

# The application of function P-sets in band information law hiding

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**Abstract.** Using the structure, law and dynamic characteristic of function P-sets, the concepts of P-information law and band information law and their generation are given. Then the attribute theorem of P-information, the hiding and hiding theorem of P-information in band information law and the recovery theorem of information law are presented. Last using the above theory results, an application example of information law hiding is given and identified by experiment.

## Introduction

Using the discrete values  $x_1, x_2, \dots, x_n$ , the function  $w(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$  is generated by interpolation method. Here there are two questions to need to be solved: (1) what kind of mathematical method is used to hide information law  $w(x)$  to prevent from being stolen and distorted by others? (2) what kind of mathematics method is used to restore information law  $w(x)$  hidden? In the paper, a novel mathematical theory will be used to give the answers about the questions, which is function P-sets<sup>[1,2]</sup> and has been applied in many research areas<sup>[3-6]</sup>. Function P-sets is the function form of P-sets<sup>[7-9]</sup>.

## The generation of band information law $\langle a, w(x)^{\bar{F}}, b, w(x)^F \rangle$

Given function set  $S = \{s_1, s_2, \dots, s_q\}$ , and  $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_k\}$  is the attribute set of  $S$ .  $y_i$  is the discrete distribution data set of  $\forall s_i \in S$ ,  $i = 1, 2, \dots, q$ , and

$$y_i = \{y_{i,1}, y_{i,2}, \dots, y_{i,n}\} \quad (1)$$

Using  $y_i$  to get the discrete data set  $y$  of  $S$ , and

$$y = \{y_1, y_2, \dots, y_n\} = \left\{ \sum_{i=1}^q y_{i,1}, \sum_{i=1}^q y_{i,2}, \dots, \sum_{i=1}^q y_{i,n} \right\} \quad (2)$$

Using the data points  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  and Lagrange interpolation function  $w(x) = \sum_{j=1}^n y_j \prod_{\substack{i=1 \\ i \neq j}}^n \frac{x - x_i}{x_j - x_i}$  to get  $w(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$ , and  $w(x)$  is called the information

law generated by  $y$ , which is shown with dotted line in Fig.1.

$\alpha_i$  is supplemented into  $\alpha$ , which constitute attribute set  $\Delta\alpha$ , then  $\alpha$  becomes  $\alpha^F$ , and

$$\alpha^F = \alpha \cup \Delta\alpha \quad (3)$$

After supplementing some attributes into  $\alpha$ ,  $S$  becomes function internal P-set  $S^{\bar{F}}$ , and

$$S^{\bar{F}} = \{s_1, s_2, \dots, s_p\} \quad (4)$$

Using the similar method to equations (1) and (2) to get the discrete data set  $y^{\bar{F}}$  of  $S^{\bar{F}}$ , and

$$y^{\bar{F}} = \{y_1^{\bar{F}}, y_2^{\bar{F}}, \dots, y_n^{\bar{F}}\} = \left\{ \sum_{i=1}^p y_{i,1}^{\bar{F}}, \sum_{i=1}^p y_{i,2}^{\bar{F}}, \dots, \sum_{i=1}^p y_{i,n}^{\bar{F}} \right\} \quad (5)$$

Using the data points  $(x_1, y_1^{\bar{F}}), (x_2, y_2^{\bar{F}}), \dots, (x_n, y_n^{\bar{F}})$  and Lagrange interpolation function to get  $w(x)^{\bar{F}} = b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \dots + b_1x + b_0$  and is called internal P-information law as shown in Fig.1.

$\alpha_j$  is deleted from  $\alpha$ , which constitutes attribute set  $\nabla\alpha$ , then  $\alpha$  becomes  $\alpha^{\bar{F}}$

$$\alpha^{\bar{F}} = \alpha - \nabla\alpha \quad (6)$$

After deleting some attributes from  $\alpha$ , general function set  $S$  becomes function outer P-set  $S^F$ ,

$$S^F = \{s_1, s_2, \dots, s_r\} \quad (7)$$

Using the similar method to equations (1) and (2) to get the discrete data set  $y^F$  of  $S^F$ , and

$$y^F = \{y_1^f, y_2^f, \dots, y_n^f\} = \left\{ \sum_{i=1}^r y_{i,1}, \sum_{i=1}^r y_{i,2}, \dots, \sum_{i=1}^r y_{i,n} \right\} \quad (8)$$

Using the data points  $(x_1, y_1^f), (x_2, y_2^f), \dots, (x_n, y_n^f)$  and Lagrange interpolation function to get  $w(x)^F = c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \dots + c_1x + c_0$  and is called outer P-information law as shown in Fig.1.

**Definition 1** The information law pair consisted of  $w(x)^{\bar{F}}$  and  $w(x)^F$  is called P-information law generated by function P-sets  $(S^{\bar{F}}, S^F)$ , and

$$(w(x)^{\bar{F}}, w(x)^F) \quad (9)$$

**Definition 2**  $\langle a, w(x)^{\bar{F}}, b, w(x)^F \rangle$  is called band information law generated by  $(w(x)^{\bar{F}}, w(x)^F)$ , if  $w(x)^{\bar{F}}$  and  $w(x)^F$  are the up-boundary and down-boundary, respectively.  $a$  and  $b$  are two different common points on x-axis of  $\langle a, w(x)^{\bar{F}}, b, w(x)^F \rangle$ , respectively.

**Theorem 1** If  $w(x)^{\bar{F}}$  is the internal P-information law generated by function internal P-set  $S_\lambda^{\bar{F}}$ , then there exists attribute set  $\Delta\alpha \neq \emptyset$  to make  $\Delta\alpha$ , attribute set  $\alpha_\lambda^F$  of  $w(x)_\lambda^F$  and attribute set  $\alpha$  of  $w(x)$  satisfy the following equation.

$$(\alpha_\lambda^F - \Delta\alpha) - \alpha = \emptyset \quad (10)$$

**Theorem 2** If  $w(x)_\lambda^F$  is the outer P-information law generated by function outer P-set  $S_\lambda^F$ , then there exists attribute set  $\nabla\alpha \neq \emptyset$  to make  $\nabla\alpha$ , attribute set  $\alpha_\lambda^{\bar{F}}$  of  $w(x)_\lambda^{\bar{F}}$  and attribute set  $\alpha$  of  $w(x)$  satisfy the following equation.

$$(\alpha_\lambda^{\bar{F}} \cup \nabla\alpha) - \alpha = \emptyset \quad (11)$$

**Theorem 3** If  $\langle a, w(x)_\lambda^{\bar{F}}, b, w(x)_\lambda^F \rangle$  is the band information law generated by  $(w(x)_\lambda^{\bar{F}}, w(x)_\lambda^F)$ , the sufficient and necessary condition of information law hidden in  $\langle a, w(x)_\lambda^{\bar{F}}, b, w(x)_\lambda^F \rangle$  is that attribute set  $\alpha_\lambda^F$  and  $\alpha$ ,  $\alpha_\lambda^{\bar{F}}$  and  $\alpha$  satisfy equations (12) and (13), respectively.

$$\alpha_\lambda^F = \alpha \cup \Delta\alpha \quad (12)$$

$$\alpha_\lambda^{\bar{F}} = \alpha - \nabla\alpha \quad (13)$$

**Theorem 4** The sufficient and necessary condition of  $\langle a, w(x)_\lambda^{\bar{F}}, b, w(x)_\lambda^F \rangle$  being restored to  $w(x)$  is that the attribute set  $\alpha$ ,  $\alpha_\lambda^F$  and  $\alpha_\lambda^{\bar{F}}$  satisfy equations (14) and (15), respectively.

$$\alpha = \alpha_\lambda^F - \Delta\alpha \quad (14)$$

$$\alpha = \alpha_\lambda^{\bar{F}} \cup \nabla\alpha \quad (15)$$

## The application of information law hiding

### The hiding-reduction of $w(x)$ in $\langle a, w(x)^{\bar{F}}, b, w(x)^F \rangle$ and its application.

Given discrete data set  $y$ , which are the discrete values at  $t=1,2,3,4,5,6$  about important information law  $w(x)$ , and  $\alpha$  is the attribute set of  $w(x)$ . The name of  $\alpha_i \in \alpha$  is omitted.

$$y = \{y_1, y_2, y_3, y_4, y_5, y_6\} = \{1.1000, 1.2600, 1.6300, 1.4000, 1.3100, 1.5800\} \quad (16)$$

$$\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\} \quad (17)$$

Given  $\eta_i \in (0, 1)$ ,  $i=1,2,3,4$ , and  $\sum_{i=1}^4 \eta_i = 1$ .  $\eta_i$  is the separation coefficient, and  $\eta_1 = 0.18$ ,  $\eta_2 = 0.27$ ,  $\eta_3 = 0.35$ ,  $\eta_4 = 0.20$ . Using  $\eta_1 \sim \eta_4$  to separate  $y_i \in y$  into  $y_{i,1} = \eta_1 y_i$ ,  $y_{i,2} = \eta_2 y_i$ ,  $y_{i,3} = \eta_3 y_i$ ,  $y_{i,4} = \eta_4 y_i$ ,  $i=1,2,3,4,5,6$ . The separation of  $y_i$  about  $\eta_i$  is listed in the Table 1.

**Table 1.** The separation of discrete value  $y_i$  about  $\eta_i$  of information law  $w(x)$  with attribute set

	$\alpha$					
$k$	1	2	3	4	5	6
$y_{i,1}$	0.198	0.227	0.293	0.252	0.236	0.284
$y_{i,2}$	0.297	0.340	0.440	0.378	0.354	0.427
$y_{i,3}$	0.385	0.441	0.571	0.490	0.459	0.553
$y_{i,4}$	0.220	0.252	0.326	0.280	0.262	0.316

The data in the Table 1 and  $y_i$  in equation (9) satisfy  $y_i = y_{i,1} + y_{i,2} + y_{i,3} + y_{i,4} = \sum_{j=1}^4 y_{i,j}$ .

$\alpha_5, \alpha_6, \alpha_7$  are supplemented into  $\alpha$ , or  $\alpha_1, \alpha_3$  are deleted from  $\alpha$ , then  $\alpha$  becomes  $\alpha_i^F, \alpha_j^F$ , respectively.

$$\alpha_i^F = \alpha \cup \{\alpha_5, \alpha_6, \alpha_7\} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\} \quad (18)$$

$$\alpha_j^F = \alpha - \{\alpha_1, \alpha_3\} = \{\alpha_2, \alpha_4\} \quad (19)$$

$y_{i,3}, y_{i,4}$  are deleted from Table 1, then  $y$  becomes  $y_i^F$ , and Table 2 is obtained.

**Table 2.** The separation of discrete value  $y_i$  about  $\eta_i$  of  $w(x)_i^F$  with attribute set  $\alpha_i^F$

$k$	1	2	3	4	5	6
$y_{i,1}$	0.198	0.227	0.293	0.252	0.236	0.284
$y_{i,2}$	0.297	0.340	0.440	0.378	0.354	0.427

From Table 2 and equation (5),  $y_i^F$  is obtained, and

$$\begin{aligned} y_i^F &= \{y_1^F, y_2^F, y_3^F, y_4^F, y_5^F, y_6^F\} \\ &= (y_{1,1} + y_{1,2}), (y_{2,1} + y_{2,2}), (y_{3,1} + y_{3,2}), (y_{4,1} + y_{4,2}), (y_{5,1} + y_{5,2}), (y_{6,1} + y_{6,2}) \\ &= \{0.4950, 0.5670, 0.7335, 0.6300, 0.5895, 0.7110\} \end{aligned} \quad (20)$$

$y_{i,5}, y_{i,6}, y_{i,7}$  are supplemented into table 1, then  $y$  becomes  $y_j^F$ , and Table 3 is obtained.

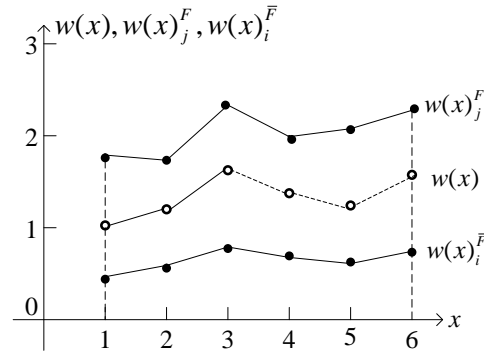
**Table 3.** The separation of discrete value  $y_i$  about  $\eta_i$  of  $w(x)_j^F$  with attribute set  $\alpha_j^F$

$k$	1	2	3	4	5	6
$y_{i,1}$	0.198	0.227	0.293	0.252	0.236	0.284
$y_{i,2}$	0.297	0.340	0.440	0.378	0.354	0.427
$y_{i,3}$	0.385	0.441	0.571	0.490	0.459	0.553
$y_{i,4}$	0.220	0.252	0.326	0.280	0.262	0.316
$y_{i,5}$	0.312	0.121	0.271	0.168	0.140	0.230
$y_{i,6}$	0.127	0.239	0.193	0.270	0.283	0.379
$y_{i,7}$	0.200	0.093	0.237	0.128	0.377	0.071

From Table 3 and equation (8),  $y_j^F$  is obtained, and

$$\begin{aligned} y_j^F &= \{y_1^F, y_2^F, y_3^F, y_4^F, y_5^F, y_6^F\} \\ &= \{(y_{1,1} + y_{2,1} + y_{3,1} + y_{4,1} + y_{5,1} + y_{6,1} + y_{7,1}), \dots, (y_{1,6} + y_{2,6} + y_{3,6} + y_{4,6} + y_{5,6} + y_{6,6} + y_{7,6})\} \\ &= \{1.7392, 1.7119, 2.3304, 1.9658, 2.1093, 2.2598\} \end{aligned} \quad (21)$$

Obviously,  $w(x)$ ,  $w(x)_i^F$  and  $w(x)_j^F$  generated by  $y_j^F$  satisfy  $w(x)_i^F < w(x) < w(x)_j^F$ , or  $w(x)$  is hidden in band information law  $< a, w(x)_i^F, b, w(x)_j^F >$ , which is shown in Fig.1.



**Fig.1**  $w(x)_i^{\bar{F}}$  and  $w(x)_j^F$  are expressed by real line. Information law  $w(x)$  is hidden in  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$ , and expressed by dotted lin, and  $a=1, b=6$ .

### The experimental verification of information law $w(x)$ reduction.

Data set  $y$  is obtained by the discretization to  $w(x)$ . Using the structure and characteristic of function P-sets: if attribute set  $\alpha$  is supplemented into some attributes, the elements in  $y$  will be reduced. If some attributes are deleted from  $\alpha$ , the elements in  $y$  will be added, so  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$  is obtained. Using theorem 4,  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$  is restored to  $w(x)$ . The error between  $w(x)$  obtained by reduction and  $w(x)$  generated by equation (16) is 0.0127%.

### Conclusion

Information law  $w(x)$  is hidden in band information law  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$ , and  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$  is not single, so it is difficult to steal  $w(x)$  from  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$ . This is because people do not know which attributes are supplemented into, and which attributes are deleted from  $\alpha$ , even the definition form and structure of  $\alpha_i$ . It is of no avail to steal  $w(x)$  from  $\langle a, w(x)_i^{\bar{F}}, b, w(x)_j^F \rangle$  by classical mathematical methods, which is proved by many experiments. The study given in the paper is a novel application of function P-sets in dynamic information system.

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