

Performance Enhancement in LDPC coded high-dimension MIMO-OFDM systems

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Abstract. Currently, low-density parity check (LDPC) coded multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems becoming very important research area in wireless communication. Especially in high throughput wireless communication, due to its capacity increasing ability in a wireless fading environment. And also it is one of the research candidates for 5G communication when it is applied in large scale MIMO systems. It has been shown that LDPC coded MIMO systems have much attention in industrialization by some development work from its theory to commercial products in the semiconductor industry. And in this paper, we applied Expectation Propagation (EP) algorithm with A message-passing algorithm (MPA) and belief propagation (BP) decoding for LDPC coded MIMO-OFDM systems. In fact, Expectation Propagation (EP) algorithm is a low-complexity algorithm. It solves the posterior marginalization problem easily which it makes efficient work with high-dimension MIMO. Therefore, our proposed work is for high throughput providing channel coding in the high-dimension wireless channel.

Introduction

Nowadays, the LDPC code is one of the most favorable applicants for high throughput environment due to its capacity approaching (increasing) performance. Formerly, the inventor of LDPC is Gallager [1]. However, its high complexity in encoding/decoding made it have not drawn much attention in implementation practical system schemes for a few decades. Amazingly, in the current time, theoretical research in LDPC coded systems have become one of the hot topics in wireless communication high throughput area. Implementation of LDPC coded MIMO system in commercial products has going to be development in the semiconductor industry. A plenty of research have already brought LDPC codes into MIMO systems to push the system closer to the capacity. And LDPC MIMO-OFDM systems is gaining more R&D attention in the wireless communication area.

In the industry, LDPC coded MIMO systems have been applied in IEEE 802.11n wireless local area networks (WLAN) [2]; IEEE 802.15c millimeter-wave 60-GHz wireless personal area networks (WPAN) [3]; and etc. Some designed binary LDPC coded MIMO systems have a significant enhancement in the data rate and reliability of wireless communication [4]-[5]. By increasing the number of transmitting and receiving elements in MIMO system can be achieved to increase capacity (throughput) and improve more channel reliability such as reduction of symbol error rate. On the other hand, there are spatial limitations for antenna distribution in the signal processing complexity of the high-dimension MIMO systems. In spite of above matters, some innovative studies [6] advise some reasonable solutions for scaling the numbers too large antennas in the practical application. As the system dimensions grow in MIMO systems, Soft-output symbol detection for soft-decoding becomes a sensitive process. In large scale $r \times m$ MIMO situation with an additive white Gaussian noise and a memoryless channel. The uniformly distributed transmitted symbols $s \in \mathcal{A}_m$ and given received symbols $r \in \mathcal{C}_r$, we need to marginalize the posterior

probability distribution $p(s|r)$ to obtain the per antenna symbol posterior probability $p(s_i|r)$ $i = 1, \dots, m$. The Gaussian tree approximation (GTA) algorithm that minimizes the Kullback–Leibler (KL) divergence with respect to $p(r|s)$ by ignoring the discrete environment of the prior $p(s)$ and computes [7]. The GTA successive interference cancellation (SIC) has more restrains in the LDPC system performance because it is not intended to provide soft-outputs. [8].

A reminder that utmost cases of system design important are the receiver side as it is most complex part of the system. Therefore, we concentrate on receiver part, we used Expectation Propagation (EP)-based soft-decisions MIMO detector algorithm [9] for receiver part of LDPC-coded MIMO systems. The EP solves marginalization problem of the symbol vector posterior $p(s|r)$ probability generalizes belief propagation (BP) in two ways. And EP efficiently works with large scale MIMO systems by giving soft outputs (SO). Furthermore, we applied LDPC and OFDM structures from IEEE 802.11n [2].

System Description

The system has to M_t transmit and M_r receive antennas. A binary word $\mathbf{b} = [b_1, b_2, \dots, b_w]$ is encoded by an (v, w) LDPC encoder giving the binary codeword $\mathbf{c} = [c_1, c_2, \dots, c_v]^T$. The OFDM mod block modulates with 16 QAM modulation codeword \mathbf{c} . The N size modulated k^{th} symbols is expressed as

$$S_k[n] = [S_k[0], \dots, S_k[N-1]]^T \quad (1)$$

The (1) symbols pass through IDFT. The output of IDFT refers to sampled discrete-time OFDM symbol is expressed as

$$s_k[n] = \sum_{n=0}^{N-1} S_k[n] e^{j \frac{2\pi n k}{N}} \quad \text{for } n = 0, 1, \dots, N-1 \quad (2)$$

where N is the IDFT size. After CP adding into OFDM symbol through inserting the end P subcarrier to the start as per the following equation

$$s_k[d] = \begin{cases} s_k[N-p+P] & p = 0, 1, \dots, P-1 \\ s_k[p-D] & D = P, P+1, \dots, P+N-1 \end{cases} \quad (3)$$

where P is the cyclic prefix length, we assume that the cyclic prefix in each OFDM symbol is longer than the maximum delay spread. The received signal at the MIMO-OFDM system receiver after proper cyclic insertion will be sent through the channel. And it can be expressed in the following input-output relation

$$\mathbf{r}_M(k) = \sqrt{\frac{\text{SNR}}{N}} \mathbf{H}_M(k) \mathbf{s}_M(k) + \mathbf{z}_M(k) \quad (4)$$

where $\mathbf{z}_M(k)$ is AWGN noise with σ_z^2 -variance and $\mathbf{H}_M(k)$ is the frequency domain channel response matrix at the n th subcarrier and m th OFDM block ($k=0, \dots, M-1$)

$$\mathbf{H}_M(k) = \sum_{l=0}^{L-1} \mathbf{A}_l^{1/2} \mathbf{H}_l(k) \mathbf{W}_l^{1/2} \quad (5)$$

and where $\mathbf{A}_l = \mathbf{A}_l^{1/2} \mathbf{A}_l^{1/2}$ and $\mathbf{W}_l = \mathbf{W}_l^{1/2} \mathbf{W}_l^{1/2}$ characterize of receive and transmit the spatial correlation matrix. Also here

$$\text{SNR} = 10 \log_{10} \left(m \log_2 M \frac{n E_b}{k \sigma_z^2} \right) \quad (6)$$

where $E_s = \log_2 M \frac{n}{k} E_b$ is the constellation average energy.

For understanding better EP algorithm, we believe that presentation of inference in the graphical model typically uses real-valued random variables, rather than complex-valued variables used in signal processing for communications. Consequently, we first reformulate the complex-valued MIMO system into a real-valued one, before presenting the EP algorithm. The system model in (1)

can be translated into an equivalent double sized real-valued representation that is obtained by considering the real $R(\cdot)$ and imaginary parts $I(\cdot)$ separately. We define $\tilde{s}_M(k) = [R(s_M(k))I(s_M(k))]$, $\tilde{r}_M(k) = [R(r_M(k))I(r_M(k))]$, $\tilde{z}_M(k) = [R(z_M(k))I(z_M(k))]$,

$$\tilde{\mathbf{H}}_M(k) = \begin{bmatrix} R(\mathbf{H}_M(k)) & -I(\mathbf{H}_M(k)) \\ I(\mathbf{H}_M(k)) & R(\mathbf{H}_M(k)) \end{bmatrix} \quad (7)$$

The channel model can now be written as follows:

$$\tilde{\mathbf{r}}_M(k) = \sqrt{\frac{\text{SNR}}{N}} \tilde{\mathbf{H}}_M(k) \tilde{\mathbf{s}}_M(k) + \tilde{\mathbf{z}}_M(k) \quad (8)$$

where the variance is $\sigma_z^2 = \sigma_z^2 / 2$ of the real and imaginary elements of the noise.

Encoding of Low-Density Parity-Check (LDPC) codes

A low-density parity check (LDPC) code is a linear block code identified by a parity check matrix. The parity check matrix \mathbf{P} of a *regular* (v, w, s, t) LDPC code of rate $r = w/v$ is a $(n-k) \times n$ matrix. The matrix has s ones in each column and $t > s$ ones in each row, where $s \ll n$, and the ones are typically placed at random in the parity check matrix. In another hand, if the number of ones in every column is not the same, the code is identified as an *irregular* LDPC code. Although deterministic construction of LDPC codes is possible, in this paper, we consider only pseudorandom constructions. The parity check matrix \mathbf{P} with code can be characterized by a bipartite graph that consists of two types of nodes—variable nodes and checks codes. Each code bit is a variable node, whereas each parity check or each row of the parity check matrix represents a check node. An edge in the graph is placed between the variable node and check node j if $P_{ij} = 1$. That is, each check node is connected to code bits whose sum modulo-2 should be zero. Irregular LDPC codes are specified by two polynomials $\lambda(x) = \sum_{i=1}^{d_{l\max}} \lambda_i x^{i-1}$ and $\rho(x) = \sum_{i=1}^{d_{r\max}} \rho_i x^{i-1}$, where λ_i is the fraction of edges in the bipartite graph that is connected to variable nodes of degree i , ρ_i and is the fraction of edges that are connected to check nodes of degree i . Correspondingly, the degree profiles can also be identified from the node perception, i.e., two polynomials $\tilde{\lambda}(x) = \sum_{i=1}^{d_{l\max}} \tilde{\lambda}_i x^{i-1}$ and $\tilde{\rho}(x) = \sum_{i=1}^{d_{r\max}} \tilde{\rho}_i x^{i-1}$, where $\tilde{\lambda}_i$ is the fraction of variable nodes of degree i , and $\tilde{\rho}_i$ is the fraction of check nodes of degree i .

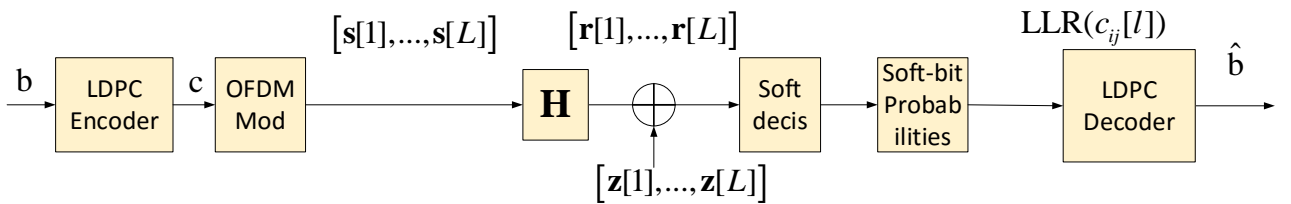


Fig. 1 System structure of an LDPC coded MIMO-OFDM system.

Soft-output detection for soft binary decoding

The system model can be observed in Fig.1. At the receiver, a soft-output detector computes the posterior probability for each antenna, which is then used to obtain soft information for the LDPC coded bits. From given the model above, we rewrite the $\mathbf{r}[k] = \mathbf{r}_M(k)$, $\mathbf{s}[k] = \mathbf{s}_M(k)$ and $\mathbf{H}[k] = \mathbf{H}_M(k)$. And the posterior probability of the transmitted symbol vector $\mathbf{s}[k]$ can be formulated as follows:

$$p(\mathbf{s}[k]|\mathbf{r}[k]) = \frac{p(\mathbf{s}[k]|\mathbf{r}[k])p(\mathbf{s}[k])}{p(\mathbf{r}[k])} \propto N(\mathbf{r} : \mathbf{H}[k]\mathbf{s}[k], \sigma_w^2 \mathbf{I}) \prod_{i=1}^{2m} \mathbf{I}_{s_i[k] \in \mathcal{A}}, \quad (9)$$

where $\mathbf{I}_{s_i[k] \in A}$ is the indicator function that takes value one if $s_i[k] \in A$ and otherwise, it is zero. Note that $p(\mathbf{r}[k]) \propto \prod_{i=1}^{2m} \mathbf{I}_{s_i[k] \in A}$, is uniform across all antenna dimensions m , although non-uniform signaling could be handled by the proposed algorithm in this paper.

In the next section, we present an approximation based on the EP algorithm. Finally, we use the standard BP algorithm to perform soft decoding fed with the vector of LLR computed (or estimated) in one shot for each LDPC-coded bit, once L symbols are received in order to obtain the n code length.

Expectation Propagation Soft-Output Detector

In [4], [5] MMSE detectors are employed in the system at the receiver side. The Expectation Propagation (EP) algorithm in the soft-output detector is proposed in [9]. Applying EP can able to achieve better performance improvements than MMSE soft-output detectors in terms one order of magnitude for rate-1/2 LDPC codes in the MIMO system. Applying the same LDPC code, EP is able to propose much more reliable estimates to the symbol posterior probabilities than of MMSE for improvement in performance. It has been presented that EP suitable as an influential and effective technique to implement the receiver detector in high-dimension MIMO systems and also considerably higher data rate.

The MMSE approximation to the true posterior distribution $\Pr(s|r)$ (as before, x is the transmitted symbols and y is the received symbols) modifications the prior above the transmitted symbols by a zero-mean component-wise independent Gaussian. Instinctively, it might make sense to select the parameters of the Gaussian prior in this way, since it equals the initial two moments of the input distribution. It is positively not the top selection, as we are interested in matching the posterior distribution. Therefore, the [9] proposes a technique, where the prior distribution is enhanced to warrant that the approximating Gaussian posterior equals the initial two moments of the posterior distribution.

The EP is a Bayesian machine learning algorithm to build controllable estimates to a specified probability distribution. In this case, we apply EP to approximate $\Pr(s|r)$ by a Gaussian distribution $q_{EP(x)} = N(x : \mu_{EP}, \Sigma_{EP})$ that equals the initial two moments of $\Pr(s|r)$, specifically

$$\mu_{EP} = E_{p(s|r)}[s] \quad (10)$$

$$\Sigma_{EP} = \text{CoVar}_{p(s|r)}[s] \quad (11)$$

This circumstance is identified as moment matching. Although the direct calculation of the $\Pr(s|r)$ moments involves specific $|A|^{2m}$ operations, an iterative solution-approaching process is established to evaluate the solution at polynomial complexity. Accordingly to [123], we can estimate the LLR for each coded symbol as following

$$\text{LLR}(c_{ji}[l]) = \log \frac{\sum_{s_i \in B_j(1)} N(s_{i,EP}, \Sigma_{i,EP})}{\sum_{s_i \in B_j(0)} N(s_{i,EP}, \Sigma_{i,EP})} \quad (12)$$

correspondingly. $s_{i,EP}$ is the i th element of the mean vector s_{EP} and $\Sigma_{i,EP}$ is the i th element of $\text{diag}(\Sigma_{EP})$. Furthermore, $B_j(c) = \text{bit in the } j\text{th location of the Gray encoding of symbol } c$.

Simulation results

In the simulation, we demonstrate the performance of the EP algorithm used to compute the hard decisions in the receiver LDPC coded MIMO-OFDM system. The EP estimates the bit posterior probabilities which applied to the LDPC decoder. We used $m = r = 32$ scenarios with a 16-QAM modulation in MIMO-OFDM system. We have applied LDPC code of length $n = 5120$ bits with equivalent to [9]. In our simulation, we have applied multipath Rayleigh fading channel matrix. Fig. 2 presented word error rate (WER) of the LDPC with rate 1/2. And we have compared the simulation of an EP detector (EPD) in the stage when we used the WER system performance of LDPC decoding

using the MMSE and EP soft-outputs. The EP algorithm runs for 20 iterations. From the result, the soft output EP detection BP decoding for LDPC coded the MIMO OFDM receiver shows the better result than MMSE with BP decoding. We have compared sum - product algorithm (SPA) and message-passing algorithm (MPA) of LDPC decoding in MIMO-OFDM system in Fig. 3. The result shows MPA has better performance than SPA in MIMO LDPC coded OFDM. These results clarify the importance of performance enhancement after the LDPC decoding stage.

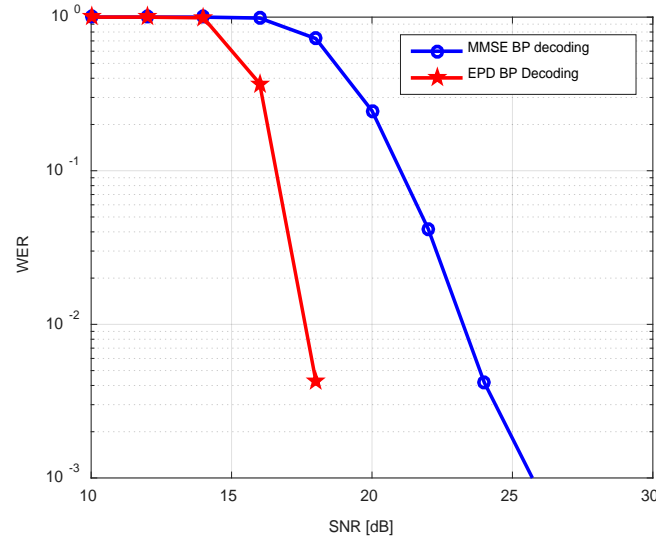


Fig. 2. EP Performance of MMSE with BP channel decoding in MIMO-OFDM System.

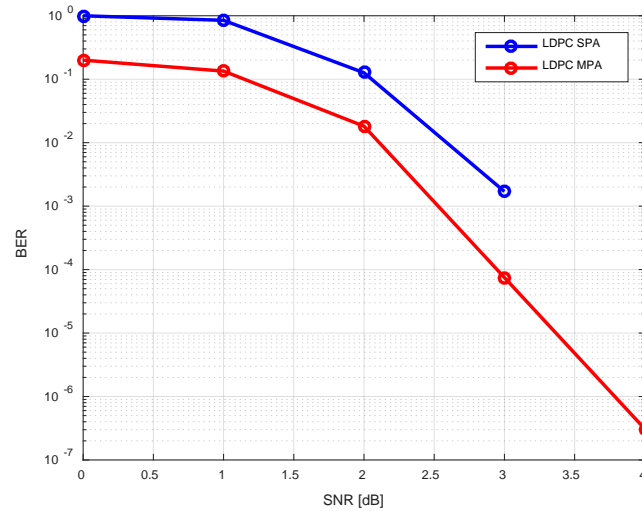


Fig. 3. Sum - Product algorithm (SPA) and message-passing algorithm (MPA) of LDPC decoding

Conclusion

In this paper, we focus on obtaining soft-decision information that can be further used for symbol detection or for decoding LDPC coded MIMO-OFDM system. The Expectation Propagation is presented as a great approximation inference technique. And it is an applicable approximation for a given probability distribution. We saw from simulation results EP approximation for the posterior probability can compute the receiver complexity and it works better in large scale antennas in LDPC coded OFDM systems. And EP also works better when we use it's output value in MPA decoding case. Overall EP able to provide enhanced performance when combined with an LDPC coded OFDM system by providing more accurate estimates to the posterior probability.

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