Study of Adaptive Sliding Mode Control Method Based on Disturbance Observer for Quadrotor Unmanned Aerial Vehicle

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Keywords: Adaptive Sliding Mode Control; Disturbance Observer; Quadrotor Unmanned Aerial Vehicle; Simulation

Abstract . Quadrotor Unmanned Aerial Vehicle is a kind of UAV, which is disturbed easily and has nonlinear and strong coupling characteristics. In the paper, the model of Quadrotor Unmanned Aerial Vehicle is presented, then the adaptive sliding mode control laws is designed based on the model. And the disturbance observer is lead in the control laws. The control laws can overcome the uncertainty and unmodelled dynamics in the model. In the end, the correctness of control laws is proved by the method of Mathematics and the simulations prove that the control laws ensure that Quadrotor UAV can flight steady.

Introduction

Quadrotor Unmanned Aerial Vehicle is a kind of UAV has simple mechanical structure and power supply, which can locate the hover and flight with low speed. This UAV can independently complete environmental detection and information gathering, such as tasks. It has huge application value and potential.

Quadrotor UAV is the kind of vehicle with nonlinear, strong coupling and interference sensitive. Study of control method is a hot issue. Stable flight control under the condition of the interference is one of the difficulties of the study. The paper designs the adaptive sliding mode control method based on disturbance observer with the presence of interference for aircraft.

The research object

Quadrotor UAV is an underactuated system. Independent of the motor drive the propellers, which distribution the crossed with completely symmetric structure, as is shown in figure 1.

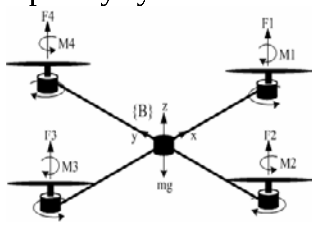


Fig.1: Quadrotor UAV model structure.

Pitch and yaw movement, roll movement and vertical movement are main movement methods of this UAV. Different ways of movement associated with the rotation direction and speed of four rotors.

The mathematical model of reference [1] is used in the paper. As shown in the equations (1) and (2).

$$\begin{cases} \dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta & \dot{r} = pq(J_X - J_Y)/J_Z + U_3/J_Z \\ \dot{\theta} = q\cos\phi - r\sin\phi & \ddot{x} = [-k_{f1}|\dot{x}|\dot{x} + (c\phi c\varphi s\theta + s\phi s\varphi)U_4]/m \\ \dot{\varphi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta & \ddot{y} = (-k_{f2}|\dot{y}|\dot{y} + c\phi s\varphi s\theta - s\phi c\varphi)U_4]/m \\ \dot{p} = qr(J_Y - J_z)/J_X + qJ_r\Omega_r/J_X + U_1/J_X & \ddot{z} = [-k_{f3}|\dot{z}|\dot{z} - mg + (c\phi c\theta)U_4]/m \\ \dot{q} = pr(J_Z - J_X)J_Y - pJ_r\Omega_r/J_Y + U_2/J_Y & \dot{\Omega}_i = -\Omega_i/\tau_M - d\Omega^2_i/\eta J_M + u_{Mi}/R_M\tau_M \end{cases} \quad (1)$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} lC_T & 0 & -lC_T & 0 \\ 0 & lC_T & 0 & -lC_T \\ C_Q & -C_Q & C_Q & -C_Q \\ C_T & C_T & C_T & C_T \end{pmatrix} \begin{pmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{pmatrix} \tag{2}$$

ϕ, θ, φ are the roll angle, pitch angle and yaw angle. p, q, r are the angular velocity component. x, y, z is location component [2]. J_x, J_y, J_z are the moment of inertia. U_1, U_2, U_3, U_4 are three torque and the total force. J_r is the total moment of inertia, Ω_r is total turning speed. k_{f1}, k_{f2} and k_{f3} are air drag coefficient, k_M is torque coefficient, k_E is electromotive force coefficient. τ_d is friction torque motor. J_M is moment of inertia of the motor, d is drag coefficient, η is motor efficiency. $\Omega_i(1,2,3,4)$ is motor speed. C_T is lift the coefficient, C_Q is reverse proportion coefficient. State variables [3] of UAV are

$$X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \varphi, \dot{\varphi}, z, \dot{z}, x, \dot{x}, y, \dot{y}] = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}] \tag{3}$$

Four control input variables of Quadrotor UAV are:

$$[M_x \quad M_y \quad M_z \quad \sum_{i=1}^{4} F_i] = [U_1 \quad U_2 \quad U_3 \quad U_4] \tag{4}$$

Writing the original system model to the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (a_1 x_4 x_6 + l x_4) / J_x + B_1(U_1 + F_1) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = (a_2 x_2 x_6 - l x_2) / J_y + B_2(U_2 + F_2) \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = (a_3 x_2 x_4) / J_z + B_3(U_3 + F_3) \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = (-k_{f1} |x_8| x_8 - mg + B_4 U_4 + F_4) / m \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = (-k_{f2} |x_{10}| x_{10} + B_5 U_4 + F_5) / m \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = (-k_{f3} |x_{12}| x_{12} + B_6 U_4 + F_6) / m \end{cases} \tag{5}$$

k_{f1}, k_{f2}, k_{f3} are the air drag coefficient [4]. F_i is the total uncertainty. $\Delta f_i(X,t) = f_i(X,t) - f_{im}(X,t)$ is the real model and nominal model uncertainty, d_i is the interference.

$$\begin{cases} F_i = \Delta f_i(X,t) + d_i \\ |F_i| \le \bar{F}, i = 1, 2, \ldots 6 \end{cases} \tag{6}$$

$$\begin{cases} a_1 = J_y - J_z, a_2 = J_z - J_x & B_4 = \mathrm{c}\, x_1 \, \mathrm{c}\, x_3 \\ a_3 = J_x - J_y, l = J_r \Omega_r & B_5 = \mathrm{c}\, x_1 \, \mathrm{s}\, x_3 \, \mathrm{c}\, x_5 + \mathrm{s}\, x_1 \, \mathrm{s}\, x_5 \\ B_1 = 1 / J_x, B_2 = 1 / J_y, B_3 = 1 / J_z & B_6 = \mathrm{c}\, x_1 \, \mathrm{s}\, x_3 \, \mathrm{s}\, x_5 - \mathrm{s}\, x_1 \, \mathrm{c}\, x_5 \end{cases} \tag{7}$$

Design for control method

After using disturbance observer, roll angle subsystem can be converted into the following form:

$$\dot{x}_2 = (a_1 x_4 x_6 + l x_4) / J_x + B_1(U_{S1} - U_{F1}) + B_1 F_1 = (a_1 x_4 x_6 + l x_4) / J_x + B_1(U_{S1} + \tilde{F}_1) \tag{8}$$

The system is compensated by disturbance observer and disturbance is from F_1 to $\tilde{F}_1$ [5].The original interference model of the roll angle is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (a_1 x_4 x_6 + l x_4) / J_x + B_1(U_{S1} + \tilde{F}_1) \end{cases} \tag{9}$$

For the new model, the paper assumes the roll expectations as x_{1d} [6], and defines tracking error variables and the sliding mode surface as equations (10) and (11).

$$\begin{cases} z_1 = x_1 - x_{1d} \\ z_2 = x_2 - \dot{x}_{1d} + c_1 z_1 \end{cases} \tag{10}$$

$$S_1 = k_1\int z_1 + z_2 \tag{11}$$

Constructing the Lyapunov function as follows:

$$V_1 = \frac{1}{2}J_x S_1^2 + \frac{1}{2\rho_1}\tilde{\theta}_1^2 + \frac{1}{2\gamma_1}\tilde{\eta}_1^2 + \frac{1}{2}\tilde{F}_1^2 \tag{12}$$

Derivativing the equation (12) to:

$$\begin{aligned}\dot{V}_1 &= J_x S_1\dot{S}_1 + \frac{1}{\rho_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{\gamma_1}\tilde{\eta}_1\dot{\hat{\eta}}_1 + \tilde{F}_1\dot{\tilde{F}}_1 = J_x S_1[c_2 z_1 + c_1(z_2 - c_1 z_1) + J_x\dot{x}_2 - J_x\ddot{x}_{1d}] + \frac{1}{\rho_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{\gamma_1}\tilde{\eta}_1\dot{\hat{\eta}}_1 + \tilde{F}_1\dot{\tilde{F}}_1 \\ &= S_1[J_x c_2 z_1 + J_x c_1(z_2 - c_1 z_1) + a_1 x_4 x_6 + l x_4 + U_{S1} + \tilde{F}_1 - J_x\ddot{x}_{1d}] + \frac{1}{\rho_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{\gamma_1}\tilde{\eta}_1\dot{\hat{\eta}}_1 + \tilde{F}_1\dot{\tilde{F}}_1\end{aligned} \tag{13}$$

Selecting state variables:

$$q_1 = \ddot{x}_{1d} - c_2 z_1 + c_1(z_2 - c_1 z_1) \tag{14}$$

Designing the roll angle control law completes state error convergence.

$$U_{S1} = \hat{\theta}_1 q_1 - a_1 x_4 x_6 - l x_4 - h_1 S_1 - \hat{\eta}_1\,\mathrm{sgn}(S_1) \tag{15}$$

Selecting parameters, and adaptive laws of the switch gain：

$$\begin{cases}\dot{\hat{\theta}}_1 = -\rho_1 q_1 S_1 \\ \dot{\hat{\eta}}_1 = \gamma_1|S_1|\end{cases} \tag{16}$$

Taking equations (14), (15), (16) into equation (13) to get the following:

$$\begin{aligned}\dot{V}_1 &= S_1[-\theta_1 q_1 + a_1 x_4 x_6 + l x_4 + U_{S1} + \tilde{F}_1] - \tilde{\theta}_1 q_1 S_1 + \tilde{\eta}_1|S_1| - bB_1\tilde{F}_1^2 = -h_1 S_1^2 + \tilde{F}_1 S_1 - \hat{\eta}_1|S_1| + \tilde{\eta}_1|S_1| - bB_1\tilde{F}_1^2 \\ &\le -h_1 S_1^2 + \eta_1|S_1| - \hat{\eta}_1|S_1| + \tilde{\eta}_1|S_1| - bB_1\tilde{F}_1^2 \\ &\le -h_1 S_1^2 - bB_1\tilde{F}_1^2\end{aligned} \tag{17}$$

The control law can ensure that the system is asymptotically stable because of $B_1 > 0$ [7].

The paper designs pitch and yaw angle disturbance observer by the same methods. The control inputs are compensated by the observations values of disturbance observer and system gets new disturbance model. Select control inputs

$$U_i = U_{Si} - U_{F_i} \tag{18}$$

Where $U_{F_i} = \hat{F}_i (i = 2,3)$ is the equivalent control input compensation[8]. Take state variables:

$$\begin{cases}q_2 = \ddot{x}_{3d} - c_4 z_3 - c_3(z_4 - c_3 z_3) \\ q_3 = \ddot{x}_{5d} - c_6 z_5 - c_5(z_6 - c_5 z_5)\end{cases} \tag{19}$$

Designing Pitch angle and Yaw angle control laws based on the system compensated by disturbance observer.

$$\begin{cases}U_{S2} = \hat{\theta}_2 q_2 - a_2 x_2 x_6 + l x_2 - h_2 S_2 - \hat{\eta}_2\,\mathrm{sgn}(S_2) \\ U_{S3} = \hat{\theta}_3 q_3 - a_3 x_2 x_4 - h_3 S_3 - \hat{\eta}_3\,\mathrm{sgn}(S_3)\end{cases} \tag{20}$$

Selecting parameters, and adaptive laws of the switch gain：

$$\begin{cases}\eta_2 = \gamma_2|S_2| \\ \hat{\theta}_2 = -\rho_2 q_2 S_2\end{cases} \qquad \begin{cases}\eta_3 = \gamma_3|S_2| \\ \hat{\theta}_3 = -\rho_3 q_3 S_3\end{cases} \tag{21}$$

Designing disturbance observer for location channel:

$$\begin{cases}\hat{F}_1 = Z + P(X) \\ \dot{Z} = -L(X)B_1[Z + P(X)] - L(X)f_1(X, U_1)\end{cases} \tag{22}$$

Selecting the equivalent control input compensation for:

$$U_{F_i} = \hat{F}_i (i = 4,5,6) \tag{23}$$

For the new disturbance model, the paper defines height error variables and the sliding surface as equations (17) and (18) before designing height control law.

$$\begin{cases}z_7 = x_7 - x_{7d} \\ z_8 = x_8 - \dot{x}_{7d} + c_7 z_7\end{cases} \tag{24}$$

$$S_4 = k_7\int z_7 + z_8 \tag{25}$$

Constructing the Lyapunov function as follows:

$$V_4 = \frac{1}{2} m S_4{}^2 + \frac{1}{2\rho_4} \tilde{m}^2 + \frac{1}{2\gamma_4} \tilde{\eta}_4{}^2 + \frac{1}{2} \tilde{F}_4{}^2 \tag{26}$$

Selecting state variables:

$$q_4 = \ddot{x}_{7d} - c_8 z_7 - c_7 (z_8 - c_7 z_7) + g \tag{27}$$

Derivativing the equation (26) to:

$$\dot{V}_4 = S_4[-mq_4 - k_{f1}|x_8|x_8 + B_4 U_{s4} + F_4 - B_4 \hat{F}_4] + \frac{1}{\lambda_1} \tilde{m}\dot{\hat{m}} + \frac{1}{\gamma_4} \tilde{\eta}_4 \dot{\hat{\eta}}_4 + \tilde{F}_4 \dot{\hat{F}}_4 \tag{28}$$

Designing height control law:

$$U_{s4} = B_4^{-1}[\hat{m} q_4 + k_{f1}|x_8|x_8 - h_4 S_4 - \hat{\eta}_4 \operatorname{sgn}(S_4)] \tag{29}$$

Selecting parameters adaptive law:

$$\begin{cases} \dot{\hat{m}} = -\lambda_1 q_4 S_4 \\ \dot{\hat{\eta}}_4 = \gamma_4 |S_4| \end{cases} \tag{30}$$

Selecting $\tilde{F}_4^* = F_4 - B_4 \hat{F}_4$, and taking equations (14), (15) and (16) into equation (13), and select proper control gain $L(X) = b, b > 0$ [9]. And b is constant. Design nonlinear function: $P(X) = bx_2$.

$$\begin{aligned} \dot{V}_4 &= S_4[(\hat{m} - m) q_4 + \tilde{F}_4^* - h_4 S_4 - \hat{\eta}_4 \operatorname{sgn}(S_4)] - \tilde{m} q_4 S_4 + \tilde{\eta}_4 |S_4| - b\tilde{F}_4^2 \\ &= S_4[\tilde{F}_4^* - h_4 S_4 - \hat{\eta}_4 \operatorname{sgn}(S_4)] + \tilde{\eta}_4 |S_4| - b\tilde{F}_4^2 \le -h_4 S_4{}^2 + (\eta_4 - \hat{\eta}_4)|S_4| + \tilde{\eta}_4 |S_4| - b\tilde{F}_4^2 \le -h_4 S_4{}^2 - b\tilde{F}_4^2 \end{aligned} \tag{31}$$

Proved through theory, the designed control laws can ensure that the height subsystem is asymptotically stable. And the control laws can effective compensate "generalized interference"[10]. Then selecting

$$\begin{cases} q_5 = \dot{x}_{9d} - c_{10} z_9 - c_9 (z_{10} - c_9 z_9) \\ q_6 = \dot{x}_{11d} - c_{12} z_{11} - c_{11} (z_{12} - c_{11} z_{11}) \end{cases} \tag{32}$$

Designing the horizontal direction location control laws.

$$\begin{cases} U_x = U_4^{-1}[\hat{m}_x q_5 + k_{f2}|x_{10}|x_{10} - h_5 S_5 - \hat{\eta}_5 \operatorname{sgn}(S_5)] \\ U_y = U_4^{-1}[\hat{m}_y q_6 + k_{f3}|x_{12}|x_{12} - h_6 S_6 - \hat{\eta}_6 \operatorname{sgn}(S_6)] \end{cases} \tag{33}$$

Selecting parameters, and adaptive laws of the switch gain：

$$\begin{cases} \dot{\hat{m}}_x = -\lambda_x q_5 S_5 \\ \dot{\hat{\eta}}_5 = \gamma_5 |S_5| \end{cases} \qquad \begin{cases} \dot{\hat{m}}_y = -\lambda_y q_6 S_6 \\ \dot{\hat{\eta}}_6 = \gamma_6 |S_6| \end{cases} \tag{34}$$

Simulation results

The paper makes the assumptions for the interferences of the model for checking observation effect of the disturbance observer.

$$\begin{cases} F_1 = 0.2 \sin t \sin(x_2{}^2) \cos(x_1) + 5(\sin \pi t + \sin 0.1\pi t) \qquad F_3 = 0.2 \cos t \sin(x_5) + 5(\sin \pi t + \sin 0.1\pi t) \\ F_2 = 0.3 e^{-t} \cos(x_3) \sin(x_3) + 5(\sin \pi t + \sin 0.1\pi t) \qquad F_i = 5(\sin \pi t + \sin 0.1\pi t), (i = 4, 5, 6) \end{cases} \tag{35}$$

Selecting the estimate values as the input compensate amplitude and assuming the value is 1.Thus observation errors is the new disturbance models after "generalized interference" compensated. Simulation results are shown as follow:

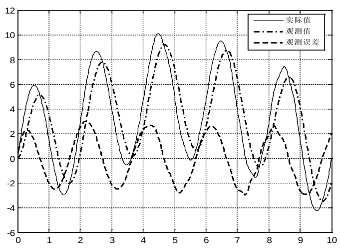


Fig.2: Disturbance estimation of roll angle.

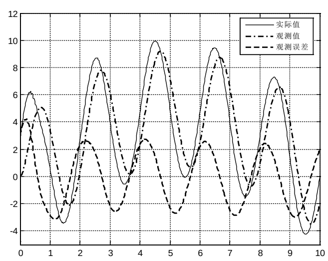
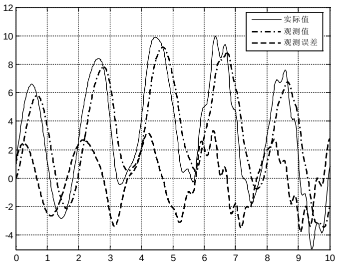


Fig.3: Disturbance estimation of pitch angle.



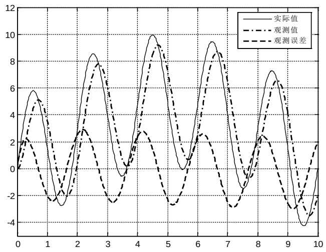
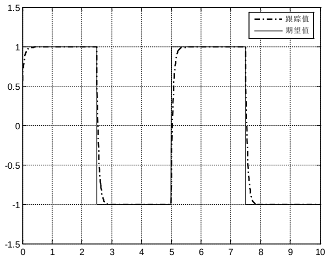
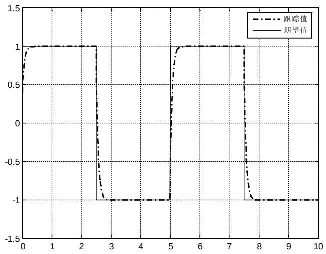


Fig.4: Disturbance estimation of yaw angle. Fig.5: Disturbance estimation of position.

The paper selects the parameters for the simulation on the control algorithm [7]. $b=15$, $\gamma_i = 20(i=1,2,\cdots 6)$, $\lambda_1 = 20$, $\lambda_x = \lambda_y = 15$, $c_i = 15(i=1,2,\cdots 12)$, $\rho_i = 20(i=1,2,3)$ $\eta_i = 5.0(i=1,2,\cdots 6)$, $\varepsilon = 0.01$, $h_i = 20,(i=1,2,\cdots 6)$, initial value of parameter estimation is 0, the initial value of parameter estimate for interference is 0, the simulation results as follows:





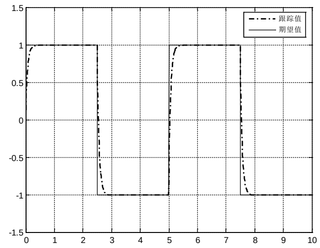
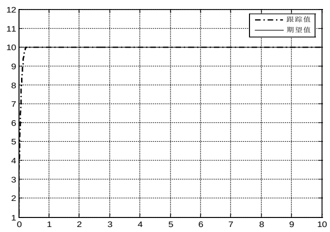
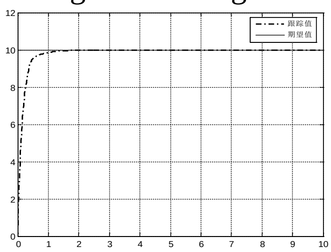


Fig.6: Roll angle curve. Fig.7: Pitch angle curve. Fig.8: Yaw angle curve.





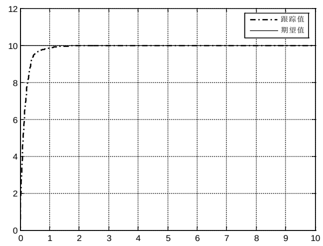


Fig.9: Altitude curve. Fig.10: The X axis Tracking. Fig.11: The Y axis Tracking.

Conclusions

Observations values gain through using disturbance observer to estimate of "generalized interference", which is used to compensate the equivalent control inputs. In the paper, the methods have the advantages in solving "generalized interference" uncertainty about the unknown. Firstly, the compensation can reduce that model uncertain neutral section effect the precision of the system, secondly, After compensation, the "generalized interference" upper bound is decreased, and when a switch gain adaptive law is designed, the switch gain range is also reduced, thus the chattering phenomenon is reduced. The simulation results prove that the control laws are right with robustness.

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