Complex Projective Synchronization of Complex System With Disturbance

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Abstract. In this paper, we propose complex projective synchronization method to realize synchronization of two chaotic complex systems with disturbance. Base on the theory of the Lyapunov stability, we design adaptive feedback controller to realize complex projective synchronization of master-slave chaotic complex systems. Complex projective synchronization means that projective proportion function is complex. Numerical simulations are provided to show the effectiveness of the proposed method.

Introduction

There appear many complex chaotic system [1-4]. Interest in the nonlinear dynamics of nonlinear systems have been increasing is that their have application in many fields [5]. It is a difficult task to research property of the chaotic complex system in the field of complex numbers. However, chaotic complex system can be converted into two real number chaotic systems by imaginary part and real part are separated. It is a easy task to research property of the chaotic complex system in real number field. All kinds of methods research chaotic system are developed [6-8].

So far, various chaos synchronization methods have been researched, such as complete synchronization [9], generalized synchronization [10], phase synchronization [11], lag synchronization [12], projective synchronization [13], etc. In this paper, we define a complex projective synchronization which mean projective proportion function is complex.

Complex System

The following is a complex system [14],

$$\begin{cases} \dot{x} = a(y-x) + yz \\ \dot{y} = cx - y - xz \\ \dot{z} = 1/2(\bar{x}y + x\bar{y}) - bz \end{cases}, \tag{1}$$

where x and y are variable, $x = u_1 + iu_2$, $y = u_3 + iu_4$, $u_i (i = 1, 2, \ldots, 5) \in R$, i is imaginary unit, $i = \sqrt{-1}$; z is real variable and $z = u_5$. The complex system (1) can be showed as follows [14],

$$\begin{cases} \dot{u}_1 = a(u_3 - u_1) + u_3 u_5 \\ \dot{u}_2 = u_4 u_5 + a(u_4 - u_2) \\ \dot{u}_3 = cu_1 - u_3 - u_1 u_5 \\ \dot{u}_4 = cu_2 - u_4 - u_2 u_5 \\ \dot{u}_5 = u_1 u_3 + u_2 u_4 - bu_5 \end{cases}. \tag{2}$$

When $a = 35,\ b = 8/3,\ c = 25$, the system (2) is chaotic state. Lyapunov index of system (2) Lyapunov dimension is [14]

$$D_L = j + \frac{1}{|L_{j+1}|} \sum_{i=1}^{j} L_i = 2 + \frac{L_1 + L_2}{|L_3 + L_4 + L_5|} = 2.011.$$

The chaotic attractor of the system (2) is shown in Fig.1.

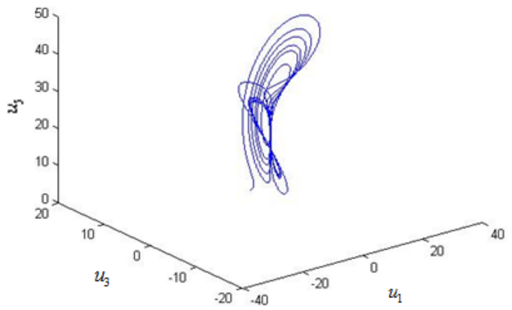


Fig.1 Chaotic attractor of the system (2)

Complex Projective Synchronization

We study the master system (3) and slave system (4),

$$\dot{x}(t) = f(t,x), \tag{3}$$

$$\dot{y}(t) = g(t,y) + u(t,x,y), \tag{4}$$

where $x(t) = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in C^n$, $y(t) = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in C^n$ are state vectors, $x_i(i=1,2,\ldots,n) \in C$, $y_i(i=1,2,\ldots,n) \in C$; $f : C^n \to C^n$ and $g : C^n \to C^n$ are continuous nonlinear vector functions, $u(t,x,y) = (u_1, u_2, \ldots, u_n)^{\mathrm{T}} \in C^n$ is the controller for synchronization between system (3) and (4).

Define 1. The system (3) and (4) can realize complex projective synchronization if there is a constant $\lambda_i(t)$ ($\lambda_i(t) \neq 0$) such that

$$\lim_{t\to\infty} \| y_i(t) - (\lambda_i(t) + \eta) x_i(t) \| = 0,$$

where $\lambda_i(t) \in C$, $\lambda_i(t) = d + if$, η is perturbed term, i is imaginary unit; $d \in R, f \in R$.

Method

We assume that the system (1) is the mater system which is showed as following

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) + y_1 z_1 \\ \dot{y}_1 = c x_1 - y_1 - x_1 z_1 \\ \dot{z}_1 = 1/2(\bar{x}_1 y_1 + x_1 \bar{y}_1) - b z_1 \end{cases}, \tag{5}$$

where $x_1 = u_{11} + iu_{12}$, $y_1 = u_{13} + iu_{14}$, $z_1 = u_{15}$, i is imaginary unit, $u_{1i}(i=1,2,\ldots,5) \in R$.

The system (5) can be showed,

$$\begin{cases} \dot{x}_1 = a u_{13} + u_{13} u_{15} - a u_{11} + i(a u_{14} - a u_{12} + u_{14} u_{15}) \\ \dot{y}_1 = c u_{11} - u_{11} u_{15} - u_{13} + i(c u_{12} - u_{14} - u_{12} u_{15}) \\ \dot{z}_1 = u_{11} u_{13} + u_{12} u_{14} - b u_{15} \end{cases}. \tag{6}$$

The slave system can be expressed as system (7),

$$\begin{cases} \dot{x}_2 = \hat{a}(y_2 - x_2) + y_2 z_2 + \xi + u_1 \\ \dot{y}_2 = \hat{c} x_2 - y_2 - x_2 z_2 + u_2 \\ \dot{z}_2 = 1/2(\bar{x}_2 y_2 + x_2 \bar{y}_2) - \hat{b} z_2 + u_3 \end{cases}, \tag{7}$$

where $x_2 = u_{21} + iu_{22}$, $y_2 = u_{23} + iu_{24}$, $z_3 = u_{25}$, i is imaginary unit, $u_{2i}(i=1,2,\ldots,5) \in R$, $\hat{a}$, $\hat{b}$ and $\hat{c}$ are parameters. $u_1 = v_1 + iv_2, u_2 = v_3 + iv_4, u_3 = v_5$, $v_i(i=1,2,\ldots,5) \in R$, i is imaginary unit. ξ is perturbed term, $\xi = \xi_1 + i\xi_2$. u_1, u_2 and u_3 are controller.

The system (7) may be written as follows,

$$\begin{cases} \dot{x}_2 = \hat{a}u_{23} + v_1 + \xi_1 - \hat{a}u_{21} + u_{23}u_{25} + i(\hat{a}u_{24} - \hat{a}u_{22} + u_{24}u_{25} + v_2 + \xi_2) \\ \dot{y}_2 = \hat{c}u_{21} - u_{23} - u_{21}u_{25} + v_3 + i(\hat{c}u_{22} - u_{24} - u_{22}u_{25} + v_4) \\ \dot{z}_2 = u_{21}u_{23} + u_{22}u_{24} - \hat{b}u_{25} + v_5 \end{cases} . \tag{8}$$

The error is given by

$$e_1 = x_2 - (\lambda_1 + \eta) x_1,$$
$$e_2 = y_2 - \lambda_2 y_1, \tag{9}$$
$$e_3 = z_2 - \lambda_3 z_1.$$

where $\lambda_1 = \lambda_2 = d + if, \lambda_3 = d\ ; d \in R, \eta \in R,\ f \in R$.

Above Eqs. (9) can be expressed as follows

$$\begin{aligned} e_1 &= x_2 - (\lambda_1 + \eta) x_1 = e_{1s} + ie_{1x} = u_{21} + iu_{22} - (d + \eta + if)(u_{11} + iu_{12}) \\ &= u_{21} - du_{11} - \eta_1 u_{11} + fu_{12} + i(u_{22} - du_{12} - fu_{11}), \\ e_2 &= y_2 - \lambda_2 y_1 = e_{2s} + ie_{2x} = u_{23} + iu_{24} - (d + if)(u_{13} + iu_{14}) \\ &= u_{23} - du_{13} + fu_{14} + i(u_{24} - du_{14} - fu_{13}), \\ e_3 &= z_2 - \lambda_3 z_1 = e_{3s} = u_{25} - du_{15}. \end{aligned} \tag{10}$$

The time derivative of Eq. (10) is given by

$$\begin{aligned} \dot{e}_1 &= \dot{x}_2 - (\lambda_1 + \eta) \dot{x}_1 \\ &= \dot{e}_{1s} + i\dot{e}_{1x} \\ &= \hat{a}u_{23} - \hat{a}u_{21} + u_{23}u_{25} + v_1 + \xi_1 - adu_{13} + adu_{11} - du_{13}u_{15} + \eta afu_{14} - afu_{12} + fu_{14}u_{15} + i(\hat{a}u_{24} - \hat{a}u_{22} \\ &\quad + u_{24}u_{25} + v_2 + \xi_2 - dau_{14} + dau_{12} - du_{14}u_{15} - au_{13}f + \eta fau_{11} - fu_{13}u_{15}), \\ \dot{e}_2 &= \dot{y}_2 - \lambda_2 \dot{y}_1 \\ &= \dot{e}_{2s} + i\dot{e}_{2x} \\ &= \hat{c}u_{21} - u_{23} - u_{21}u_{25} + v_3 - cdu_{11} + du_{13} + du_{11}u_{15} + fcu_{12} - fu_{14} - fu_{12}u_{15} + i(\hat{c}u_{22} - u_{24} \\ &\quad - u_{22}u_{25} + v_4 - dcu_{12} + du_{14} + du_{12}u_{15} - fcu_{11} + fu_{13} + fu_{11}u_{15}), \\ \dot{e}_3 &= \dot{z}_2 - \lambda_3 \dot{z}_1 \\ &= \dot{e}_{3s} \\ &= u_{21}u_{23} + u_{22}u_{24} - \hat{b}u_{25} - du_{11}u_{13} - du_{12}u_{14} + dbu_{15} + v_5. \end{aligned} \tag{11}$$

The controller are designed to realize synchronization,

$$\begin{aligned} v_1 &= -\hat{a}u_{23} + \hat{a}u_{21} - u_{23}u_{25} + \hat{a}du_{13} - \hat{a}du_{11} + du_{13}u_{15} - \hat{a}fu_{14} + \hat{a}fu_{12} - fu_{14}u_{15} - k_1 e_{1s}, \\ v_2 &= -\hat{a}u_{24} + \hat{a}u_{22} - u_{24}u_{25} + d\hat{a}u_{14} - d\hat{a}u_{12} + du_{14}u_{15} + \hat{a}u_{13}f - f\hat{a}u_{11} + fu_{13}u_{15} - k_2 e_{1x}, \\ v_3 &= -\hat{c}u_{21} + u_{23} + u_{21}u_{25} + \hat{c}du_{11} - du_{13} - du_{11}u_{15} - f\hat{c}u_{12} + fu_{14} + fu_{12}u_{15} - k_3 e_{2s}, \\ v_4 &= -\hat{c}u_{22} + u_{24} + u_{22}u_{25} + d\hat{c}u_{12} - du_{14} - du_{12}u_{15} + f\hat{c}u_{11} - fu_{13} - fu_{11}u_{15} - k_4 e_{2x}, \\ v_5 &= -u_{21}u_{23} - u_{22}u_{24} + \hat{b}u_{25} + du_{11}u_{13} + du_{12}u_{14} - d\hat{b}u_{15} - k_5 e_{3s}, \end{aligned} \tag{12}$$

where $k_i (i = 1, 2, \ldots, 5) > 0$.

Substituting Eqs. (12) in Eqs. (11) we get

$$\begin{aligned} \dot{e}_{1s} &= \tilde{a}(du_{13} - du_{11} - fu_{14} + fu_{12}) - k_1 e_{1s}, \\ \dot{e}_{1x} &= \tilde{a}(du_{14} - du_{12} + fu_{13} - fu_{11}) - k_2 e_{1x}, \\ \dot{e}_{2s} &= \tilde{c}(du_{11} - fu_{12}) - k_3 e_{2s}, \\ \dot{e}_{2x} &= \tilde{c}(du_{12} + fu_{11}) - k_4 e_{2x}, \\ \dot{e}_{3s} &= -d\tilde{b}u_{15} - k_5 e_{3s}, \end{aligned} \tag{13}$$

where $\tilde{a},\ \tilde{b},\ \tilde{c}$ are parameter estimation error, $\tilde{a} = \hat{a} - a,\ \tilde{b} = \hat{b} - b,\ \tilde{c} = \hat{c} - c.$

The parameters update law are,

$$\dot{\hat{a}} = -e_{1s}(du_{13} - du_{11} - fu_{14} + fu_{12}) - e_{1x}(u_{14}d - u_{12}d + fu_{13} - fu_{11})$$

$$\dot{\hat{b}} = de_{3s}u_{15} \qquad (14)$$

$$\dot{\hat{c}} = -e_{2s}(du_{11} - fu_{12}) - e_{2x}(du_{12} + fu_{11}).$$

Theorem 1. The rive system (5) and response system (7) can realize complex projective synchronization by the controller (12) with parameters update rules (14).

Proof : Choosing the Lyapunov function

$$V = \frac{1}{2}(e_{1s}^2 + e_{1x}^2 + e_{2s}^2 + e_{2x}^2 + e_{3s}^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2).$$

Then

$$\dot{V} = e_{1s}\dot{e}_{1s} + e_{1x}\dot{e}_{1x} + e_{2s}\dot{e}_{2s} + e_{2x}\dot{e}_{2x} + e_{3s}\dot{e}_{3s} + \tilde{a}\dot{\hat{a}} + \tilde{b}\dot{\hat{b}} + \tilde{c}\dot{\hat{c}} . \qquad (15)$$

Inserting Eqs. (13) and Eqs. (14) into Eq. (15) is,

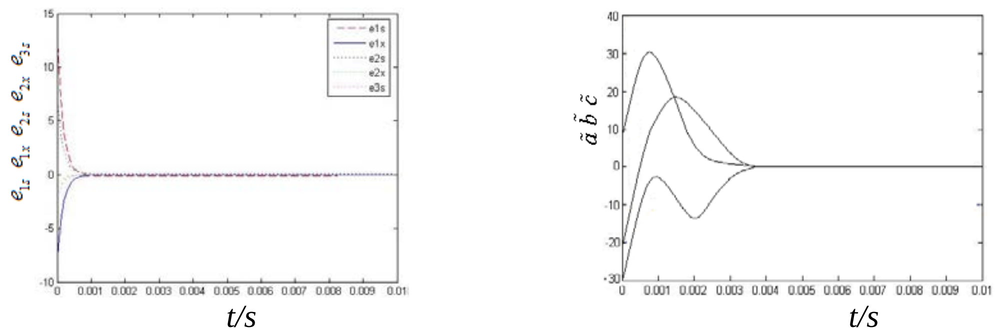
$$\dot{V} = -k_1e_{1s}^2 - k_2e_{1x}^2 - k_3e_{2s}^2 - k_4e_{2x}^2 - k_5e_{3s}^2,$$

$k_i(i = 1, 2, \ldots, 5) > 0$, $\dot{V} \le 0$. We have $e_{1s}, e_{1x}, e_{2s}, e_{2x}, e_{3s} \to 0$ as $t \to \infty$, so $e_1, e_2, e_3 \to 0$ as $t \to \infty$.

Test results

We use Fourth-order Runge-Kutta method to solve system (5) and system (7) with time step size 0.00001. The parameters of the system (5) are $a = 30,\ b = 8/3,\ c = 21$. The initial values of the estimated parameters are $\hat{a} = 5,\ \hat{b} = 12,\ \hat{c} = 4$. The initial values of the system (5) are $x_1(0) = u_{11} + iu_{12} = 2 + 2i$, $y_1(0) = u_{13} + iu_{14} = 1 + 3i$, $z_1(0) = u_{15} = 4$, and those of the system (7) are $x_2(0) = u_{21} + iu_{22} = 6 + i$, $y_2(0) = u_{23} + iu_{24} = 1 + 7i$, $z_2(0) = u_{15} = 7$. Moreover, the complex are chosen as $\lambda_1 = 4 + 3i,\ \ \lambda_2 = 3i$ and $\lambda_3 = 2$. The control gains are $k_i = 5000$（$i = 1, 2, 3, 4, 5$）. $\eta = \xi = \sin\theta$.

Fig. 2 shows the system (5) and system (7) realize synchronization and parameters identification.



(a) Synchronization error (b) Parameter estimation error

Fig. 2 Results of synchronization

Conclusion

The contribution of this paper is that we define complex projective synchronization, which synchronization proportion function is complex numbers. Base on Lyapunov stability theory, we design adaptive controller to realize synchronization and parameters identification.

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