An Improved Fractional Differential Edge Detection Algorithm

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Abstract. In order to extract detailed edge information, a multi-scale fractional differential edge detection algorithm is proposed in this paper. Firstly, the G-L factional differential is applied to enhance image with two different fraction differential orders (one is small and the other is big), then, the edges can be gotten by subtraction the two enhanced images. Experiments and results showed that the proposed method can not only efficiently detect the edges information of simple objects, but also can detect the edges of complex objects.

Introduction

Edge detection aims at identifying points in a digital image at which the image brightness changes sharply or more formally has discontinuities. The points at which image brightness changes sharply are typically organized into a set of curved line segments termed edges. Edge detection can eliminate the irrelevant information, and can greatly reduce the amount of data, retain the image of important structural properties. Edge detection is not only the basic problem of image processing, but also an important part of computer recognition and understanding. It plays an important role in human and computer vision [1] [2].

The edge indicates the end of a feature region and the beginning of another feature region that contains most information of image. The purpose of edge detection is to identify the pixels whose gray level changes sharply within a spatial neighborhood. The differential methods, which are based on derivatives of the function with respect to position, are the natural choice for edge detection. Roberts, Sobel, Prewitt and Krish operators are edge detection operators based on the first order differential [3]. But, the first differential or gradient based operators can produce a wide response near the edge, which will affect the accuracy of the edge detection. As for the fact that the gradient or first order differential will have a wider response in the vicinity of the image edge, this will affect the accuracy of the edge location. Then, the 2nd order differential is applied to image edge detection, compared with the first order differential, the zero-crossings of the second order directional derivative or the extreme points are the edges [1] [3].

The edge and the noise are high frequency information in frequency domain. The noise is enhanced while the image is enhanced. Thus, the appropriate smoothing method is used before edge detection. The Laplacian of Gaussians (LoG) operator operator is based on the detection of zero-crossings of the Laplacian operator applied to a Gaussian-smoothed image. The difference of Gaussians (DoG) operator with strong robustness for noise is mentioned by Lowe has been shown to yield aesthetically pleasing edge lines without postprocessing [3]. The XDoG(eXtended Difference of Gaussians) operator presented by Winnem¨oller Obtains a range of subtle artistic effects, such as ghosting, speed-lines, negative edges, indication, and abstraction[1] [4]. SUSAN (Small Univalue Segment Assimilating Nucleus) operator based on local gray level difference with strong robustness for noise is an acronym standing for smallest univalue segment assimilating nucleus. The Canny edge detector is arguably the most popular such operator, due to its widespread use in the field of computer vision [5].

Fractional differential processing is not only nonlinearly enhancing the signal's high-frequency and middle-frequency components, but also nonlinearly keeping signal's low-frequency and direct current components [6] [7]. Many findings show that fractional differentiation is the powerful approaches for dealing with texture details [6] [7]. Mathieu presented an edge detector based on non-integer (fractional) differentiation which can improve the criterion of thin detection, or detection selectivity in the case of parabolic luminance transition [8]. A G-L fractional differentiation edge detector which can effectively extract edge information and has higher SNR than traditional operators is presented by YANG [9].

Though the fact that the fractional order differential operators have achieved good results in edge detection, the operators have a poor anti-noise ability and have weak edge detection of texture region. In order to overcome these shortcomings, an improved fractional differential edge detection algorithm was proposed.

Fractional Differential Edge Detection

It is necessary to remove the noise before using fractional differential methods to extract edge. The Gaussian kernel function has good local character in frequency and time domain, and it is the only kernel function that is multi-scale. The Gauss function is strong robustness for noise, and it is defined as

$$G_\sigma(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \tag{1}$$

Suppose the original image is $f(x, y)$, and the image filtered by Gaussian function is $g(x, y)$, then, we have

$$g(x, y) = G_\sigma(x, y) * f(x, y) \tag{2}$$

But too much smoothing will make the detection of edge difficult. It is necessary to enhance the image before edge detection. According to the differential properties of convolution, we have

$$\frac{d}{dt}[h(t) * f(t)] = \frac{d}{dt}\int f(\tau)h(t-\tau)d\tau = f(t) * \frac{d}{dt}h(t) \tag{3}$$

Using the fractional differential definition and (3), we have

$$D^v[h(t) * f(t)] = \frac{1}{\Gamma(-v)}\int_a^x (x-u)^{-v-1}\int f(\tau)h(t-\tau)d\tau du = f(t) * D^v(h(t)) \tag{4}$$

Part of the edge information is smoothed by Gaussian filter, these edges is very difficult to detect. And the fractional differential method can perfectly enhance image, and it can well recover the smoothed edge and texture information. So, it is necessary to enhance the smoothed information using fractional differential before edge extraction. Grünwald–Letnikov definition of derivatives of arbitrary order v is defined as [10]

$${}_aD_t^v = \lim_{h->0} \frac{1}{h^v} \sum_{m=0}^{\frac{t-a}{h}} \frac{(-1)^m \Gamma(v+1)}{m!\,\Gamma(v-m+1)} f(t-mh) \tag{5}$$

Where, the Gamma function is $\Gamma(n) = \int_0^\infty e^{-t}t^{n-1}dt = (n-1)!$. Taking the duration of signal $h \equiv 1$, the dispersion expression of 1-D signals fractional differential is expressed as

$$\frac{d^v f(t)}{dt^v} \approx f(t) + (-v)f(t-1) + \frac{(-v)(-v+1)}{2} f(t-2) + \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} f(t-n) \tag{6}$$

The rule of image processing with a nonlinear filter consists is to move the filter mask from point to point in an image. The linear filtering operation is given by expression [7]

$$g(x, y) = \sum_{s=-p}^{a} \sum_{t=-q}^{b} w(s, t) f(x+s, y+t) \tag{7}$$

Where $f(x, y)$ and $w(s, t)$ are a value of pixel and mask respectively. And the filtering mask is shown in figure1.

c_2	0	c_2	0	c_2
0	c_1	c_1	c_1	0
c_2	c_1	c_0	c_1	c_2
0	c_1	c_1	c_1	0
c_2	0	c_2	0	c_2

Figure 1. Fractional differential mask for image enhancement

Where $c_0 = 8$, $c_1 = -v$, $c_2 = \left(v^2 - v\right)/2$, to make the sum of the fractional differential mask be in $[0,1]$, each of the coefficient in figure 1 must be divided by $c = 8 - 12v + 4v^2$, and the mask is traditional fractional differential edge detection method. The fractional differential is a low pass filter; the original image is especial case when fractional differential order $v = 0$, and then the order can be extended to arbitrary order. Let $0 \le v_1 < 1$, $0 \le v_2 < 1$ and $v_2 > v_1$, to get more precise edges, the fractional differential mask is twice employed to enhance edge and texture with order v_1 and v_2 respectively, and the subtractive result of the two enhanced image is the edge of the image, and the process can be expressed as the following formula.

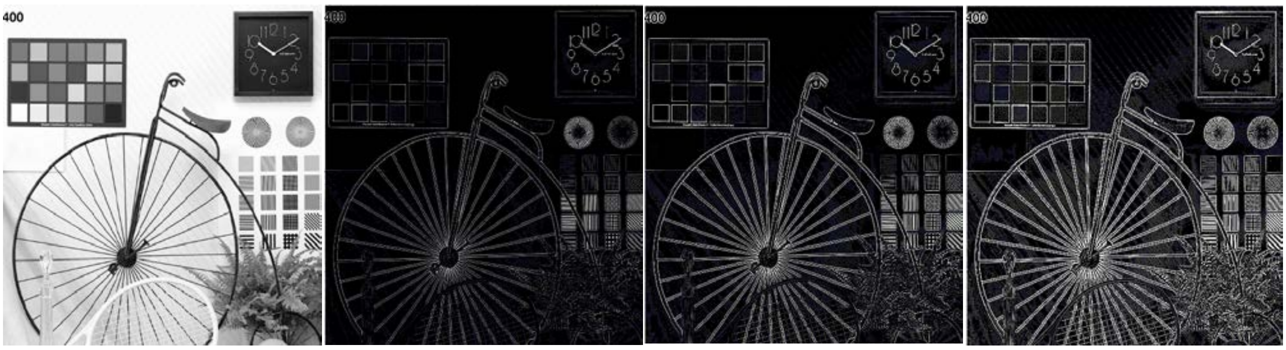
$$Df(x, y) = D^{v_2} f(x, y) - D^{v_1} f(x, y) \tag{8}$$

The traditional fractional differential based edge detection method is a special case of equation (8) when $v_1 = 0$.

Experiments and Result Analysis

According to figure 1, we adopt 5×5 mask. Fig.2 are the results when differential order v_1 = 0.3, and order v_2 changes from 0.7 to 0.9.

It can be seen that the fractional differential gives a quite different approach to edge detection. The edges are becoming more clear-cut, and the complex textural details are becoming clearer than before after employing the fractional differential mask. The edges become cleaner and cleaner with increasing of fractional order. The edges in these domains that the gray changes greatly are easy to detect, but the edges in these domain that the gray changes little are difficult to detect. When order v_2 is 0.6, the edges of most objects can be detected, but the edges in white background cannot be detected. When order v_2 is 0.9, part edges in white background can be detected. The bigger the fractional differentials order is, the more edges can be detected. Therefore, the edge detection based on fractional differential is multi-scale, and edge detection can be performed at different scales. Some examples of edge detection with fractional differential is given in figure 3.



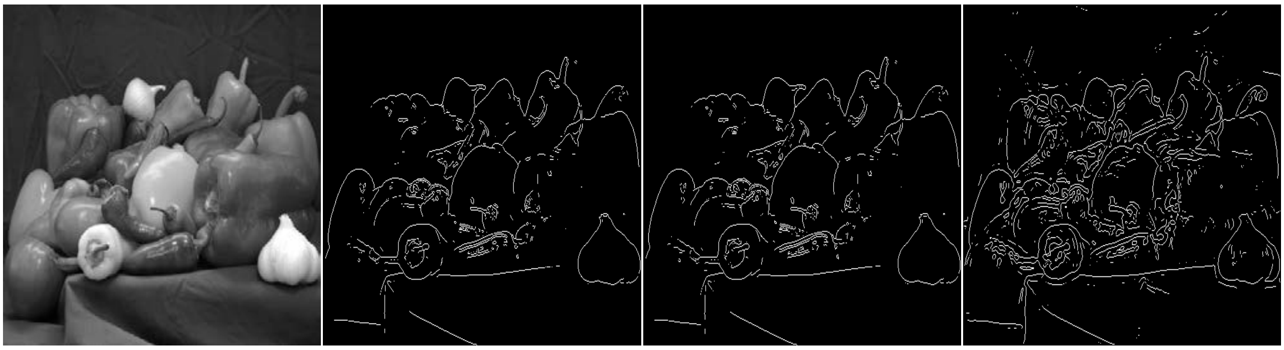
(a)Original image (b) v=0.7 (c) v=0.8 (d) v=0.9

Fig.1 Edge detection with different differential orders v=0.0, 0.7, 0.8, and 0.9, respectively

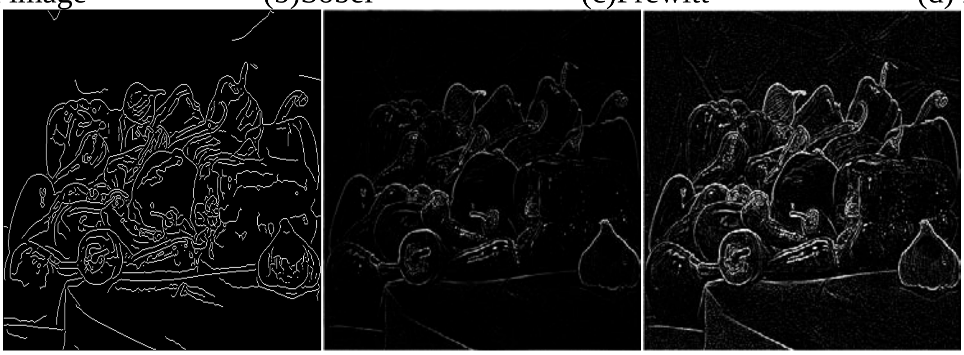
(a)Original image (b)Edge detection result (c) Original image (d))Edge detection result

Fig.2 Edge detection results by the proposed method with order v_1 =0.2, v_2=0.92

In our next experiment, the proposed method compares with Sobel operator, Prewitt operator, LoG operator, Laplacian operator, and Canny operator. The results of comparison are shown in Fig.4. It can be seen from Fig. 4 that all algorithms mentioned above can effectively detect the edges in the domain that gray changes greatly. The two garlic gross edges can effectively be detected, but lots of texture edges cannot be detected by the first-order differential operators, such as Sobel and Prewitt operator. The second-order differential operators, such as LoG and Laplacian operator can detect more edges than the first-order differential operators, and can detect part edges of garlic and other subject in the image. Canny operator is a good edge detection algorithm; it can detect more texture edges than the second-order differential operators, but cannot detect the edge of background object. The proposed method can not only detect the edges in area that gray changes greatly, but also can detect the edge of texture area, and it is an effective method for edge detection.



(a)Original image (b)Sobel (c)Prewitt (d) LoG



(e)Canny (f) Laplacian (g) The proposed method

Fig.3 Edge detection comparison

Conclusion

In this paper, a fractional differential method for edge detection is presented. The method has good edge detection ability. It can detect the edge in the area that gray changes greatly, at the same time, it also can detect the edge of complex object and texture. The method is multi-scale, and the two small fractional orders are needed when to detect rough edge information, but two fractional orders are needed when to detect a detailed edge information.

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