Micro-damage Identification Method of Metal Plate Based on Wavelet Packet Energy Spectrum

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ABSTRACT: In this paper, the dynamic model of the intelligent plate with the pasted piezoelectric material is built. Through the dynamic model, the dynamic response signal is simulated when the damage has slightly changes. The response signal is decomposed by using wavelet packet analysis and a series of sub signals can be obtained. The energy spectrum can be obtained based on sub signals. By comparing with the full structure energy spectrum, the rate of energy spectrum can be obtained. Through the rate of energy spectrum, the detection of damage degree of the intelligent plate is finally realized.

KEYWORD: Dynamic model; Dynamic response signal; Wavelet packet analysis; Energy spectrum

1 INTRODUCTION

Because the initial damage of the structure is relatively small and the impact on the structural dynamic performance is very small, real-time monitoring of the small damage is difficult. At present, most nondestructive testing methods cannot detect small damage, such as the fundamental frequency method. The excitation frequency of the piezoelectric crystal can be up to several hundred KHz, so it has great potential for the micro damage detection. Due to the simple structure, low cost and miniaturization, the piezoelectric crystal is widely used for damage detection. The key of structural damage detection is that the structure model with high frequency dynamic performance including the piezoelectric crystal and high precision damage identification algorithm are built.

Wavelet analysis is a new method for structural damage identification. Wavelet analysis technology has the function of local analysis and refinement. It more fully reveals the data information. Therefore, as a tool for signal analysis and processing, wavelet analysis technology has obvious advantages in improving the accuracy of structural damage identification, and it has been widely used in structural damage identification. Al-khalidy et al. successfully identify the damage in spring mass damper structure by wavelet analysis (Al-khalidy A, 1997). Surace et al. give a numerical simulation of the dynamic behavior of the beam for cracks of varying depths at different locations along the beam (Surace C, 1994). It is found that the outline of the wavelet decomposition coefficient will change obviously with the depth of the crack. Melhem et al. find that the wavelet transform can be more sensitive to two kinds of structural damage identification than Fourier transform (MelhemH, 2003). Wavelet analysis technology has been widely used, but the high frequency signal decomposition accuracy is obviously insufficient. Mallat (Li D. J, 2005) put forward the multi-resolution analysis theory. The high frequency signal can realize a more detailed decomposition and it can make up for the lack of wavelet analysis to a certain extent. Wavelet packet analysis is used to decompose the wavelet space through the multi-layer partition of frequency band, and the high frequency part of the multi resolution analysis is analyzed in detail. The wavelet packet energy spectrum is very sensitive to the micro damage of the structure, so the time-frequency resolution of the signal is greatly improved. As a kind of high precision analysis method, wavelet packet technology has become a hot spot of structural damage identification in recent years. The dynamic characteristics of simply supported beam model are analyzed by wavelet packet analysis (Shi C. X., 2009). The results show that the wavelet packet analysis has a high sensitivity in the damage identification of beam structure. Acceleration response of the beam model under two types of excitations respectively is analyzed (Cui J., 2010). Through the rate of energy change, the recognition of the damage of the beam model is realized. Z. L. Wang et al. achieve a simple beam damage model in the early warning and positioning based on the wavelet packet energy change rate (Wang Z. L., 2008). Although the nondestructive testing technology has been rapid development through the wavelet packet analysis technology, the plate model analysis based on the wave packet is relatively small. The wavelet packet analysis technology is mainly focused on identifying the existence and location of damage, and the damage degree and characteristics are relatively small. In this paper, the detection and identification of the board structure model damage degree and change based on wavelet packet analysis technology are studied.

In this paper, the dynamic model of the intelligent plate with the pasted piezoelectric material based on the finite element theory is built. Through the dynamic model, the dynamic response signal is obtained when the damage has slightly changes. The response signal is decomposed by using wavelet packet analysis and the energy spectrum can be obtained based on sub signals. By comparing with the full structure energy spectrum, the rate of energy spectrum can be obtained. Through the rate of energy spectrum, the identification and detection of damage category and damage degree of the intelligent plate is finally realized.

2 DYNAMIC MODEL OF PIEZOELECTRIC MATERIAL COUPLED PLATE STRUCTURES

2.1 *Dynamic relationship of substrate*

The displacement field of the substrate in the time domain follows the Mindlin plate theory, and the displacement field can be ignored. The displacement field can be expressed as,

$$u(x,y,z,t) = -z\theta_x(x,y,t) \tag{1a}$$

$$v(x,y,z,t) = -z\theta_y(x,y,t) \tag{1b}$$

$$w(x,y,z,t) = w(x,y,t) \tag{1c}$$

Where $w(x,y,t), \theta_x(x,y,t)$ and $\theta_y(x,y,t)$ denote the transverse displacement of the neutral plane, around the Y axis angle, and X axis angle respectively.

The relationship between strain and displacement is expressed as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial \theta_x}{\partial x} \tag{2a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = -z\frac{\partial \theta_y}{\partial y} \tag{2b}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\theta_y + \frac{\partial w}{\partial y} \tag{2c}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\theta_x + \frac{\partial w}{\partial x} \tag{2d}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -z\frac{\partial \theta_x}{\partial y} - z\frac{\partial \theta_y}{\partial x} \tag{2f}$$

$$\varepsilon_b = \{\varepsilon_x \varepsilon_y \gamma_{xy}\}^T, \varepsilon_s = \{\gamma_{xz}\gamma_{yz}\}^T \tag{3}$$

Where ε_b is bending strain and ε_s is shear strain.

When the substrate is isotropic material, the relationship between stress and strain is,

$$\sigma_b = \{\sigma_x \sigma_y \tau_{xy}\}^T = D_b\varepsilon_b, \sigma_s = \{\tau_{xz}\tau_{yz}\}^T = D_s\varepsilon_s \tag{4}$$

Where,

$$D_b = \frac{E}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}, D_s = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}$$

Where σ_b is bending stress, σ_s is shear stress, E is Young modulus, G is shear modulus, μ is Poisson's ratio.

2.2 *The stress-strain relationship of piezoelectric actuators and piezoelectric sensor*

The piezoelectricity crystal is PZT-5A material. When the piezoelectricity crystal acts as actuators, piezoelectric coupling relationship can be expressed as,

$$\sigma_p = Q_p\varepsilon_p - e^T E \tag{5a}$$

$$e = dQ_p \tag{5b}$$

Where σ_p is stress field and ε_p is strain field. The PZT thickness is very small, so formula (5) can be expressed as,

$$\sigma_p = \{\sigma_x^p \, \sigma_y^p \tau_{xy}^p\}^T \;,\; \varepsilon_p = \{\varepsilon_x^p \, \varepsilon_y^p \gamma_{xy}^p\}^T \tag{6}$$

When the substrate and PZT is in coupling state, the PZT strain field ε_p is equal to the substrate surface bending strain ε_b due to thin PZT, $\varepsilon_p = \varepsilon_b = \{\varepsilon_x \varepsilon_y \gamma_{xy}\}^T$.

E is the piezoelectric stress constant and d is the piezoelectric strain constant. Q_p is the elastic matrix.

$$Q_p = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{7}$$

Where, $Q_{11} = Q_{22} = \frac{E_p}{1-\mu_p^2}, Q_{11} = Q_{22} = \frac{E_p\mu_p}{1-\mu_p^2}$, $Q_{33} = \frac{E_p}{2(1+\mu_p)}$

E_p is the elastic modulus and μ_p is Poisson's ratio.

The external electric field E as,

$$E = \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \tag{8}$$

For the PZT as a sensor, the piezoelectric equations are as follows,

$$D = \{D_1 \quad D_2 \quad D_3\}^T = e\varepsilon_p + \bar{\varepsilon}E \tag{9}$$

Where D is the potential field generated by the sensor. D_1, D_2, D_3 are the components for the direction of x, y, and z, $\bar{\varepsilon}$ is dielectric constant of piezoelectric plate. Because the sensor is affected by the external electric field E is 0.

$$D = e\varepsilon_p = dQ_p\varepsilon_b \tag{10}$$

2.3 *Finite element modeling*

4 node and 12 degree of freedom rectangular element model are chosen. Interpolation of each node,

$$w = \sum_{i=1}^4 N_i(\xi,\eta)w_i \tag{11a}$$

$$\theta_x = \sum_{i=1}^4 N_i(\xi,\eta)\theta_{xi} \tag{11b}$$

$$\theta_y = \sum_{i=1}^4 N_i(\xi,\eta)\theta_{yi} \tag{11c}$$

Where $w_i, \theta_{xi}, \theta_{yi}$ （i=1,2,3,4） are displacement components for each node, $N_i(\xi,\eta)$ is the shape function matrix,

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) \tag{12a}$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta) \tag{12b}$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) \tag{12c}$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta) \tag{12d}$$

$N_i(\xi,\eta)$ are put them into the formula (1),(2) and (3),

$$\varepsilon_b = -\frac{1}{ab} z B_b d^e,$$

$$\varepsilon_s = \frac{1}{ab} B_s d^e \tag{13}$$

Where,

$$Z = \begin{bmatrix} -z & 0 & 0 \\ 0 & -z & 0 \\ 0 & 0 & -z \end{bmatrix}$$

$$B_b = \{B_{b1} B_{b2} B_{b3} B_{b4}\}$$

$$B_{bi} = \begin{bmatrix} b\frac{\partial N_i}{\partial \xi} & 0 & 0 \\ 0 & a\frac{\partial N_i}{\partial \eta} & 0 \\ a\frac{\partial N_i}{\partial \eta} & b\frac{\partial N_i}{\partial \xi} & 0 \end{bmatrix} \quad \text{(i=1,2,3,4)}$$

$$B_s = \{B_{s1} B_{s2} B_{s3} B_{s4}\} B_{si} = \begin{bmatrix} -abN_i & 0 & b\frac{\partial N_i}{\partial \xi} \\ 0 & -abN_i & a\frac{\partial N_i}{\partial \eta} \end{bmatrix}$$

$$d^e = \{d_1 d_2 d_3 d_4\}^T$$

$$d_i = \{\theta_{xi} \theta_{yi} w_i\}$$

Where a is the unit length and b is the unit width.

2.4 *Structure dynamic control equation*

For the Mindlin plate element, the mass matrix and the dynamic stiffness matrix are obtained by the Hamilton principle,

$$k_b = \int_V B_b^T D_b B_b dV + \int_V B_s^T D_s B_s dV \tag{14a}$$

$$m_b = \int_V \rho N^T N dV \tag{14b}$$

For the PZT coupling unit, the additional element stiffness matrix is generated.

$$k_p = \int_V B_b^T Q_p B_b dV \tag{15}$$

For the excitation unit, PZT node effect is expressed as:

$$F^e = \int_V B_b^T Q_p dE dV \tag{16}$$

For the sensing unit, the charge generated by the PZT in the wave driven,

$$q^e = \int_V D\, dV = \int_V dQ_p \varepsilon_b\, dV = -\int_V dQ_p z B_b d^e\, dV \tag{17}$$

Through the finite element assembly of each unit, the time domain control equation of the whole structure is obtained.

$$M\ddot{d}^e + C\dot{d}^e + Kd^e = F^e(t) \tag{18}$$

Where K is the global dynamic stiffness matrix C is the damping matrix,M is the mass matrix,$F^e(t)$ is the equivalent nodal force matrix generated by the actuator., d^e is the nodal displacement matrix. By solving the governing equations, the dynamic response of the structure, which is prepared for the analysis, is obtained.

3 SIGNAL ANALYSIS AND DAMAGE INFORMATION EXTRACTION

3.1 *Wavelet packet analysis theory*

Through the multi-level division of signal frequency, wavelet packet analysis makes up signal frequency resolution defects, which has greatly improved the time-frequency resolution. According to the different scale actor j, $L^2(R)$ is divided into subspaces $W_j (j \in Z)$ orthogonal sum.

$$L^2(R) = \oplus W_j (j \in Z) \tag{19}$$

Where W_j is $\{\psi_{j,k}\}_{k \in Z}$ wavelet subspace

The subspace V_j and wavelet subspace W_j are used in the unified representation of the subspace U_j^n, and $U_j^0 = V_j, U_j^1 = W_j$, j ∈ Z, the U_j^n decomposition unifies the orthogonal decomposition $V_{j+1}^0 = V_j \oplus W_j$.

$$U_{j+1}^0 = U_j^0 \oplus U_j^1, j \in Z \tag{20}$$

SubspaceU_j^n is defined as a child control functions $u_n(x)$, and so to meet the two-scale equation:

$$u_{2n}(x) = \sum_{k \in Z} h_k u_n (2x-k) \tag{21a}$$

$$u_{2n+1}(x) = \sum_{k \in Z} g_k u_n (2x-k) \tag{21b}$$

Among them $g_k = (-1)^k h_{k-1}$, and the two coefficient orthogonal.

When n=0, then

$$u_0(x) = \sum_{k \in Z} h_k u_0 (2x-k), \{h_k\} \in l^2 \tag{22a}$$

$$u_1(x) = \sum_{k \in Z} g_k u_0 (2x-k), \{g_k\} \in l^2 \tag{22b}$$

Promote to $n \in Z^+$:

$$U_{j+1}^n = U_j^{2n} \oplus U_j^{2n+1} \tag{23}$$

The sequence $\{u_n(x)\}(n \in Z^+)$ generated by the formula (21) and the formula (22) is called the wavelet packet based on the basis function$u_0(x) = \varphi(x)$.

3.2 *Damage information extraction*

The structure response signal R_00 (t) that decomposed by wavelet packet is expressed as,

$$R_{00}(t) = \sum_{j=1}^{2^{k-1}} R_{kj}(t) \tag{24}$$

Among them, $R_{kj}(t)$ is the decomposed sub signal of $R_{00}(t)$, and j, k is the node number and number of layers of the wavelet packet decomposition tree. Each sub-band signal energy is expressed as,

$$\Psi_{kj} = \int \left|R_{kj}(t)\right|^2 dt \tag{25}$$

So the signal energy variation of the board structure before and after the injury response in each subsignal energy is expressed as,

$$\Gamma_j = 1 - \frac{\Psi_{kj}^0}{\Psi_{kj}} \ , j = 1,2,3,\ldots,2^{k-1} \tag{26}$$

Among them, Ψ_{kj}^0 and Ψ_{kj} are the j order subsignal energy before and after the structural damage, and Γ_j is the amount of change.

Therefore, the amount of energy change of all the sub signals in the structural damage before and after is expressed as,

$$\Lambda = \{\Gamma_1, \Gamma_2, \ldots, \Gamma_{2^{k-1}}\} \tag{27}$$

4 NUMERICAL SIMULATION RESULTS AND ANALYSIS

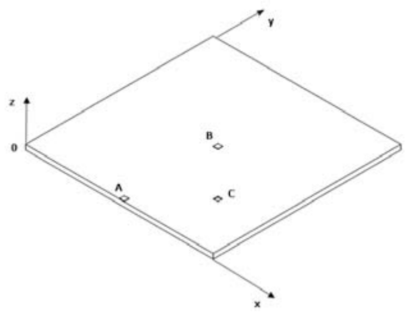

Fig. 1 Schematic diagram of structure

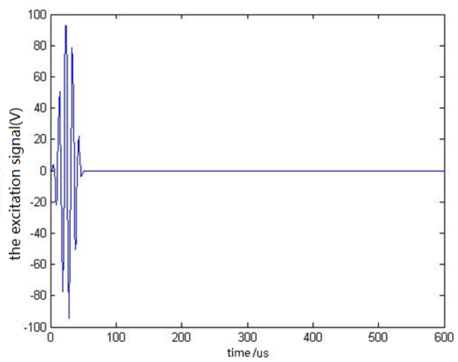

Fig. 2 the excitation signal

The structure of this paper is shown in Fig.1; the substrate is aluminum, Dimensions: length 40cm, width 40cm, height 1cm. A, B, C, respectively for the actuators, sensors and injuries, among them, A, B of the same size, as long 1cm, wide 1cm, high 0.1cm. The vertex coordinates of the bottom left corner of A, B, C in the xy plane are: (20cm, 0cm), (20cm, 20cm) and (30cm, 10cm). The excitation voltage is the center frequency of 100kHZ, the five peaks of the maximum amplitude modulation wave Hanning window 95V, waveform as shown in Fig. 2.

To study the effect of the response signal when the damage occurs subtle changes in depth, this paper simulated the grooves injury response signal in Fig. 1 in the presence of long and wide are 1cm, the depth of 0.5mm, 1mm, 1.5mm, 2mm, and 2.5mm, as shown in Fig. 3(a). And the complete structure of the response signal is compared, the response signals of each damage structure are calculated respectively, and the change rate of the energy spectrum of the energy spectrum is relatively complete under the wavelet packet decomposition, as shown in Fig. 3(b). As seen from Fig. 3(a), it is difficult to extract the feature of damage only by the response signal, and from Fig. 3(b) can see the significantly different of the energy spectrum of damage. 8, 9, 11, 14-scale wavelet packet signal energy is more sensitive to the depth of 0.5mm damage, and the energy spectrum amplitude of scale 9 is the highest. The amplitude of the energy spectrum of 1mm deep damage corresponds to the increasing trend, and the maximum is reached at the scale of 14. The energy spectrum corresponding to the 1.5mm deep damage is larger than that of the scale 1, 4 and 9, and the other scale changes are smaller, and the amplitude is the highest at the scale of 9. The peak of the amplitude of the energy spectrum of 2mm deep damage appears at the scale of 11, and the energy spectrum amplitude at the scale 5, 9 is sensitive to the other scales. The energy spectrum of 2.5mm deep damage is higher than that of the 8,14, and the rest is generally small, and the majority is negative. In summary, wavelet packet of the energy bands spectrum of plate type structure in a tiny area grooves injury is more sensitive, and with the slight changes in the depth of damage, the energy spectrum distribution and amplitude changes significantly. Therefore, the information of the structure concave damage and its depth can be characterized by the wavelet packet energy spectrum.

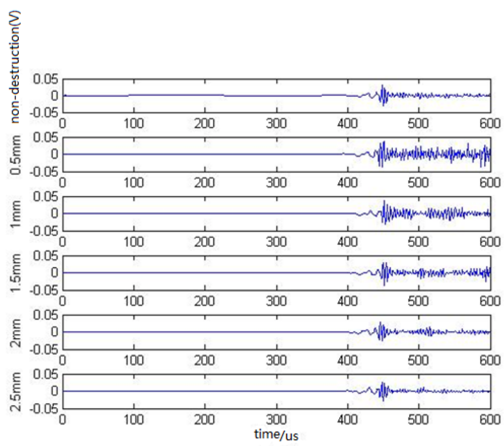

Fig.3a the depth of damage and the integrity of the structural response signal

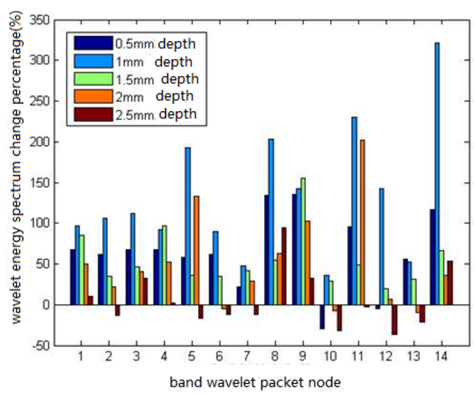

Fig.3b wavelet packet energy spectra of various depth damage relative to full structure (%)

Secondly to study the effects of subtle changes in the damage area to the response signal, this paper simulated the square grooves injury response signal in Fig. 1 containing 1mm in depth and length were 1cm, 1.1, 1.2, 1.3, 1.4 cm and 1.5 mm, as shown in Fig. 4(a). The energy spectrum of the response signal corresponding to the damage in the wavelet packet decomposition is shown in Fig. 4(b),

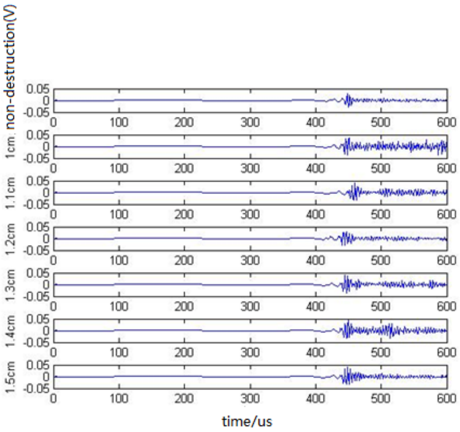

Fig.4a the different length of injured and intact structure response signal

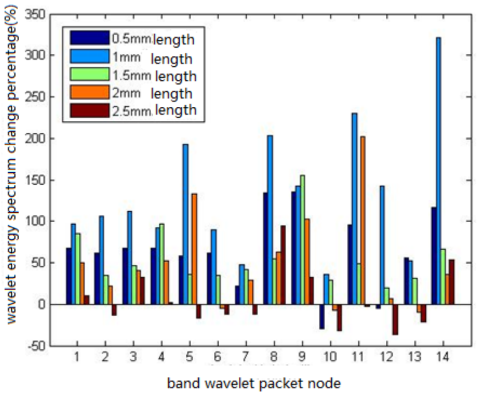

Fig.4b the length of damage relatively complete structure of wavelet packet energy spectrum graph percentage change (%)

Fig. 4(a) is also very difficult to identify the characteristics of the damage, and in Fig. 4(b) with significant differences. Response signal corresponding to the concave injury of 1.1 cm in length at scale 1, 4 and 9 of the energy spectrum amplitude is obvious, and scale 9 reached its peak. 1.2cm edge length that corresponds to the total signal energy spectrum amplitude variation is relatively stable, and the amplitude is small, only the scale 8, 9 relatively to other scales is larger, and in scale 9 reached its peak. The energy spectrum of 1.4cm edge is similar to that of 1.3cm, and is slightly higher than that of 1.3cm. 1.5 cm in length of the corresponding signal of each order energy spectrum in scale 2, 6, 9, 13 changes in the amplitude of compared with other scale is larger, and the four scales of the corresponding energy spectrum amplitude difference is small, the scale 9 amplitude slightly higher. Overall, with the damage area gradually increasing, the variation of the energy bands spectrum appear decline first, corresponding to 1.2 cm length of the square concave injury response signal to minimum, thereafter, gradually increased. And the energy spectrum of the response signal corresponding to the damage also has a significant difference in the distribution and amplitude of the response signal. To sum up, the same depth of concave damage in the presence of small differences can also be well recognized by the wavelet packet energy spectrum.

5 CONCLUSION

Through the establishment of pasted on the surface of the pressure material for electrical coupling of a smart structure dynamics model, in the plate surface in the presence of a small area of the dent damage and the extent of the damage occurred in small changes to the respectively to simulate the model, get the response signal of the sensor. by extracting each contain the damage of the structure of the response signal of wavelet packet energy spectrum, and compared with complete structure, and according to the energy spectrum of the index plate structure in the characterization of different types, different degrees of damage, so as to realize the damage identification of plate structure. The results show that the wavelet packet energy spectrum can be very intuitive to characterize the impact of structural damage contained in the distribution and magnitude of the energy of different frequency bands. Thus, the damage degree and the category of the plate structure are well recognized, which is the traditional method of detection based on modal analysis and natural frequency cannot be achieved. Can foresee, with the continuous improvement and development of the theory of wavelet analysis and structural damage, wavelet packet analysis technology will be more and more widely used in the damage detection of engineering structures. Finally, the research results of this paper will provide a theoretical basis and reference index for structural health monitoring and damage diagnosis in practical engineering applications.

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