Strong Coupled Agglomeration Multigrid for Navier-Stokes Equations on Unstructured Grids

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***Abstract*-This paper focused on a series of agglomeration multigrid techniques for the numerical solution of the 2D/3D Reynolds Averaged Navier-Stokes equations on cell-centered unstructured grids, which conform with FVM discretization scheme. We explore a new agglomeration strategy to automated generate coarse grids for multigrid methods, which can conduct to 2D/3D unstructured/hybrid grids. The algorithm based on face weighting, fuse the strongest coupled neigh-bor face to form a new cell in coarsen grid once at a time. The numerical results which conduct on NACA0012 airfoil indicate that nearly optimal computational complexity, this method also shows better complexity than the listed method owing to the better quality of the coarse grid.**

Keywords-Strong coupled agglomeration, Unstructured grids, Cell-centered, Multigrid.

I. INTRODUCTION

The development of robust and efficient algorithms for explicit and implicit solution of Reynolds Averaged Navier-Stokes equations in unstructured grids is an active research topic. Multigrid methods are among the fastest numerical algorithms for the solution of large sparse systems equations. A key issue for using multigrid is the generation of the coarse grid levels.

Over the last few years, three different approaches can be adopted for generate unstructured multilevel girds process. The first one begins with a coarse mesh to generate finer grid by refinement [1]; the second approach uses non-nested unstructured grids as coarse grids [2,3], which are generated independently using any given gird generation strategy; For complex geometries, especially in 3D, the above two methods construct coarse grids that faithfully represent the complex geometries can become a difficult proposition. The third to generate coarse grids through agglomeration, which is first introduced by Lallemand [4] and Smith [5], circumvents this problem by using heuristics to fuse the control volumes for the cell or vertex. Several improvements [6-9] have been made to agglomeration grids in order to optimize the fused cells according to a surface to volume ratio and coarsening ratio between different grid levels. Other improvements [10,11] need additional knowledge about the cell types of the primary grid for the agglomerated volumes. For inviscid transonic flow, the theoretically convergence rate for isotropic agglomeration multigrid can be achieved. However. for high Reynolds number Navier-Stokes problems with highly stretched grids, the efficiency of multigrid will broken down. Mavriplis [12] has demonstrate an algorithm coupling with directional-coarsening and line-implicit smoothing to improve viscous conver-gence rate. Directional agglomeration restrict merging directions with edge coefficients on highly stretched grids. Many reference [12-15] demonstrate the best features for directional-coarsening grid generator and the method in present work is also inspired by this techniques.

In this manuscript, we develop a heuristic strong coupled agglomeration technique, designed for cell-centered control volumes and finite volume scheme multigrid method, which can produce better quality in coarse grid shape for 2D and can be easily extended to 3D. Finally, the approach is compared with the isotropic [4], surface to volume ratio [6] based improvement and directional coarsened [12] multigrid algorithm for solving Navier-Stokes equations, which provide better conver-gence acceleration.

II. GOVERNING EQUATIONS AND DISCRETIZATION

The Reynolds Averaged Navier-Stokes equations in integral form for 3D compressible steady flows can be expressed as

$$\frac{\partial}{\partial t}\iiint \mathbf{Q}\cdot d\mathbf{V}+\oiint\left(\mathbf{F}_i-\mathbf{G}_i\right)\cdot n_i d\mathbf{S}=0 \qquad (1)$$

Where $\mathbf{V}$ is the control volume, $\mathbf{S}$ is cell's surface area and $\mathbf{n}$ is the unit outward vector normal to the surface area. $\mathbf{Q}$ is the vector of the conservation, $\mathbf{F}$ is inviscid flux vector, $\mathbf{G}$ is viscous flux vector, are defined by

$$\mathbf{Q}=\begin{bmatrix}\rho\\ \rho u_1\\ \rho u_2\\ \rho u_3\\ e\end{bmatrix},\ \mathbf{F}_i=\begin{bmatrix}\rho u_i\\ \rho u_1u_i+p\delta_{1i}\\ \rho u_2u_i+p\delta_{2i}\\ \rho u_3u_i+p\delta_{3i}\\ (e+p)u_i\end{bmatrix},\ \mathbf{G}_i=\begin{bmatrix}\rho u_i\\ \tau_{1i}\\ \tau_{2i}\\ \tau_{3i}\\ u_k\tau_{ki}-q_i\end{bmatrix} \qquad (2)$$

Here ρ, p, e denote the density, pressure, total energy, respectively, δ_{ij} is the Kronecker delta and u_i is the velocity of the flow in the coordinate direction x_i. the corresponding viscous stress tensor and the averaged heat flux is defined by

$$\tau_{ij}=-\frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}+\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\ q_i=-k\frac{\partial T}{\partial x_i} \qquad (3)$$

Here T is the temperature of the fluid, k is coefficient of heat exchange, μ is the viscosity coefficient. Using Cell-Centered type system, equation (1) can be re-written in semi-discrete form as.

$$\frac{\Delta\mathbf{Q}_i^{\,n}}{\Delta t}V_i=-\sum_{j=1}^{nf}\left[\left(\bar{\mathbf{F}}-\bar{\mathbf{G}}\right)\Delta\bar{s}\right]_j^{n+1}=-\mathbf{R}^{n+1} \qquad (4)$$

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Where nf is total faces number of the cell i, $\overline{\mathbf{F}}$ and $\overline{\mathbf{G}}$ are spatially discretized inviscid flux vector and viscous flux vector, $\overline{s}$ is normal area of the cell face. Equation (4) can be linearized in time as

$$\left(\frac{V_i}{\Delta t}\mathbf{I}+\sum_{j=1}^{nf}\left(\frac{\partial \mathbf{R}^n}{\partial \mathbf{Q}}\cdot\overline{s}\right)_j\right)\Delta\mathbf{Q}_i^n +\sum_{j=1}^{nf}\left(\frac{\partial \mathbf{R}^n}{\partial \mathbf{Q}}\cdot\overline{s}_{ij}\cdot\Delta\mathbf{Q}_j^n\right)=-\mathbf{R}^n \quad (5)$$

III. Strong Coupled Agglomeration Method

For equation (5), high Reynolds number Navier-Stokes problems with high stretched grids, which conducts huge difference in the coefficient of $\Delta\mathbf{Q}_j^n$. In the algebraic multigrid sense, which conduct anisotropic problems, will reduce the efficiency of isotropic agglomeration. Directional coarsening techniques can be considered for anisotropic problem involving highly stretched meshes, However, the memory usage of a directional coarsening combine with fully implicit solver grows rapidly with an increasing number of nodes, particularly in 3D. Except this, there will be more than one direction with the same weight, the user may be required to determine the merge direction.

The present agglomeration algorithm uses a face weight take absolute value of face normal area $\overline{s}_{ij}$ between cell i and cell j, which represents the degree of coupling in the discretization. For cell-centered finite volume scheme, this coarsening algorithm can be given as follows.

Algorithm 1 (Strong Coupled Agglomeration)

Input: Primary grid in cell set C
Strong/weak coupled threshold Γ

Output: Coarsen grid in cell set G

Initialization: For all i computing each face weight w_{ij}
For all i, $S_i=\{j\in C\setminus i \mid w_{ij}>\Gamma\max w_{ik}\}$
Sort set C by wall, far field, inner cell.

Algorithm: While $C\neq\varnothing$ do
Select $i\in C$, set $N_i=\{j\in C\setminus i \mid \exists w_{ij}\}$
Select $j\in N_i$ and $w_{ij}=\max_{k\in N_i} w_{ik}$
If $j\in S_i$, $G=G\cup\{(i,j)\}$
Otherwise, $G=G\cup\{(i)\}$
Update, $C=C\setminus G$

In 2D ideally each coarse grid cell will be made up of exactly four cells, this approach will repeat in 2D to achieve favorite optimal performance of multigrid methods. The choice of selecting cell $i\in C$ is frontal type, begin with one of solid wall boundary, which we can choose the smallest volume cell, second by far field boundary. For singletons, which will fuse with its coarse neighbors. One efficient strategy for this process is agglomerating with the smallest coarsening ratio neighbor. If more than one candidate remains, the singleton agglomerate to the coarse grid with the smallest volume.

IV. Results

We implement the present algorithm and compare with three agglomeration strategies. The notation is:

Strategy I: The original algorithm which developed by Lallemand et al [4].

Strategy II: The geometrical algorithm developed by Venkatakrishnan and Mavriplis [6].

Strategy III: The directional coarsening developed by Mavriplis [12].

Strategy IV: The present new agglomeration algorithm.

We test four algorithms on NACA0012 airfoil hybrid mesh with 18,501 points, 38,342 faces, 19,841 cells in 2D, which is shown in Figure.1.

Four agglomeration approaches can produce four different types coarse levels. Figure.2-5 show three levels coarse grid produced by four algorithms.

Strategy I implies that agglomerated cell is direct neighbours of the seed cell to produce a new coarsen grid cell. Strategy II implies that less stretched out control volumes are generated. Strategy III implies that cell agglomerate with the direct neighbours and anisotropic cell agglomerate with the most coupled cell. Strategy IV implies that the seed cell agglomerate with the most strongest coupled direct neighbour cell, the next seed selected by the frontal of the last choose one.

As we know all, each coarse grid cell will be made up of exactly four fine grid cells in 2D ideally. Figure.6 depict the frequency distribution of the number of coarsen control volumes versus the coarsening ratio for four strategies.

It is clear that Strategy I and Strategy III give poor control of the coarsening ratio, Strategy II and Strategy IV have a similar dispersion of the coarsening ratio, which all the levels of them with almost the same coarsening ratio. For more details about the four Strategies can refer to the Table I, which N stand for number of the cells at each levels, respectively, CR stand for the coarsening ratio. However, for Strategy II which produce the coarse grids without considering the anisotropic problems conducted by highly stretched grids, at this point Strategy III and Strategy IV will be very suit for high Reynolds number Navier-Stokes and highly stretched grids problems, but Strategy III have the memory usage grows rapidly with an increasing number of cells, particularly, in 3D situation the shape of cells conduct more complicate direction, So, Strategy IV relatively have the optimal computational complexity

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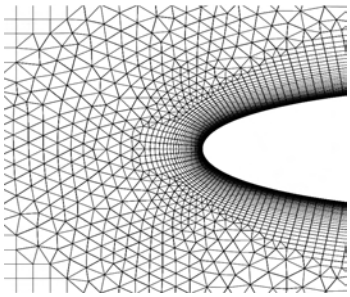

Figure 1. NACA0012 2D airfoil initial hybrid grid

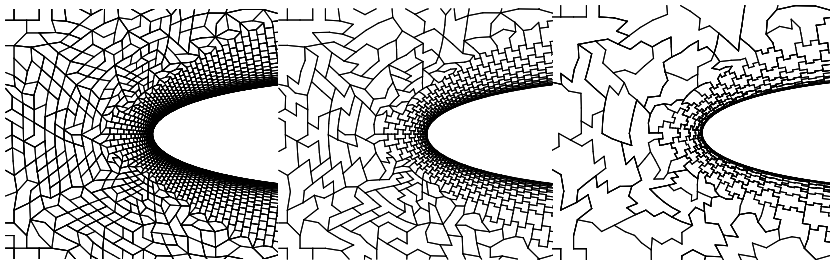

Figure 2. Three levels of coarse grids generated by the agglomeration strategy I

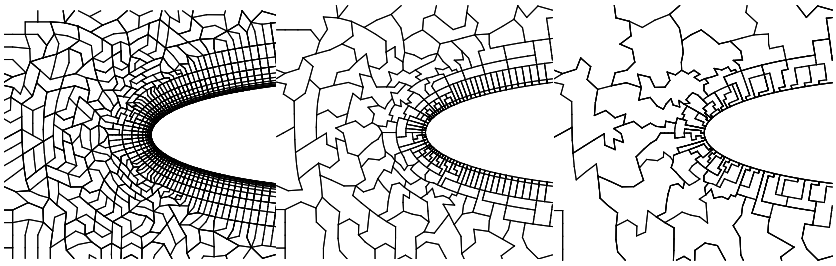

Figure 3. Three levels of coarse grids generated by the agglomeration strategy II

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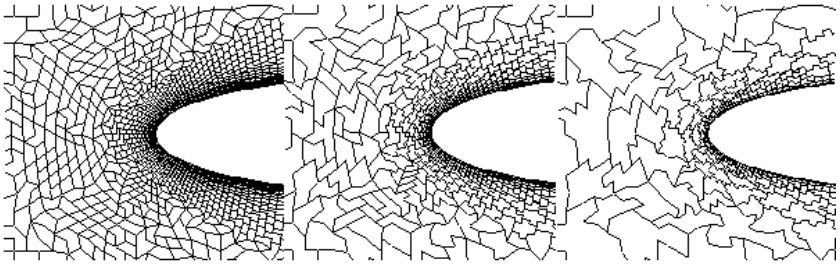

Figure 4. Three levels of coarse grids generated by the agglomeration strategy III

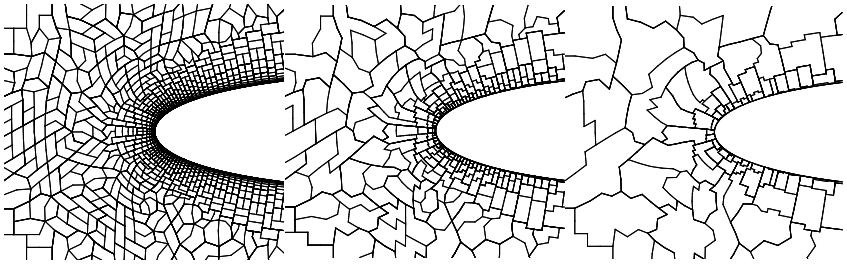

Figure 5. Three levels of coarse grids generated by the agglomeration strategy IV

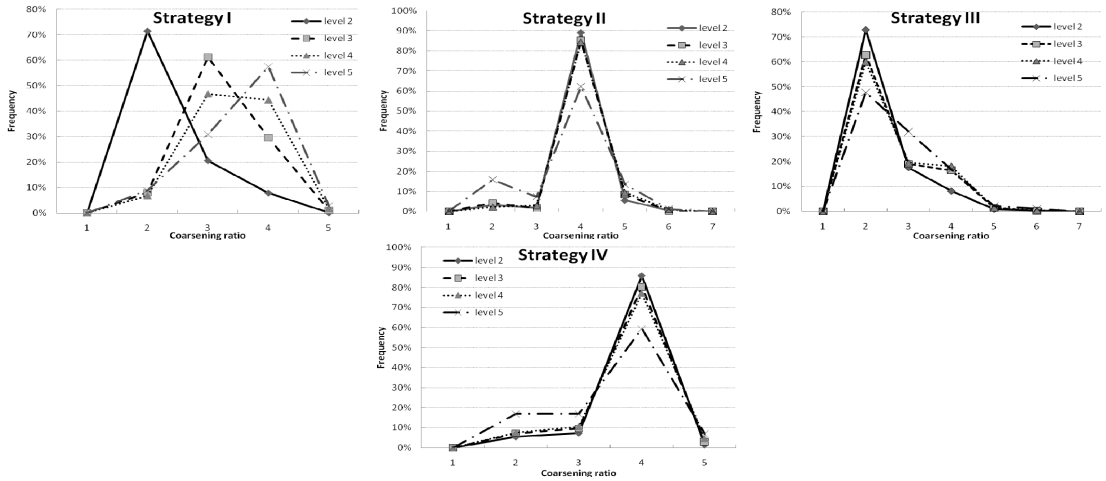

Figure 6. The frequency distribution of the number of coarse control volumes versus the coarsening ratio for four strategies

TABLE I. DETAILS ABOUT FOUR AGGLOMERATION STRATEGIES

Levels	Strategy I		Strategy II		Strategy III		Strategy IV	
	N	CR	N	CR	N	CR	N	CR
2	8378	2.37	4981	3.98	8331	2.38	5177	3.83
3	2587	3.24	1248	3.99	3238	2.57	1365	3.79
4	758	3.41	309	4.04	1230	2.63	360	3.79
5	214	3.54	82	3.77	444	2.77	101	3.56

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