Odd-elegant Labeling Algorithm of Generalized Ring Core Networks

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Abstract. In the network design, the choice of the network topology plays a decisive role for the realization of the function of computer network efficiency. The labeling problem of the computer network topology directly affects the network design and communication costs, etc. Generalized ring network topology is a very important hybrid network topology structure, and generalized ring core network is its base. In this paper, based on the requirements of research of generalized ring network addressing, the author designs the *GRN-OEL-algorithm* when $n_1, n_2, \cdots, n_m \equiv 0 (\bmod 4)$, proves odd-elegant of the generalized ring core network, works out the corresponding software, and tests the practical effectiveness of this algorithm with our experimental data.

Introduction

Computer network is the result of the close combination, mutual penetration, mutual promotion, and common development between computer technology and communication technology. In the network design, the choice of the network topology plays a decisive role for the realization of the function of computer network efficiency. The labeling problem of the computer network topology directly affects the network design and communication costs, etc. The labeling of network topology refers to a mapping of integer set to the node or edge of network topology, and it satisfies certain conditions. According to different conditions, the researchers defined several types of labeling of network topology and put forward many conjectures. In 1981, Chang put forward the concept of elegant labeling and guessed that [1]: all tree topology are elegant. In 2013, Zhou et al., defined the concept of odd-elegant labeling and put forward the hypothesis that [2]: all tree topology are odd-elegant. Bus topology, star topology and ring network topology are the three basic network topologies, and the labeling problems of them lays a necessary theoretical basis for the design of the computer network system. However, the singleness of their structure seriously affects the extension of network function. Therefore, the new topology structure has been organic combined with two or more a single topology structure, which, has become an important research topic for researchers of computer theory and application, especially for the web worker[3-6]. In 2007, Gao Zhenbin proved the odd-graceful of three union structure $\bigcup P_n$, $\bigcup S_n$ and $\bigcup C_n$,(n is a multiple of 4) [7]. In 2009, Barrientons proved the odd-graceful of topology structure of tree with its diameter of no more than five [8]. In 2013, Zhou et al. proved the odd-elegant of lobster -----a hybrid topology structure [2].

Generalized ring network is a very important hybrid network topology structure, which refers to a number of closed rings formed of nodes linked together point to point and end to end in network and each nodes of rings has a line nodes(we call line nodes *leaves*). The information both can be translated in a direction between nodes in the loop, and can be translated between ring nodes and leaves. After the leaves of generalized ring network are removed, we call the rests the generalized ring core network. Obviously, generalized ring core network is the basis of generalized ring network. In this paper, based on the requirements of research of generalized ring network addressing, the author designs the *GRN-OEL algorithm* when $n_1, n_2, \cdots, n_m \equiv 0 (\bmod 4)$, proves odd-elegant of this network topology, works out the corresponding software, and tests the practical effectiveness of this algorithm with our experimental data.

Preliminary knowledge

We begin with simple, finite and undirected network topology $G=(V,E)$ with V for nodes set and E for edges set. For the network topology G, positive integer p is called as the nodes number of G, and q is called as the edges number of G. For the sake of simplicity, the shorthand symbol $[m,n]$ stands for an integer set $\{m,m+1,\cdots,n\}$, where m and n are integers with $0\le m<n$.

Definition 1 [2]**.** A function f is called odd-elegant labeling of a graph G if $f:V\to[0,2q-1]$ is injective and the induced function $f^*:E(G)\to\{1,3,\cdots,2q-1\}$ defined as $f^*(e)=|f(u)+f(v)| \mod 2q$, $(e=uv)$ is bijection. The graph which admits odd-elegant labeling is called an odd-elegant graph.

Definition 2. Let m is a integer not less than 2, and let $n_1,n_2,\cdots,n_m\equiv 0(\mathrm{mod}\,4)$ are all positive integer. The nodes in each ring C_{n_i} $(i\in[1,m])$ will be ordered into $v_{i1},v_{i2},\cdots,v_{in_i}$ in the same direction. And let the node $v_{i(n_i-1)}$ of n_i-1 of ring C_{n_i} $(i\in[1,m])$ is coincided with first node $v_{(i-1)1}$ of ring $C_{n_{i-1}}$, then a network topology $G=\omega_{n_1n_2\cdots n_m}$ including m rings is defined, which is called as generalized ring core network if $n_1,n_2,\cdots,n_m\equiv 0(\mathrm{mod}\,4)$ (figure 1).

Odd-elegant labeling algorithm of generalized ring core network

Let $G=\omega_{n_1n_2\cdots n_m}$ be a generalized ring core network if $m,n_1,n_2,\cdots,n_m\in N^*$ and $n_1,n_2,\cdots,n_m\equiv 0\ (\mathrm{mod}\,4)$, then: $V=\{v_{11}(v_{2(n_2-1)}),v_{12},\cdots,v_{1(n_1-1)},v_{1n_1},v_{21}(v_{3(n_3-1)}),\ v_{22},\cdots,v_{2(n_2-2)},\ v_{2n_2},\cdots,v_{(m-1)1}(v_{m(n_m-1)}),\ v_{(m-1)2},\ v_{(m-1)3},\cdots,v_{(m-1)(n_{m-1}-2)},v_{(m-1)n_{m-1}},\ v_{m1},v_{m2},\cdots,v_{m(n_m-2)},v_{mn_m}\}$, and $p=\sum_{k=1}^{m}n_k-m+1,\ \ q=\sum_{k=1}^{m}n_k$.

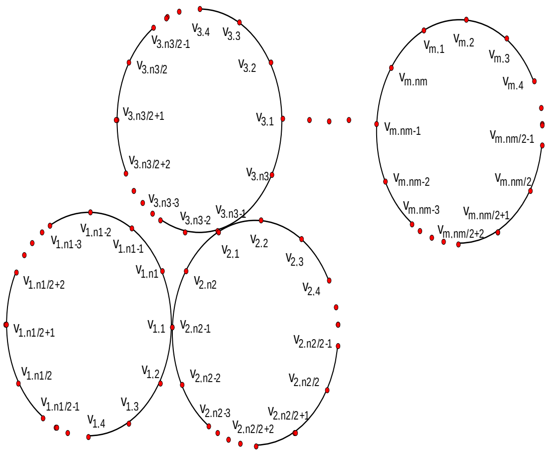

Fig. 1. Generalized ring core network with labeled vertices

The node been overlap labeling is regarded as two different nodes $v_{(i-1)1}$ and $v_{i(n_i-1)}$ $(i\in[2,m])$. All nodes can be divided into three parts:

$V_1=\{v_{i(2j-1)}\mid i\in[1,m],\ j\in[1,\frac{n_i}{4}]\}$, $V_2=\{v_{i(2j-1)}\mid i\in[1,m],\ j\in[\frac{n_i}{4}+1,\frac{n_i}{2}]\}$, $V_3=\{v_{i(2j)}\mid i\in[1,m],\ j\in[1,\frac{n_i}{2}]\}$.

Edge set $E=\{v_{ij}v_{i(j+1)},\ v_{in_i}v_{i1}\mid i\in[1,m],\ j\in[1,\ n_i-1]\}$ according to the location of ring can be divided into m parts:

$E_1=\{v_{11}v_{12},\ v_{12}v_{13},\cdots,\ v_{1(n_1-1)}v_{1n_1},v_{11}v_{1n_1}\}$, $E_2=\{v_{21}v_{22},\ v_{22}v_{23},\cdots,\ v_{2(n_2-1)}v_{2n_2},v_{2n_2}v_{21}\}$, ……, $E_m=\{v_{m1}v_{m2},\ v_{m2}v_{m3},\cdots,\ v_{m(n_m-1)}v_{mn_m},v_{mn_m}v_{m1}\}$.

On the basis of the above classification, we construct an odd-elegant labeling algorithm of generalized ring core network $\omega_{n_1n_2\cdots n_m}$ (here referred to as *GRN-OEL- ALGORITHM*) and it is as follows (Table 1).

Table 1. GRN-OEL-ALGORITHM

Algorithm: *GRN-OEL- ALGORITHM*

Input: The number of rings m of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ .The number of nodes n_i ($i = 1, 2, \cdots, m$) of each ring C_{n_i} .

Output: Odd-elegant labeling of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ **.**

1. Construct a generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ with *p* nodes and *q* edges ($p = \sum_{k=1}^{m} n_k - m + 1,\ \ q = \sum_{k=1}^{m} n_k$) and let $n_{m+1} = 0$.

2. Label the node $v_{i(2j-1)}(i \in [1, m],\ j \in [1, \frac{n_i}{4}])$ according to the labeling function $f(v_{i(2j-1)}) = \sum_{k=i+1}^{m+1} n_k + 2(j-1)$.

3. Label the node $v_{i(2j-1)}(i \in [1, m],\ j \in [\frac{n_i}{4}+1, \frac{n_i}{2}])$ according to the labeling function $f(v_{i(2j-1)}) = \sum_{k=i+1}^{m+1} n_k + 2j$.

4. Label the node $v_{i(2j)}(i \in [1, m],\ j \in [1, \frac{n_i}{2}])$ according to the labeling function $f(v_{i(2j)}) = \sum_{k=i+1}^{m+1} n_k + 2j - 1$.

5. Label the edge $uv \in E$ of $\omega_{n_1 n_2 \cdots n_m}$ according to the function $g(uv) = | f(u) + f(v) | \bmod 2q$ for $\forall u, v \in V(G)$, $u \neq v$.

6. Output a odd-elegant labeling of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$.

Then let's prove the correctness of *GRN-OEL- ALGORITHM*.

Theorem 1: Let $\omega_{n_1 n_2 \cdots n_m}$ be a generalized ring core network if $m, n_1, n_2, \cdots, n_m \in N^*$ and $n_1, n_2, \cdots, n_m \equiv 0 (\bmod 4)$, then the *GRN-OEL- ALGORITHM* determines a odd-elegant labeling of $\omega_{n_1 n_2 \cdots n_m}$.

Proof: For every generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ if $m, n_1, n_2, \cdots, n_m \in N^*$ and $n_1, n_2, \cdots, n_m \equiv 0 (\bmod 4)$, it has $p = \sum_{k=1}^{m} n_k - m + 1$ nodes and has $q = \sum_{k=1}^{m} n_k$ edges. Let $n_{m+1} = 0$ and classify the nodes of $\omega_{n_1 n_2 \cdots n_m}$ into three sets V_1, V_2, V_3 , then for every node of three sets label according to the labeling function f proposed in the *GRN-OEL- ALGORITHM* :

$$\begin{cases} f(v_{i(2j-1)}) = \sum_{k=i+1}^{m+1} n_k + 2(j-1), & i \in [1, m],\ j \in [1, \frac{n_i}{4}]; \\ f(v_{i(2j-1)}) = \sum_{k=i+1}^{m+1} n_k + 2j, & i \in [1, m],\ j \in [\frac{n_i}{4}+1, \frac{n_i}{2}]; \\ f(v_{i(2j)}) = \sum_{k=i+1}^{m+1} n_k + 2j - 1, & i \in [1, m],\ j \in [1, \frac{n_i}{2}]. \end{cases}$$

First, according to the labeling function f, we are able to calculate all node labeling of $\omega_{n_1 n_2 \cdots n_m}$:

$f(V_1) = \{ \sum_{k=i+1}^{m+1} n_k + 2(j-1) \mid i \in [1, m],\ j \in [1, \frac{n_i}{4}] \} = \{ \sum_{k=2}^{m} n_k, \sum_{k=2}^{m} n_k + 2, \cdots, \sum_{k=2}^{m} n_k + \frac{n_1}{2} - 2,\ \sum_{k=3}^{m} n_k,\ \sum_{k=3}^{m} n_k + 2,\ \cdots,$ $\sum_{k=3}^{m} n_k + \frac{n_2}{2} - 2,\ \cdots\cdots,\ n_m,\ n_m + 2,\ \cdots,\ n_m + \frac{n_{m-1}}{2} - 2,\ 0, 2, 4, 6,\ \cdots,\ \frac{n_m}{2} - 4, \frac{n_m}{2} - 2 \}$.

$f(V_2) = \{ \sum_{k=i+1}^{m+1} n_k + 2j \mid i \in [1, m],\ j \in [\frac{n_i}{4}+1, \frac{n_i}{2}] \} = \{ \sum_{k=2}^{m} n_k + \frac{n_1}{2} + 2, \sum_{k=2}^{m} n_k + \frac{n_1}{2} + 4, \cdots, \sum_{k=2}^{m} n_k + n_1 - 2,$ $\sum_{k=2}^{m} n_k + n_1, \sum_{k=3}^{m} n_k + \frac{n_2}{2} + 2,\ \sum_{k=3}^{m} n_k + \frac{n_2}{2} + 4,\ \cdots, \sum_{k=3}^{m} n_k + n_2 - 2, \sum_{k=3}^{m} n_k + n_2,\ \cdots\cdots;\ n_m + \frac{n_{m-1}}{2} + 2,\ n_m + \frac{n_{m-1}}{2} + 4,$ $\cdots,\ n_m + n_{m-1} - 2,\ n_m + n_{m-1},\ \frac{n_m}{2} + 2,\ \frac{n_m}{2} + 4, \cdots,\ n_m - 2,\ n_m \}$.

$f(V_3) = \{ \sum_{k=i+1}^{m+1} n_k + 2j - 1 \mid i \in [1, m],\ \ j \in [1, \frac{n_i}{2}] \} = \{ \sum_{k=2}^{m} n_k + 1, \sum_{k=2}^{m} n_k + 3, \cdots, \sum_{k=2}^{m} n_k + n_1 - 1,\ \sum_{k=3}^{m} n_k + 1, \sum_{k=3}^{m} n_k + 3, \cdots,\ , \cdots,$ $\sum_{k=3}^{m} n_k + n_2 - 1,\ n_m + 1, n_m + 3, \cdots, n_m + n_{m-1} - 1,\ 1, 3, \cdots,\ n_m - 1 \}$.

Let the node label sets of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ is $f(V)$, then $f(V)=\bigcup_{j=1}^{3} f(V_j)$. Easy to know from expressions $f(V_j),\ j\in[1,3]$ that other any two sets are disjoint sets except $f(v_{i1}) = f(v_{(i+1)(n_{i+1}-1)})$ ($i\in[1,m-1]$) in number set $f(V_1)$ and $f(V_2)$. So, label function f is injective from nodes set V to number set $[0, 2\sum_{k=1}^{m} n_k - 1]$ $=[0,2q-1]$ of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$.

Second, for $\forall e=uv\in E(G)$, $u\neq v$ under label function f, let $g(e)=|f(u)+f(v)| \bmod 2q$, then for $\forall i\in[1,m]$, we have: $g(v_{i1}v_{i2}) = f(v_{i1}) + f(v_{i2}) \pmod{2q} = 2\sum_{k=i+1}^{m} n_k + 1$. Therefore, we have $g(E) = \bigcup_{i=1}^{m} g(E_i) =$ $\{1,3,5,\cdots,2q-1\}$.

So, labeling function f determines a odd-elegant labeling of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$. Therefore, *GRN-OEL- ALGORITHM* can determine a odd-elegant labeling of $\omega_{n_1 n_2 \cdots n_m}$.

Obviously, we can get the following conclusion from theorem 1:

Theorem 1 Generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ is a odd-elegant network topology for $\forall m, n_1, n_2, \cdots, n_m \in N^*$, $n_1, n_2, \cdots, n_m \equiv 0 \pmod 4$.

The implementation of odd-elegant labeling algorithm of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$

Using Matlab language, we compile the *GRN-OEL- ALGORITHM* program. We finish the label experiment for sixty-eight generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ of *p* node (*p*=18,37, 44, 47, 52,56,59, 77, 167,274,345, 452,571, 721,889,1068,1149,1319,1477, 1595, 1793,1912). Limited space, only the results of the label of general ring core network $\omega_{8,8,4}$, $\omega_{12,4,12,12}$, $\omega_{12,4,12,12,8}$, $\omega_{12,4,8,12,8,8}$, $\omega_{8,12,8,12,16}$, $\omega_{12,12,8,12,16}$, $\omega_{8,8,4,12,8,24}$, $\omega_{8,8,4,8,12,16,4,24}$ is given here (shown in figure 2 to figure 9). The experiment statistics CPU time of labeling and mapping for above eighteen generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ and analyze development trend of CPU time of labeling and mapping for generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$. Statistic and analysis results of experimental are shown in table 2 and figure 10, figure 11.

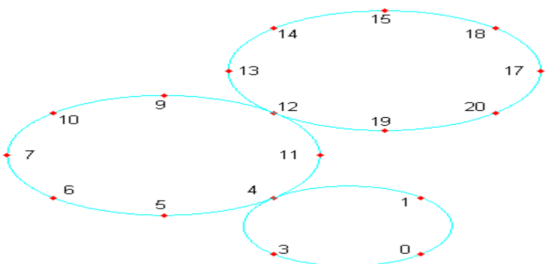

Fig. 2. Odd-elegant labeling of $\omega_{8,8,4}$

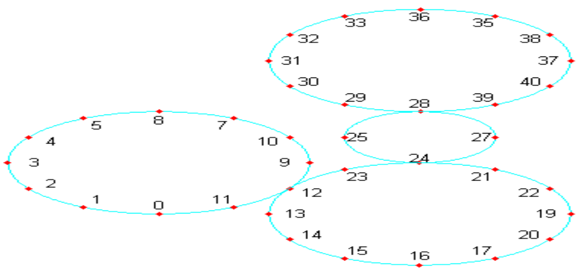

Fig. 3. Odd-elegant labeling of $\omega_{12,4,12,12}$

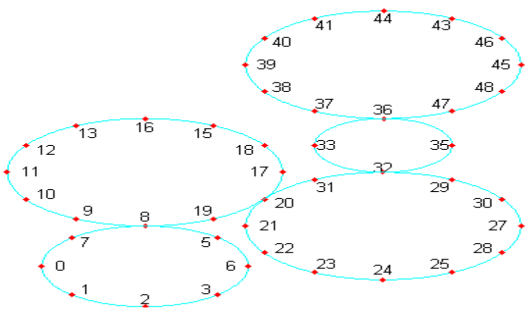

Fig. 4. Odd-elegant labeling of $\omega_{12,4,12,12,8}$

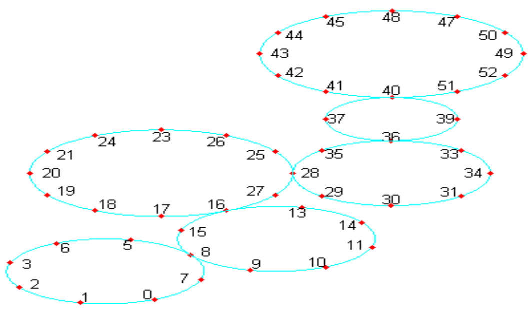

Fig. 5. Odd-elegant labeling of $\omega_{12,4,8,12,8,8}$

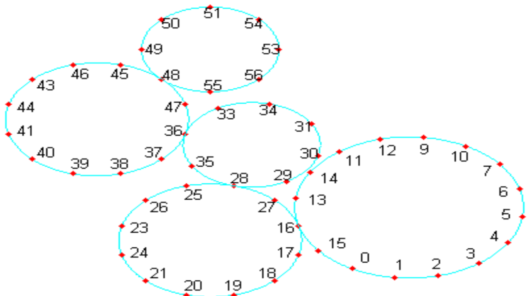

Fig. 6. Odd-elegant labeling of $\omega_{8,12,8,12,16}$

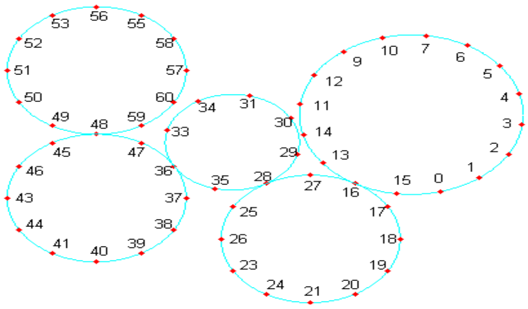

Fig. 7. Odd-elegant labeling of $\omega_{12,12,8,12,16}$

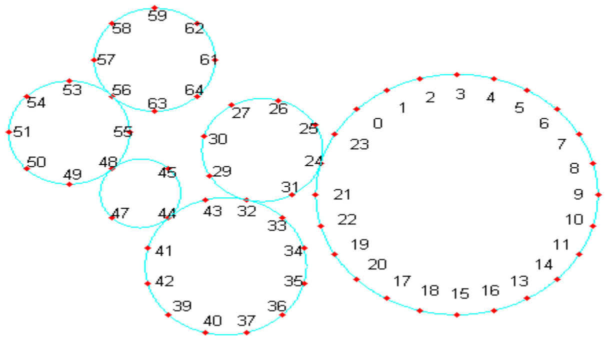

Fig. 8. Odd-elegant labeling of $\omega_{8,8,4,12,8,24}$

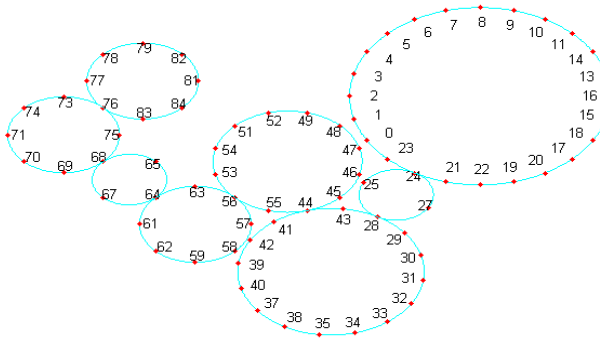

Fig. 9. Odd-elegant labeling of $\omega_{8,8,4,8,12,16,4,24}$

TABLE 2. CPU TIME STATISTICS OF ODD-ELEGANT LABELING AND MAPPING LABELING OF GENERALIZED RING CORE NETWORK $G=\omega_{n_1 n_2 \cdots n_m}$

vertex number of *G*	**generalized ringcore network *G***	**CPU time of labeling of *G***	**CPU time of mapping *G***
18	$\omega_{8,8,4}$	0.000043	0.182989
37	$\omega_{12,4,12,12}$	0.000053	0.204855
44	$\omega_{12,4,12,12,8}$	0.000057	0.195147
47	$\omega_{12,4,8,12,8,8}$	0.000057	0.201063
52	$\omega_{8,12,8,12,16}$	0.000060	0.196301
56	$\omega_{12,12,8,12,16}$	0.000059	0.201579
59	$\omega_{8,8,4,12,8,24}$	0.000063	0.202551
77	$\omega_{8,8,4,8,12,16,4,24}$	0.000072	0.217822
167	$\omega_{8,8,4,8,12,16,16,24,40,40}$	0.000095	0.257083
274	$\omega_{8,8,4,8,12,16,4,24,40,80,80}$	0.000188	0.278325
345	$\omega_{8,8,4,8,12,16,16,24,60,120,40,40}$	0.000153	0.304310
452	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80}$	0.000184	0.338296
571	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,120}$	0.000278	0.381158
721	$\omega_{8,8,4,8,12,16,16,24,60,120,60,80,60,80,100,80}$	0.000265	0.449175
889	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,120,120,200}$	0.000383	0.483294
1068	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,120,120,200,180}$	0.000390	0.548901
1149	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,100,100,140,100,140,120,100}$	0.000421	0.670272
1319	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,100,100,140,100,140,120,100,132,40}$	0.000495	0.776347
1477	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,100,100,140,100,140,120,100,132,40,120,40}$	0.000557	0.904812
1595	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,100,100,140,100,140,120,100,132,40,120,40,40,80}$	0.000615	1.03266
1793	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,100,100,140,100,140,120,100,132,40,120,40,40,80,80,120}$	0.000695	1.19125
1912	$\omega_{8,8,4,8,12,16,4,24,40,80,80,100,80,100,100,140,100,140,120,100,132,40,120,40,40,80,80,120,120}$	0.000757	1.27448

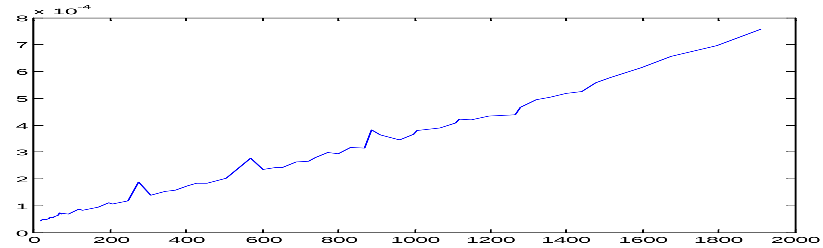

Fig. 10. The CPU time trend chart of odd-elegant labeling of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$

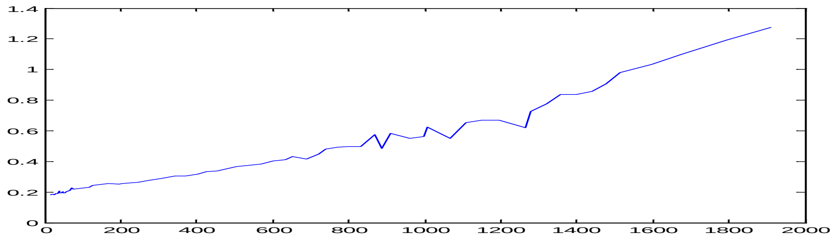

Fig. 11. The CPU time trend chart of mapping odd- elegant labeling graph of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$

According to the change trend of figure 10, we find that function relation between CPU time of labeling for generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ and vertex number *p* of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ is in an approximate linear relationship. We also find that function relation between CPU time of mapping odd-elegant labeling graph of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ and vertex number *p* of generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$ is in an approximate linear relationship by figure 11. Therefore, we may safely draw the conclusion that the *GRN-OEL- ALGORITHM* is effectiveness.

Conclusion

The computer implementation of odd-elegant labeling algorithm of special network topology structure has practical guiding significance to computer communication network system design of functional, reliability, low communication cost. In this paper, the author defines generalized ring core network $\omega_{n_1 n_2 \cdots n_m}$, designs the GRN-OEL- ALGORITHM when $n_1, n_2, \cdots, n_m \equiv 0 (\mathrm{mod}\, 4)$, proves odd-elegant of this network topology, works out the corresponding software.

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