the Gevers-Wouters algorithm [5]. According to (19), the innovations $\varepsilon(t)$ can be computed recursively as

$$\varepsilon(t) = C(q^{-1})\mathrm{y}(t) - D_1\varepsilon(t-1) - \cdots - D_{n_d}\varepsilon(t-n_d),\ t = n_d, n_d+1, \cdots\cdots, \tag{21}$$

with the initial values $(\varepsilon(0), \cdots\cdots, \varepsilon(n_d - 1))$.

From formula (19), (20) and document [5], we have

Lemma 3. System (11), (14) is completely observable, that is, ($M_2^+\Phi_2$, $E\Phi_2$) is a completely observable pair.

For the descriptor system (1) and (2), under Assumptions 1~3, we have the following white noise estimators which have the wiener filter form

$$\begin{cases} \hat{w}(t \mid t+N) = L_N^w \tilde{C}\tilde{D}^{-1}y(t+N) \\ \hat{v}(t \mid t+N) = L_N^v \tilde{C}\tilde{D}^{-1}y(t+N) \end{cases}, \tag{22}$$

where C , D are given by (19) and $\tilde{C}$, $\tilde{D}$ are determined by the left coprime factorization as

$$D^{-1}C = \tilde{C}\tilde{D}^{-1}, \tag{23}$$

where $\tilde{D}_0 = I_m$, for $N < -(\tau \vee 0)$, we define that $L_N^w = 0$, $L_N^v = 0$, for $N \geq -(\tau \vee 0)$ we define

$$L_N^w = \sum_{i=-(\tau\vee 0)}^{N} \Pi_i^w Q_\varepsilon^{-1} q^{i-N},\ L_N^v = \sum_{i=-(\tau\vee 0)}^{N} \Pi_i^v Q_\varepsilon^{-1} q^{i-N}, \tag{24}$$

and $(\tau \vee 0) = \max(\tau, 0)$, while

$$\begin{cases} \Pi_i^w = Q_w F_{i+(\tau\vee 0)}^{\mathrm{T}} + S G_{i+(\tau\vee 0)}^{\mathrm{T}} \\ \Pi_i^v = Q_v G_{i+(\tau\vee 0)}^{\mathrm{T}} + S^{\mathrm{T}} F_{i+(\tau\vee 0)}^{\mathrm{T}} \end{cases}, \tag{25}$$

where F_i and G_i can be computed recursively as

$$\begin{cases} F_i = -D_1 F_{i-1} - \cdots - D_{n_d} F_{i-n_d} + \overline{B}_i \\ F_i = 0\,(i<0)\,, \overline{B}_i = 0\,(i > n_{\overline{b}}) \end{cases}, \tag{26}$$

$$\begin{cases} G_i = -D_1 G_{i-1} - \cdots - D_{n_d} G_{i-n_d} + \overline{C}_i \\ G_i = 0\,(i<0)\,, \overline{C}_i = 0\,(i > n_{\overline{c}}) \end{cases}, \tag{27}$$

where we define $\overline{B} = Bq^{(\tau\wedge 0)}$, $\overline{C} = Cq^{(-\tau\wedge 0)}$, $(\tau \wedge 0) = \min(\tau, 0)$, $(-\tau \wedge 0) = \min(-\tau, 0)$.

Wiener State Estimators

Consider the system (11) and (14), according to ($M_2^+\Phi_2$, $E\Phi_2$) is a completely observable pair, then there exists an $(n-m)\times p$ matrix T_0 such that $M_2^+\Phi_2 + T_0 E\Phi_2$ is nonsingular [6]. Premultiplying (14) by T_0, and adding it with (11) yield

$$x_2(t+1) + (M_2^+ + T_0 E)(M_1 - q^{-1}\Phi_1)[y(t+1) - v(t+1)] - (M_2^+ + T_0 E)\Gamma w(t)$$
$$= (M_2^+\Phi_2 + T_0 E\Phi_2)x_2(t)\,. \tag{28}$$

Let $\Psi = M_2^+\Phi_2 + T_0 E\Phi_2$, we know Ψ^{-1} is existed, then the equation (28) can be rewritten as

$$x_2(t) = \Psi^{-1}x_2(t+1) + \Psi_1[y(t+1) - v(t+1)] - \Psi_2 w(t)\,, \tag{29}$$

where $\Psi_1 = \Psi^{-1}(M_2^+ + T_0 E)(M_1 - q^{-1}\Phi_1)$, $\Psi_2 = \Psi^{-1}(M_2^+ + T_0 E)\Gamma$.

From (14) and (29), we have

$$\begin{bmatrix} E\Phi_2 \\ E\Phi_2\Psi^{-1} \\ \vdots \\ E\Phi_2(\Psi^{-1})^{\beta-1} \end{bmatrix} x_2(t) =$$
$$\begin{bmatrix} E(M_1 - q^{-1}\Phi_1)[y(t+1) - v(t+1)] - E\Gamma w(t) \\ E(M_1 - \Phi_2\Psi_1 - q^{-1}\Phi_1)[y(t) - v(t)] - E(\Gamma - \Phi_2\Psi_2)w(t-1) \\ \vdots \\ E(M_1 - \Phi_2\Psi_1 - q^{-1}\Phi_1)[y(t-\beta+2) - v(t-\beta+2)] - E(\Gamma - \Phi_2\Psi_2)w(t-\beta+1) \end{bmatrix}, \tag{30}$$

where β is the observability index.

Let

$$\Theta = \begin{bmatrix} E\Phi_2 \\ E\Phi_2\Psi^{-1} \\ \vdots \\ E\Phi_2(\Psi^{-1})^{\beta-1} \end{bmatrix}, \tag{31}$$

It can be proved similarly to Lemma 2 that (Ψ^{-1}, $E\Phi_2$) is a completely observable pair according to ($M_2^+\Phi_2$, $E\Phi_2$) is completely observable, then Θ is full column rank matrix, that is, $\Theta^{\#} = [\Theta^{\mathrm{T}}\Theta]^{-1}\Theta^{\mathrm{T}}$ is existed.

$\Theta^{\#}$ can be divided into

$$\Theta^{\#} = \begin{bmatrix} \Theta_0 & \Theta_1 & \cdots & \Theta_{\beta-1} \end{bmatrix}, \tag{32}$$

where Θ_i $(i = 1, 2, \cdots \beta - 1)$ are $(n-m)\times p$ matrices.

Premultiplying (30) by $\Theta^{\#}$ yields the non-recursive state expression

$$x_2(t) = \Theta_0\{E(M_1 - q^{-1}\Phi_1)[y(t+1) - v(t+1)] - E\Gamma w(t)\}$$

$$+\sum_{i=1}^{\beta-1}\Theta_i\{E(M_1-\Phi_2\Psi_1-q^{-1}\Phi_1)[y(t+1-i)-v(t+1-i)]-E(\Gamma-\Phi_2\Psi_2)w(t-i)\}, \quad (33)$$

Taking the projection operation for (33) yields

$$x_2(t\,|\,t+N)=\Theta_0\{E(M_1-q^{-1}\Phi_1)[y(t+1)-\hat{v}(t+1\,|\,t+N)]-E\Gamma\hat{w}(t\,|\,t+N)\}$$

$$+\sum_{i=1}^{\beta-1}\Theta_i\{E(M_1-\Phi_2\Psi_1-q^{-1}\Phi_1)[y(t+1-i)-\hat{v}(t+1-i\,|\,t+N)]-E(\Gamma-\Phi_2\Psi_2)\hat{w}(t-i\,|\,t+N)\}, \quad (34)$$

where $N\geq 1$.

Substituting (22) into (34) yields

$$x_2(t\,|\,t+N)=\Theta_0\{E(M_1-q^{-1}\Phi_1)[y(t+1)-L_{N-1}^{v}\tilde{C}\tilde{D}^{-1}y(t+N)]-E\Gamma L_N^{w}\tilde{C}\tilde{D}^{-1}y(t+N)\}+\sum_{i=1}^{\beta-1}\Theta_i\times$$

$$\{E(M_1-\Phi_2\Psi_1-q^{-1}\Phi_1)[y(t+1-i)-L_{i+N-1}^{v}\tilde{C}\tilde{D}^{-1}y(t+N)]-E(\Gamma-\Phi_2\Psi_2)L_{i+N}^{w}\tilde{C}\tilde{D}^{-1}y(t+N)\}, \quad (35)$$

that is

$$x_2(t\,|\,t+N)=\{\Theta_0E[(M_1-q^{-1}\Phi_1)(q^{1-N}\tilde{D}-L_{N-1}^{v}\tilde{C})-\Gamma L_N^{w}\tilde{C}]$$

$$+\sum_{i=1}^{\beta-1}\Theta_iE[(M_1-\Phi_2\Psi_1-q^{-1}\Phi_1)(q^{1-N-i}\tilde{D}-L_{i+N-1}^{v}\tilde{C})-(\Gamma-\Phi_2\Psi_2)L_{i+N}^{w}\tilde{C}]\}\tilde{D}^{-1}y(t+N). \quad (36)$$

Let

$$K_N^{(2)}=\Theta_0E[(M_1-q^{-1}\Phi_1)(q^{1-N}\tilde{D}-L_{N-1}^{v}\tilde{C})-\Gamma L_N^{w}\tilde{C}]$$

$$+\sum_{i=1}^{\beta-1}\Theta_iE[(M_1-\Phi_2\Psi_1-q^{-1}\Phi_1)(q^{1-N-i}\tilde{D}-L_{i+N-1}^{v}\tilde{C})-(\Gamma-\Phi_2\Psi_2)L_{i+N}^{w}\tilde{C}]. \quad (37)$$

then

$$\hat{x}_2(t\,|\,t+N)=K_N^{(2)}\tilde{D}^{-1}y(t+N)\ (N\geq 1). \quad (38)$$

In addition, from (9) we have

$$\hat{x}_1(t\,|\,t+N)=y(t)-\hat{v}(t\,|\,t+N). \quad (39)$$

Substituting (22) into (39) yields

$$\hat{x}_1(t\,|\,t+N)=(q^{-N}\tilde{D}-L_N^{v}\tilde{C})\tilde{D}^{-1}y(t+N). \quad (40)$$

Let

$$K_N^{(1)}=q^{-N}\tilde{D}-L_N^{v}\tilde{C}, \quad (41)$$

then

$$\hat{x}_1(t\,|\,t+N)=K_N^{(1)}\tilde{D}^{-1}y(t+N). \quad (42)$$

In summary, we can obtain the following theorem by considering the stability of $\tilde{D}$

Theorem. The non-regular descriptor system (1) and (2) have the asymptotically stable Wiener state estimator (38) and (42) under Assumptions 1~3.

Conclusion

Using modern time series analysis method and based on ARMA innovation model and white noise estimators, Wiener state estimators for non-regular descriptor systems are presented in this paper. The presented Wiener state estimators have more practical value owing to the fact that they include regular descriptor systems as a special case and use degree reduction algorithm to reduce computations and avoid solving Riccati equations and Diophantine equations at the same time.

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