Natural transverse vibration characteristics of jack-up riser and its applied research

Fengde Wang[1,a]，Wensheng Xiao[1], Hongyan Wang[2] Junguo Cui[1]，Jinsheng Song[1]，DingLi Wang[1]

[1]College of Mechanical and Electronic Engineering in China University of Petroleum，Qingdao 266580，China；

[2] College of Mechanical and Electronic Engineering in Qingdao University of Science & Technology, Qingdao 266061，China.

[a]wangfengde110@163.com

Keywords: jack-up rig, riser, resonance, frequency equation

Abstract. In order to resolve the resonance problem of jack-up riser with drill string, the frequency equation of natural transverse vibration with compressive axial load and the length of the riser above the seabed as independent variable was derived in this paper. Then the frequency equation was solved by the Newton method, and the accuracy of the frequency calculation method without axial load was analyzed. In addition, a resonant rotating speed table of top-drive system (TDS) was made according to the relationship between rotating speed and frequency. The results show that: the method proposed in this paper is more accurate than the Rayleigh method; equation(22) is available when the length of the riser above the seabed is shorter than 30 meters while axial weight less than 90 tons ; there is a nonlinear relationship between the frequencies of the riser and the length of the riser above the seabed, and the frequencies decrease with the increasing length; under the general working conditions of jack-up , the relationship between the frequencies of the riser and axial compressive load is almost linear, and the frequencies decrease with the increasing axial compressive load; the resonant frequency range of the riser with drill string depends on the length of the riser above the seabed, and the resonant frequency range is extended by the increasing length.

Introduction

During the drill process of jack-up rig，one end of marine drill riser is fixed on the seabed and the other end is hinged to the jack-up platform [1, 2]. Since there is a BOP stack installed on the hinged end of the riser, which generates an axial compressive load to the riser. Therefore, the dynamic model of jack-up riser can be defined as a fixed-hinged uniform Euler-Bernoulli beam with axial compressive load. At present, the research of the jack-up drill riser is mainly concentrated in two aspects: natural transverse vibration characteristics of riser and its elastic stability [3, 4].

When jack-up rig was operated in the Bohai Sea, a phenomenon of violent vibration in the hinged end of the riser was discovered, which led to the failure of the top screw[4], in order to solve the above-mentioned problem, natural transverse vibration characteristics of riser was studied by the Rayleigh method in reference [4]. However, the Rayleigh method can only work out approximate results of the natural transverse vibration frequency, and the Rayleigh method also has a bigger error in calculating high-order frequencies [5]. The kinematic equation of the uniform beam with axial load was derived by F.J. Shaker [6], based on the kinematic equation, the axial load impact on natural frequency and mode of beam were researched. The free vibration characteristics analysis of a beam subjected to an axial tensile load with an attached in-span mass-spring-mass system was carried out by O. R. Barry [7]. The natural transverse vibration of a continuous beam with elastic supports subjected to axial load was researched with the Laplace Transform by Junqiang Li [8]. The post-buckling behavior of a slender beam with axial load in a circular tube was studied by M. G. Munteanu [9].The governing equation of transverse vibration of a free-free axially moving Timoshenko beam under an axial compressive load is established by Hong-yong CHEN [10], and the

vibration characteristics of the beam are obtained by the analytical solution and Differential Quadrature Method (DQM).

Aiming at the problem proposed in reference [4], on the base of related research results and actual working conditions of jack-up, the frequency equation of natural transverse vibration with axial load and the length of riser above the seabed as independent variable was derived in this paper. The method proposed in this paper is more accurate than the Rayleigh method in computing frequency. In addition, the impact of axial compressive load and the length of the riser above the seabed were studied, on the base of that, a resonant rotation speed table of rig was made. The computing method of natural transverse vibration frequency of riser and the resonant rotating speed table can be used to guide the drill process of jack-up.

Natural transverse vibration characteristics of the fixed-hinged beam with axial force

Since the slenderness ratio of the riser is large enough, therefore the effects of shear deformation and rotary inertia can be neglected, and hence the Euler–Bernoulli beam theory is appropriate to be used in this study. Natural frequencies and vibration mode functions of the beam can be obtained through the free vibration analysis, the free transverse vibration equation of uniform beam with axial load is [6]:

$$m\frac{\partial^2 u(x,t)}{\partial t^2} + F\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial^2}{\partial x^2}[EI\frac{\partial^2 u(x,t)}{\partial x^2}] = 0 \tag{1}$$

In Eq.(1), t is the time, x is the coordinate measured along the beam axis, E is the modulus of elasticity, I is the moment of inertia of the beam, m is per unit length mass, F is axial force, $u(x,t)$ is the transverse deflection of the beam axis.

$$u(x,t) = \phi(x)q(t) \tag{2}$$

$\phi(x)$ is the mode of the vibration, which does not change with time; and $q(t)$ is the amplitude of the vibration .

By using variables separation method, Eq.(1) can be transferred to Eq.(3):

$$EI\frac{\phi''''(x)}{\phi(x)} + F\frac{\phi''(x)}{\phi(x)} = -m\frac{\ddot{q}(t)}{q(t)} \tag{3}$$

Where $\phi''''(x)$, $\phi''(x)$ is the fourth-order and second-order differential of x, $\ddot{q}(t)$ is the second-order differential of t . Eq.(3) can be divided into two independent ordinary differential equations as follows:

$$\ddot{q}(t) + \omega^2 q(t) = 0 \tag{4-a}$$

$$EI\phi''''(x) + F\phi''(x) - m\omega^2\phi(x) = 0 \tag{4-b}$$

Two symbols are introduced in this study, one is frequency coefficient a^4, and another one is axial force impact factor p.

$$a^4 = \frac{m\omega^2}{EI} \; ; \quad p = \frac{F}{EI} . \tag{5}$$

Where ω is natural frequency of the beam.

By substituting a^4 and p into Eq.(4-b), Eq.(4-b) can be transferred to Eq.(6):

$$\phi''''(x) + p\phi''(x) - a^4\phi(x) = 0 \tag{6}$$

On the base of the related research, the solution form of Eq.(6) can be defined as follows:

$$\phi(x) = Ce^{sx} \tag{7}$$

Where, C is a constant.

By substituting Eq.(7) into Eq.(6), Eq.(6) can be transferred to Eq.(8):

$$(s^4 + ps^2 - a^4)Ce^{sx} = 0 \tag{8}$$

Roots of the characteristic equation of Eq. (8) can be obtained as follows:

$$s_{1,2} = \pm i\alpha ; \quad s_{3,4} = \pm \beta . \tag{9}$$

Where:

$$\alpha = \sqrt{\sqrt{a^4 + \frac{p^2}{4}} + \frac{p}{2}} \tag{10-a}$$

$$\beta = \sqrt{\sqrt{a^4 + \frac{p^2}{4}} - \frac{p}{2}} \tag{10-b}$$

By substituting the solution into Eq.(7), the general solution of Eq.(4-b) can be obtained as follows:

$$\phi(x) = C_1 e^{i\alpha x} + C_2 e^{-i\alpha x} + C_3 e^{\beta x} + C_4 e^{-\beta x} \tag{11}$$

In Eq.(11), C_1、C_2、C_3、C_4 are constant. The exponential functions in above equation are substituted for trigonometric functions and hyperbolic functions, and then Eq.(11) can be transferred to Eq.(12) as follows:

$$\phi(x) = A\sin(\alpha x) + B\cos(\alpha x) + C\sinh(\beta x) + D\cosh(\beta x) \tag{12}$$

Where A、B、C、D are constant.

The boundary conditions of the fixed end (x = 0) are as follows:

$$\phi(0) = 0 \tag{13-a}$$

$$\phi'(0) = 0 \tag{13-b}$$

Substitute Eq.(13) into Eq.(12), the relationship among coifficients in Eq.(12) can be obtained as follows:

$$B + D = 0 \tag{14-a}$$

$$A\alpha + C\beta = 0 \tag{14-b}$$

The boundary conditions of the hinged end (x = L) are as follows:

$$\phi(L) = 0 \tag{15-a}$$

$$\phi''(L) = 0 \tag{15-b}$$

Where, L is the length of the riser above the seabed.

Eq.(16) can be obtained by substituting Eq.(15-a) into Eq.(12), as follows:

$$A\sin(\alpha L) + B\cos(\alpha L) + C\sinh(\beta L) + D\cosh(\beta L) = 0 \tag{16}$$

Eq.(17) can be obtained by substituting Eq.(15-b) into Eq.(12), as follows:

$$-A\alpha^2 \sin(\alpha L) - B\alpha^2 \cos(\alpha L) + C\beta^2 \sinh(\beta L) + D\beta^2 \cosh(\beta L) = 0 \tag{17}$$

The equation set of modal coefficient can be obtained by substituting Eq.(14) into Eq.(16)、(17):

$$[\sinh(\beta L) - \frac{\beta}{\alpha} \sin(\alpha L)]C + [\cosh(\beta L) - \cos(\alpha L)]D = 0 \tag{18-a}$$

$$[\beta^2 \sinh(\beta L) + \alpha\beta \sin(\alpha L)]C + [\beta^2 \cosh(\beta L) + \alpha^2 \cos(\alpha L)]D = 0 \tag{18-b}$$

In order to make the above equations have a nonzero solution, the value of coefficient determinant have to be zero, therefore the frequency equation of the beam can be obtained as follows:

$$\sqrt{\beta^2 + p} \tanh(\beta L) = \beta \tan(L\sqrt{\beta^2 + p}) \tag{19}$$

The parameters of L and p in Eq.(19) can be obtained by the working conditions of jack-up rig and the parameters of the riser. Because the closed-form expression of Eq.(19) can not be obtained at present, the parameter of β is solved by the Newton method in this paper. On the base of that, by substituting the value of β into the Eq.(10-b) and combining the Eq.(5), the natural frequencies of the riser can be obtained.

2. Transverse vibration characteristic analysis of the riser under the working conditions

According to the actual operating water depth of jack-up drilling platform, the regular range of the riser length above the seabed is 10 ~ 130 m. A typical riser was analyzed in this paper, whose external diameter is 762.0 mm, inner diameter is 711.2 mm and linear density is 461kg/m (E=2.06e11Pa). The

axial force is expressed by impact factor p, whose range of values is $1\times10^{-4}\sim11\times10^{-4}$ m^{-2}. Within the scope of the above working conditions, the natural frequencies of the riser are computed by using equation (19)、(10-b)、(5-a) and (5-b). The results show that the minimum value of the sixth order of the natural frequency is 40.3268 rad/s, which far more than the frequency range of the TDS. Therefore, only the first-order frequency to fifth-order frequency have to be calculated, which are shown in table 1 to table 5.

On the base of frequencies data, the functional relationship between the fundamental frequency of the riser and the axial compressive load has been graphed by fig.1; and the relationship between the fundamental frequency of the riser and the length of riser above the seabed has been depicted in fig.2. As shown in fig.1, the relationship between the frequency of riser and the axial compressive load is almost linear under the general working conditions of jack-up rig, and the frequencies decrease with the increasing axial compressive load. As shown in fig.2, there is a nonlinear relationship between the frequencies of the riser and the length of the riser above the seabed, and the frequencies decrease with the increasing length.

Table 1 Fundamental frequency of the riser

ω_1	L=10	L=30	L=50	L=70	L=90	L=110	L=130
p = 1e-4	206.3464	22.8701	8.2005	4.1400	2.5036	1.6568	1.1739
p = 3e-4	206.1650	22.7593	8.0837	4.0653	2.3959	1.5481	1.0579
p = 5e-4	205.8789	22.6489	8.0089	3.9491	2.2813	1.4438	0.9352
p = 7e-4	205.6979	22.5390	7.8944	3.8355	2.1601	1.3043	0.7928
p = 9e-4	205.5171	22.4981	7.8010	3.7374	2.0517	1.1624	0.6144
p = 11e-4	205.3365	22.3888	7.6694	3.6146	1.9173	1.0030	0.3505

Table 2 Second-order frequency of the riser

ω_2	L=10	L=30	L=50	L=70	L=90	L=110	L=130
p = 1e-4	668.3369	74.3400	26.6465	13.5970	8.1797	5.4743	3.8929
p = 3e-4	667.7146	74.0330	26.5538	13.4617	8.0837	5.3360	3.7826
p = 5e-4	667.2815	73.8524	26.4614	13.3806	7.9481	5.2333	3.6614
p = 7e-4	667.0375	73.6721	26.2949	13.2473	7.8147	5.1004	3.5302
p = 9e-4	666.7935	73.5548	26.1293	13.1153	7.7223	4.9703	3.4025
p = 11e-4	666.5496	73.4376	26.1120	13.0102	7.6114	4.8577	3.2655

Table 3 Third-order frequency of the riser

ω_3	L=10	L=30	L=50	L=70	L=90	L=110	L=130
p = 1e-4	1394.6	156.4411	55.7051	28.4091	17.1684	11.4565	8.1797
p = 3e-4	1393.9	154.7511	55.5660	28.2703	16.9999	11.3187	8.0837
p = 5e-4	1393.7	154.7028	55.4816	28.1322	16.8926	11.2067	7.9481
p = 7e-4	1392.5	154.4729	55.3432	27.9947	16.7563	11.0718	7.8147
p = 9e-4	1392.1	154.3340	55.2050	27.8961	16.6797	10.9620	7.6832
p = 11e-4	1391.7	154.2859	55.0672	27.7597	16.5450	10.8297	7.5728

Table 4 Fourth-order frequency of the riser

ω_4	L=10	L=30	L=50	L=70	L=90	L=110	L=130
p = 1e-4	2384.6	264.6962	95.3048	48.5339	29.3512	19.6409	14.0309
p = 3e-4	2384.0	264.5919	95.1531	48.4638	29.2473	19.5157	13.8913
p = 5e-4	2383.1	264.4877	95.0016	48.3431	29.1438	19.3594	13.7800
p = 7e-4	2382.5	264.3835	94.9214	48.2226	29.0015	19.2676	13.6694
p = 9e-4	2381.9	264.2794	94.7702	48.1025	28.8210	19.1447	13.5333
p = 11e-4	2381.3	264.1753	94.6193	47.9324	28.7577	19.0225	13.4245

Table 5 Fifth-order frequency of the riser

ω_5	L=10	L=30	L=50	L=70	L=90	L=110	L=130
p = 1e-4	3637.2	404.1983	145.3954	74.0882	44.8397	29.9878	21.4609
p = 3e-4	3636.9	404.0379	145.2646	74.0330	44.6799	29.8814	21.3576
p = 5e-4	3636.6	403.8777	145.1340	73.9151	44.5693	29.7356	21.2214
p = 7e-4	3635.9	403.7175	145.0034	73.7973	44.4589	29.6696	21.0527
p = 9e-4	3635.6	403.5574	144.8730	73.6798	44.3488	29.4855	20.9514
p = 11e-4	3635.3	403.3973	144.7427	73.5000	44.1907	29.3416	20.8504
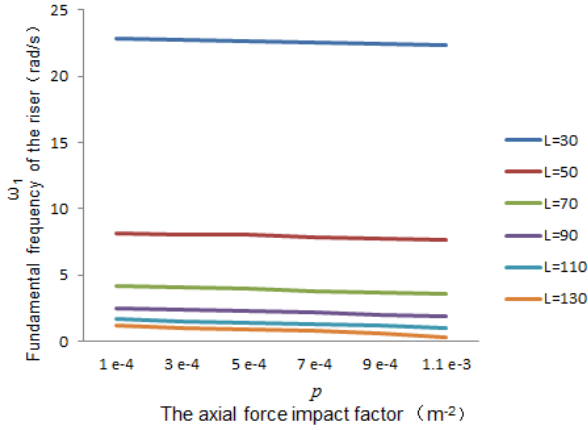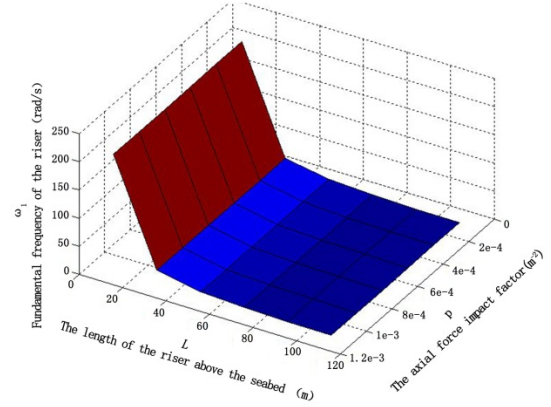
Fig.1 The relationship between ω_1 and p Fig.2 The relationship between ω_1 and L、p

When taking no account of the axial load, the frequency equation of the fixed-hinged beam is:

$$\tanh(aL) = \tan(aL) \tag{20}$$

This transcendental equation can be solved as follows:

$$a_n L \approx (n + \frac{1}{4})\pi \quad (n = 1,2,3...) \tag{21}$$

By substituting the Eq. (5-a) into the above equation, natural frequencies of beam without axial load can be obtained as follows:

$$\omega_n = \frac{(n + \frac{1}{4})^2 \pi^2}{L^2} \sqrt{\frac{EI}{m}} \quad (n = 1,2,3,...) \tag{22}$$

In order to evaluate the accuracy of the results by Eq. (22), a comparative analysis of results was carried out, and a conclusion can be obtained as follows: when the length of riser above the seabed is shorter than 30m while axial weight less than 90 tons, the error of the results by Eq. (22) are less than 2.5%, therefore Eq. (22) is suitable for calculating the natural frequency of the riser in these working conditions.

3. Resonant Rotation Speed Table

A typical rotating speed range of the TDS used in offshore drilling is up to 273r/min, the frequency range of the TDS can be obtained by equation (23):

$$\omega = \frac{n\pi}{30} \tag{23}$$

and the frequency range of the TDS is 0～28.5885 rad/s, on the base of that, a resonant rotating speed table under different working conditions can be made (as shown in table 6). When a jack-up rig operated under a certain working condition, the engineers can avoid resonant rotating speed by this table. In addition, the results in table 6 show that the orders of the marine riser's resonant frequency is

mainly decided by the length of the riser above the seabed, which increases as the increasing length of riser above the seabed.

Table 6 Resonant rotating speed of jack-up rig under different working conditions

The length of the riser above the seabed	Orders of resonant rotation speed	p=1e-4	p=3e-4	p=5e-4	p=7e-4	p=9e-4	p=11e-4
L=30	n_{30}^1	218.3934	217.3353	216.2811	215.2316	214.8410	213.7973
L=50	n_{50}^1	78.3090	77.1936	76.4794	75.3860	74.4941	73.2374
	n_{50}^2	254.4553	253.5701	252.6878	251.0978	249.5164	249.3512
L=70	n_{70}^1	39.5341	38.8208	37.7111	36.6263	35.6895	34.5169
	n_{70}^2	129.8418	128.5498	127.7753	126.5024	125.2419	124.2383
	n_{70}^3	271.2869	269.9615	268.6427	267.3297	266.3881	265.0856
L=90	n_{90}^1	23.9076	22.8792	21.7848	20.6274	19.5923	18.3089
	n_{90}^2	78.1104	77.1936	75.8988	74.6249	73.7425	72.6835
	n_{90}^3	163.9461	162.3371	161.3124	160.0109	159.2794	157.9931
L=110	n_{110}^1	15.8213	14.7833	13.7873	12.4551	11.1001	9.5779
	n_{110}^2	52.9728	50.9550	49.9743	48.7052	47.4629	46.3876
	n_{110}^3	109.4015	108.0856	107.0161	105.7279	104.6794	103.4160
	n_{110}^4	187.5568	186.3612	184.8687	183.9920	182.8184	181.6515
L=130	n_{130}^1	11.2099	10.1022	8.9305	7.5707	5.8671	3.3470
	n_{130}^2	37.1745	36.1212	34.9638	33.7109	32.4915	31.1832
	n_{130}^3	78.1104	77.1936	75.8988	74.6249	73.3692	72.3149
	n_{130}^4	133.9852	132.6521	131.5893	130.5332	129.2335	128.1945
	n_{130}^5	204.9365	203.9501	202.6494	201.0385	200.0711	199.1067

Note: n_L^b, where b is the order of the rotating speed.

Summary

The frequency equation of jack-up riser's natural transverse vibration was derived in this paper, which takes axial compressive load and the length of the riser above the seabed as independent variable. Then the frequency equation was solved by the newton method. On the base of that, natural transverse vibration characteristics of jack-up riser were analyzed in detail and a resonant rotating speed table was made, which can be used to guide the drill process of jack-up rig. From this study, the following conclusions are reached:

1. The frequency calculation method proposed in this paper is more accurate than the Rayleigh method; if the length of the riser above the seabed is shorter than 30 meters while axial weight less than 90 tons, Eq.(22) is accurate enough to compute the frequencies.
2. There is a nonlinear relationship between the frequency of riser and the length of the riser above the seabed, and the frequencies decrease with the increasing length.
3. Under the general working conditions of jack-up, the relationship between the frequencies of the

riser and axial compressive load is almost linear, and the frequencies decrease with the increasing axial compressive load.

4. The resonant frequency range of the riser with drill string depends on the length of the riser above the seabed, and the resonant frequency range is extended by the increasing length.

References

[1] Jin Yang, Shujie Liu, Jianliang Zhou, et al. Strength and security calculation on riser with the effect of wind, wave and current. China Offshore Oil and Gas, 2006, 18(3): 198-200.

[2] Qinfeng Hu. Wellhead stability analysis of offshore self-elevating exploratory well in drilling platform. Oil Drilling & Production Technology, 2007, 29(6): 10-12.

[3] Wei Jiang. Study on critical load and elastic stability of drilling riser. China Offshore Oil and Gas, 2006, 18(6): 403-407.

[4] Wei Jiang. Lateral vibration performance of drilling riser and its impact to drilling operation. China Offshore Oil and Gas, 2007, 19(6): 394-397.

[5] Jingbo Liu, Xiuli Du. Structural Dynamics. Beijing: China Machine Press, 2014: 207.

[6] F.J. Shaker. Effect of axial load on mode shapes and frequencies of beams. Lewis Research Center Report NASA-TN-8109, December 1975.

[7] O. R. Barry, Y Zhu, J.W. Zu, et al. Free vibration analysis of a beam under axial load carrying a mass-spring-mass. ASME 2012 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. 2012: 791-796.

[8] Junqiang Li, Fang Tong. The natural transverse vibration of a continuous beam with elastic supports subjected to axial load. Journal of XI'AN Shiyou University(Natural Science Edition), 1999(4): 70-72.

[9] M. G. Munteanu, Baracco. Post-buckling behavior of a slender beam in a circular tube under axial load. Computational Methods & Experimental Measurements, 2007 XIII (46):547-556.

[10] H Y Chen, H B Chen. Vibration characteristics of free-free moving beam under axial compressive loads. Engineering Mechanics, 2015, 32(3): 233-240.

[11] S. Timoshenko. Vibration problems in engineering. Hu R L, tran. Beijing: People's Railway Press, 1978: 291.

[12] Mosheng Liao. Offshore oil drilling engineering technology and equipment. Beijing: China Petrochemical Press, 2010: 148.