# Reliability Calculation of Product Failure Data (Part Ⅰ: Theory)

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**Abstract.** The failure data of products are random and discrete, showing obvious uncertainty, which bring great inconvenience for reliability calculation. In this, the experience value of median rank, Weibull distribution, lognormal distribution and maximum entropy probability distribution are proposed to describe the distribution information of product failure data. The model of Weibull distribution includes two and three parameters Weibull distribution, and lognormal distribution includes two parameters and three parameters lognormal distribution. Each calculation model reflects different aspects of the information, whose merits shouldn't be blindly judged, but we can analyze a specific product or a group failure data is suitable for what kind of reliability model.

## Introduction

Due to the increasing complexity of engineered products, especially for the aerospace, high-speed rail, nuclear reactors and precision instrument systems, whose reliability is related to the fatal results even national security. Therefore, in order to ensure the system security and stability of products running, it is very important to set up reliability evaluation models to reflect the operation condition of product components [1-4]. The reliability analysis aimed to look for a failure mechanism that can exactly reflect the product component or a failure distribution regular that can quite fit the analysis results of failure data, then the evaluation and prediction of the reliability is realized. Otherwise, the distribution function of products failure time is the basis to research the reliability. The research about product failure are mostly the Weibull distribution [5-7], lognormal distribution [8-10], gamma distribution and binomial distribution reliability analysis methods, especially the Weibull distribution and lognormal distribution are widely used in reliability theory analysis.

The failure data sequence is used to simulate the failure distribution based on the median rank experience [11], lognormal distribution, Weibull distribution and maximum entropy probability distribution [12, 13], then the reliability function of each failure model can be acquired. Among them, the experience value model is simple and reliable without parameter estimation, but the true value of reliability estimation result is discrete and wavy, difficult to provide the continuous assessment. Moreover, when there is a repeat failure data, the formula can not accurately assess. The fitting curves of lognormal distribution and Weibull distribution are smooth and continuous, but need for accurate parameter estimation. The probability distribution model of maximum entropy does not consider the probability distribution, directly to fit the original failure data, whose graph intuitively reflects the failure data distribution, and its biggest feature is applicable to the probability distribution, trend change, prior information unknown information- poor problems [14-15], but the reliability function expression is relatively complex with a gap in the engineering practical application.

## Reliability Calculation Methods

**Experience Value of Reliability.** Suppose $\mathbf{X_0}$ as the failure data sequence of product components

 

with not equal and not repeating, is given by

$$\mathbf{X}_0 = \{x_{0i}\}; x_{01} < x_{02} < ... < x_{0i} < ... < x_{0n}; i = 1,2,...,n \tag{1}$$

where $\mathbf{X_0}$ is a vector of failure data; $x_{0i}$ is the $i$th failure data of this group data sequence; $n$ is the number of this group failure data.

Under the condition of the probability distribution or distribution parameters unknown of product failure data, whose reliability can be described by experience formula of Johnson's median rank with non-parameter estimation, expressed as

$$\mathbf{R} = \{r(x_{0i})\}; i = 1,2,...,n \tag{2}$$

where $\mathbf{R}$ is the experience value vector of reliability.

Calculating formula for experience of the median rank value can be given by

$$r(x_{0i}) = 1 - \frac{i - 0.3}{n + 0.4}; i = 1,2,...,n \tag{3}$$

**Lognormal Distribution and Weibull Distribution of Two-Parameter.** Two kinds of distribution are more common models of the reliability in the engineering application, especially the Weibull distribution is widely used in the bearing failure analysis.

The probability density function of two-parameter lognormal distribution as follows

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t} \exp\left\{-\frac{[\ln t - \mu]^2}{2\sigma^2}\right\} \tag{4}$$

The reliability function

$$f(t) = 1 - \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma t} \exp\left\{-\frac{[\ln t - \mu]^2}{2\sigma^2}\right\} \mathrm{d}t \tag{5}$$

where $t$ is the random variable of lifetime; $\mu$ is the scale parameter; $\sigma$ is the shape parameter, and $t > 0$, $\sigma > 0$.

The probability density function of two-parameter Weibull distribution as follows

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right) \tag{6}$$

The reliability function

$$f(t) = \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right) \tag{7}$$

where $t$ is the random variable of lifetime; $\eta$ is the scale parameter; $\beta$ is the shape parameter, and $t > 0$, $\eta>0$, $\beta>0$.

**Lognormal Distribution and Weibull Distribution of Three-Parameter.** The probability density function of three-parameter lognormal distribution as follows

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t} \exp\left\{-\frac{[\ln(t - \tau) - \mu]^2}{2\sigma^2}\right\} \tag{8}$$

The reliability function

$$f(t) = 1 - \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma t} \exp\left\{-\frac{[\ln(t - \tau) - \mu]^2}{2\sigma^2}\right\} \mathrm{d}t \tag{9}$$

where $t$ is the random variable of lifetime; $\mu$ is the scale parameter; $\sigma$ is the shape parameter; $\tau$ is the location parameter, and $t > 0$, $\sigma > 0$, $\tau >0$.

The probability density function of three-parameter Weibull distribution as follows

$$f(t;\eta,\beta,\tau)=\frac{\beta}{\eta}\left(\frac{t-\tau}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{t-\tau}{\eta}\right)^{\beta}\right) \qquad t\geq\tau>0;\ \beta>0;\ \eta>0 \tag{10}$$

The reliability function

$$R(t;\eta,\beta,\tau)=1-F(t)=\exp(-(\frac{t-\tau}{\eta})^{\beta}) \tag{11}$$

where $t$ is the random variable of lifetime; $\eta$ is the scale parameter; $\beta$ is the shape parameter; $\tau$ is the location parameter, and $t > 0$, $\eta > 0$, $\beta > 0$, $\tau > 0$.

**The Reliability Model of Maximum Entropy.** The vector of discrete failure frequency: According to the statistical theory, via the vector $R$ of reliability experience value in the Eq. (2), the probability vector $\mathbf{F}_1$ of discrete cumulative failure can be obtained as follows

$$\mathbf{F}_1=\{f_i\}=1-R=\{1-r(x_{0i})\};\ i=1,2,...,n \tag{12}$$

where $r(x_{0i})$ is the $i$th failure data reliability of the raw data $\mathbf{X}_0$ in Eq. (1); $n$ is the number of original data.

Suppose that the discrete failure probability of each failure lifetime is $p_{0i}$, viz., for the first data, namely $i$=1, let its failure probability $p_{01} = f_1$. From the second data, i.e. $i$= 2, 3,...,$n$, the failure probability of each failure lifetime can be given by the IAGO of elements in the vector $\mathbf{F}_1$, viz., $p_{0i} = f_i - f_{i-1}$, $i$ = 2, 3,..., $n$.

Therefore the discrete frequency vector of failure data can be given by

$$\mathbf{P}_{0i}=\{p_{0i}\}=\begin{Bmatrix}\varphi_1=f_1, & i=1\\ \varphi_i=\{f_i-f_{i-1}\}, & i=2,3,...,n\end{Bmatrix} \tag{13}$$

where $\varphi_i$ is the discrete frequency of the $i$th failure data; $n$ is the number of failure data.

The probability density function of maximum entropy:

The probability density function is defined as

$$f(t)=\exp\left(c_0+\sum_{k=0}^{m}c_k x^k\right) \tag{14}$$

where $m$ is the order of origin moment, generally $m$=3~8; $c_k$ is the $k$th Lagrange multiplier, $k$=0,1,…,$m$.

The first Lagrange multiplier $c_0$ is given by

$$c_0=-\ln\left(\int_S\exp\left(\sum_{k=1}^{m}c_k x^k\right)dx\right) \tag{15}$$

The others Lagrange multipliers should meet that

$$g_k=g(c_k)=1-\frac{\int_S x^k\exp\left(\sum_{j=1}^{m}c_j x^j\right)dx}{m_k\int_S\exp(c_k x^k)dx}=0 \qquad k=1,2,...,m \tag{16}$$

According to the integral of density function $f(x)$ for the maximum entropy distribution in the interval $S$= [$x_{01}$, $x_{0n}$], the function $F$ of cumulative failure probability can be acquired as follows

$$F=\int_S f(x)dx \tag{17}$$

So the estimate true value function for the reliability of the maximum entropy can be expressed as

$$R(x)=1-F \tag{18}$$

## Summary

(1) The reliability calculation of product failure data can be completed by above several models, but

each model has itself limitations. We can't jump to conclusions only relying on a single model's reliability in the practical application.

(2) Each reliability calculation model has its own advantages and disadvantages, if we evaluate the method is good or not, it is necessary to define other indicators or fuse the several models to consider different aspects of all information.

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