

The Influence area of a subsonically expanding ellipsoidal inclusion

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ABSTRACT: The expression for the elastic field of an expanding ellipsoidal inclusion in an infinite isotropic elastic medium is presented. The influence area of a subsonically expanding ellipsoidal inclusion is given in the form of expressions and graphs. And the difference of the influence area between ellipsoidal inclusion and spherical inclusion under subsonically expanding is discussed.

KEYWORD: Influence area; Subsonic expansion; ellipsoidal; inclusion

1 INTRODUCTION

The elastic field of an infinite isotropic elastic medium is investigated when an ellipsoidal portion of the medium experiences a dynamic phase transformation. The phase transformation is nucleated from a point and expanding at a speed of time profile, where Ω (Fig.1) is given by

$$x_1^2/a_1^2(t) + x_2^2/a_2^2(t) + x_3^2/a_3^2(t) \leq 1 \quad (1)$$

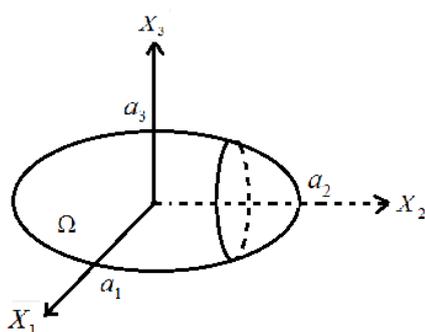


Fig.1 The expanding ellipsoidal inclusion

$$u_{im}^G(X-X', t-t') = \begin{cases} \frac{1}{4\pi\rho} \left\{ (t-t') \frac{\partial^2}{\partial x_i \partial x_m} \frac{1}{x} + \frac{1}{c_2} \delta[x-c_2(t-t')] \frac{\partial^2}{\partial x_i \partial x_m} x + \frac{(x_i-y_i)(x_m-y_m)}{c_1 x^3} \delta[x-c_1(t-t')] \right\} \\ \text{, for } \frac{x}{c_1} \leq t-t' \leq \frac{x}{c_2} \\ 0, \text{ otherwise} \end{cases} \quad (3)$$

Where c_1 and c_2 are longitudinal wave speed and shear wave speed accordingly:

$$c_1 = \sqrt{\frac{2\mu + \lambda}{\rho}}, \quad c_2 = \sqrt{\frac{\mu}{\rho}}. \quad (4)$$

2 INFLUENCE AREA

To solve the integration in Eq.(2), we introduce the concept of influence areas (paper[1]). For some fixed space point X (observation point), the influence area is defined as the collection of source points from which stress waves are emitted influencing the displacement of the observation point.

The influence areas at time t with respect to the longitudinal wave and shear wave are defined as:

For longitudinal wave

$$A_\alpha = \{X' \mid |X'| \leq \Omega(t), |X - X'| \leq c_1(t - t'), 0 \leq t' \leq t\} \quad (5)$$

For shear wave

$$A_\beta = \{X' \mid |X'| \leq \Omega(t), |X - X'| \leq c_2(t - t'), 0 \leq t' \leq t\} \quad (6)$$

Here X' indicate the points on boundary of $\Omega(t)$.

The shape and size of the influence area is influenced by the position of the observation point X and

time t . The integration in Eq. (2) then should be performed on the influence areas $A_\alpha - A_\beta$. To simplify the problem, the observation point is set on the x_3 -axis with coordinates $(0, 0, L)$. As mentioned before, two special shapes as oblate spheroid and prolate spheroid will be discussed below.

3 INFLUENCE AREA OF OBLATE SPHEROID

Here, the elliptical shape factors are given as:

$$a_1(t) = a_2(t) > a_3(t), \quad (7)$$

According to the arrival sequence of the longitudinal wave and shear wave as well as whether the observation point is included into the inclusion or not, the following cases are discussed (Fig.2), somewhat alike to the figure of influence area of an expanding spherical inclusion in paper[1]:

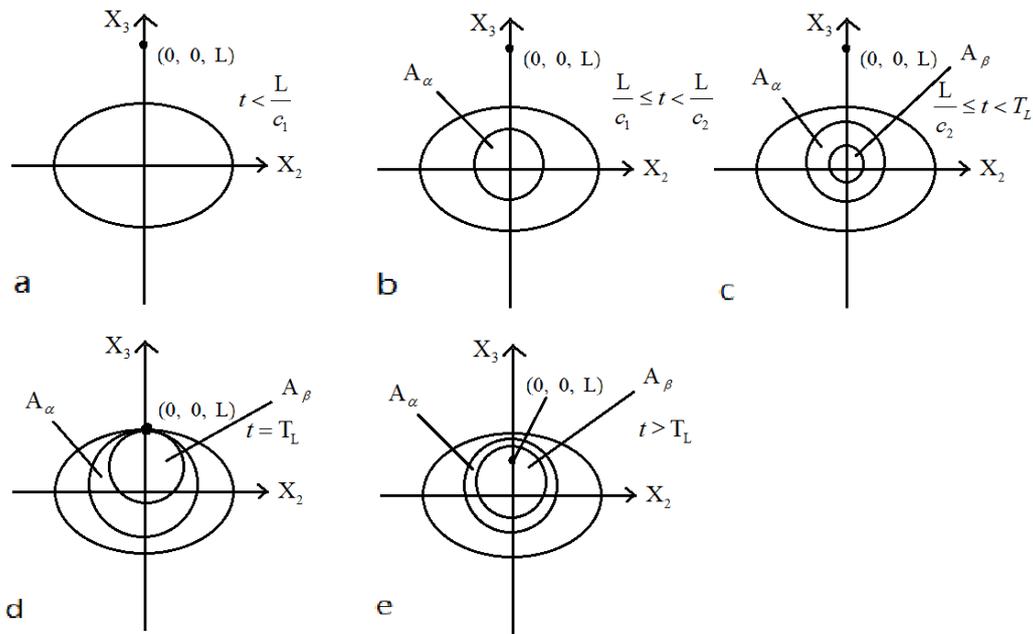


Fig.2 The influence area of a subsonically expanding oblate spheroid inclusion at different time stage

$$(a) t < \frac{L}{c_1},$$

This is the case that both waves have not arrived at the observation point, which means neither A_α nor A_β arise, as shown in Fig.2a.

$$(b) \frac{L}{c_1} \leq t < \frac{L}{c_2},$$

This is the case that some of the longitudinal waves have arrived at the observation point, but the shear waves have not. The absolute value of the intersection of A_α with the x_3 axis are denoted as

$a_{31} = a_3(t_1)$, $a_{32} = a_3(t_2)$, with the x_2 axis denoted as $a_{23} = a_2(t_3)$, $a_{24} = a_2(t_4)$, as shown in Fig.2b. It is easy to show that a_{31} , a_{32} , a_{23} and a_{24} satisfy the following equations:

$$(t - t_1)c_1 = L + a_{31} = L + a_3(t_1), \quad (8.1)$$

$$(t - t_2)c_1 = L - a_{32} = L - a_3(t_2), \quad (8.2)$$

$$(t - t_3)c_1 = \sqrt{L^2 + a_{23}^2} = \sqrt{L^2 + a_2^2(t_3)}, \quad (8.3)$$

$$(t-t_4)c_1 = \sqrt{L^2 + a_{24}^2} = \sqrt{L^2 + a_2^2(t_4)}. \quad (8.4)$$

We can easily learn that $a_{31} < a_{32}$, but we cannot draw a certain conclusion whether $a_{31} < a_{23}$, it depends on t , t_1 and t_3 . Only when the equation $(t-t_1)c_1 - L = \sqrt{(t-t_3)^2 c_1^2 - L^2}$ is satisfied, a_{31} equals to a_{32} .

$$(c) \frac{L}{c_2} \leq t < T_L, (a_3(T_L) = L),$$

At that time, A_β comes into being. Accordingly, the absolute value of the intersection of A_β with the x_3 , x_2 axis are denoted as $a'_{31} = a_3'(t_1')$, $a'_{32} = a_3'(t_2')$, $a'_{23} = a_2'(t_3')$ and $a'_{24} = a_2'(t_4')$. The equations which a'_{31} , a'_{32} , a'_{23} and a'_{24} satisfy have the similar form as equations (8.1)-(8.4) for A_α .

$$(d) t \geq T_L.$$

As shown in Fig.2d and e, the observation point is now included inside the inclusion. Particularly when $t = T_L$, both a_{32} and a'_{32} are equal to L . After that, the equations for a_{32} and a'_{32} change to:

$$(t-t_2)c_1 = a_{32} - L = a_3(t_2) - L, \quad (9.1)$$

$$(t-t_2')c_1 = a'_{32} - L = a_3(t_2') - L. \quad (9.2)$$

4 INFLUENCE AREA OF PROLATE SPHEROID

Here, the elliptical shape factor are given as:

$$a_1(t) = a_2(t) < a_3(t) \quad (10)$$

The influence area of an expanding prolate spheroid is similar to the one of an expanding oblate spheroid in relative location with the inclusion while much narrower in shape.

5 COMPARING WITH SPHERICAL CASE

In order to make clear the difference between the influence areas of spheroid and spherical case in paper[1], some specific point pairs both on the boundary of A_α at a specific time T but different in shape of inclusion are made comparison. The radius of an expanding sphere is defined as $R(t)$.

As shown in Fig.3, we assume that $a_1(t) = a_2(t) = R(t)$, the dashed represents sphere inclusion at t_1' , t_2' and t_3' , and the X marks P_1, P_2, P_3 indicate points on the boundary of A_α at time T , while the full line represents spheroid and the spots indicate the corresponding point of spheroid case. The spherical and spheroid inclusion which overlap on X_2 axis are expanding at the same time of t_n' .

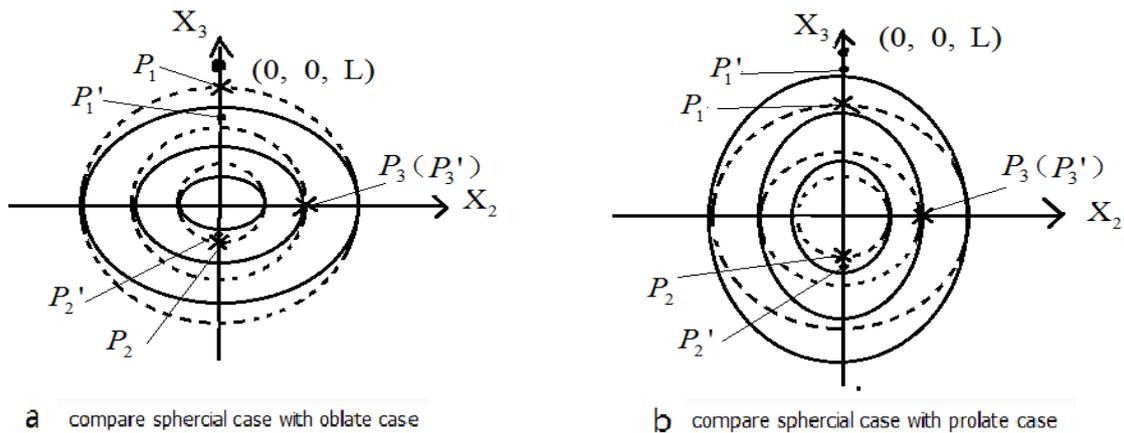


Fig.3 Specific points on the boundary of A_α on spherical and spheroid inclusion

In Fig.3a, P_1 is given as the intersection of the X_3 axis with the surface of $R(t_1')$, also on the boundary of A_α . But the intersection of the X_3 axis with the surface of $\Omega(t_1')$ in oblate case is no longer included in A_α . In fact, the point P_1' which somewhat lower in figure and earlier at emitted time t' could be

on the boundary of A_α at time T . Under this train of thought, when a sphere inclusion changes into an oblate spheroid, P_2 changes into P_2' , which also below the intersection on surface of $\Omega(t_2')$. Besides, P_3 and P_3' overlap due to locating on the X_2 axis. We can learn that the influence area of an ex-

panding oblate spheroid has a trend of far away from the observation point except the points on X_2 axis, compared to the influence area of an expanding sphere.

As for prolate case, the opposite trend is found, as shown in Fig.3b.

6 CONCLUSION

The elastic field of an expanding ellipsoidal inclusion in an infinite isotropic elastic medium is presented with Green's function method. The influence area of the ellipsoidal inclusion expanding in subsonic speed is discussed and compared with that of spherical case.

ACKNOWLEDGEMENTS

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