# Welfare analysis based on the frequency resource allocation

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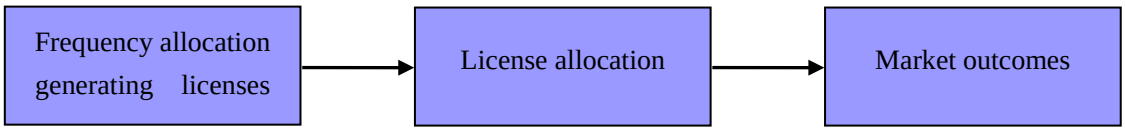
**Abstract.** The paper proposed the welfare analysis based on the frequency resource allocation by analyzing the basic framework of the frequency resource allocation policy evaluation. And analyzed the changes of social welfare caused by the extraneous variable of the Cournot model, which built contains frequency allocation policy, at the same time combined the HHI index. At last, the optimal number of solving frequency licenses obtained by rigorous mathematical derivation and complex theoretical analysis can balance the marginal costs and input costs.

## 1. Introduction

As a limited strategic resource, frequency resources have the characteristics of limitation, exclusiveness, reusability, non-exhaustiveness, inherent transmission and stain. It is national scarce resources just like air, water, land and mineral resources. Think globally, the development of radio service, market opening, and economic cultural and legal system in each country is not the same, so is the allocation way of radio frequency resources.

The frequency resource is scarce, so the management of frequency resources is very important. International frequency allocation strategy tends to be diversified and market oriented. Therefore, the analysis social welfare of the frequency resource allocation policies is necessary.

## 2. The basic framework of the frequency resource allocation policy evaluation




Frequency allocation herein is divided into three steps, as shown in Figure 1.

Figure1 Three steps of frequency allocation

Assuming it exits a planner, intent to maximize the social welfare. Based on this, the following objectives are analyzed in this paper:

(1)Allot the frequency resource to maximize the service efficiency;

(2)Select an allocation mechanism to maximize the social welfare;

(3)Based on these constraints to maximize the government revenue.

Among them, the first goal is decided before distributing the license, it contains the generation process of the license. In this process, the government allocate the frequency resources well, and then assigned to the individual. By establishing rules to discipline the market structure and performance, then affect the expected income by maximum extent.

The second goal is to maximize the general welfare by allotting the license.

The third goal is to focus on improving the social benefits applicable. In the paper, assuming that the process is a pure rent transfer. Under the circumstances, the high yield is popular than low yield beyond all question.

## 3. Theoretical model of the frequency resource allocation

In this section, the paper establishes an evaluation model as the basis for analysis. Variables in the model are included in the social welfare analysis of the frequency allocation policy. Suppose there are N companies able to offer wireless service in the market, the output level of each company

i（$i=1,2,3,...,N$）is $q_i$. Therefore, the combination of the output is $Q=\sum_{i=1}^{N} q_i$. Associated with the output market price is $p(Q)$. Each company has the form of a cost function:

$$C_i(q_i)=c(K_i,S_i)q_i \tag{3-1}$$

This formula shows that a given certain capital level of $K_i$ and numbers of frequency $S_i$, the marginal costs are fixed. When the decision of numbers are made, capital and frequency become fixed value. The price determined by the resources has been a fixed value.

Assume that the competition subject to the Cournot competition, leading to the price rules is

$$p(Q)=c(K_i,S_i)\left[1+\frac{s_i}{\varepsilon(Q)}\right]^{-1} \tag{3-2}$$

wherein， $s_i=\frac{q_i}{Q}$， $\varepsilon(Q)=\frac{1}{\frac{dp}{dQ}\frac{Q}{p}}$

The government allocates frequency resources according to the license under competition, the frequency resources assigned to a licensed is $S_i=\phi_i S$ , $0<\phi_i\le 1$. Thus, can be obtained:

$$p(Q)=c(K_i,\phi_i S)\left[1+\frac{s_i}{\varepsilon(Q)}\right]^{-1} \tag{3-3}$$

When the allocation of frequency resource is based on equal competition license, to get

$$p(Q)=c(K,\frac{S}{N})\left[1+\frac{s_i}{N\varepsilon(Q)}\right]^{-1} \tag{3-4}$$

As can be seen, Herfindahl-Hirschman index (HHI) in a case of where N companies is 1 / N. By balancing equation（3-3）can be obtained, market price depends on the elasticity of demand $\varepsilon(Q)$ the level of investment K the number of allocated frequency resource S and focus test.

When licenses are allocated according to the auction, successful bidders i need to pay for B. Among them

$$B_i\le PV=\frac{\pi_i}{r} \tag{3-5}$$

Among them, $B_i$ is to pay the company i's; $\pi_i$ is the expected net income; r is the breakage rate.

That is, the maximum payment for a license is determined by the company's expected revenue. This paper assumes that income is constant, the payment cap for that bid is

$$B_i=\frac{\alpha_i\pi_i}{r} \tag{3-6}$$

Among them， $0 \le \alpha_i \le 1$ 。

Given a fixed marginal cost, the expected net income is

$$\pi_i = \left[ p(Q) - c(K_i, \phi_i S) \right] q_i - rK \tag{3-7}$$

You can also get

$$B_i = \alpha_i \left[ \frac{(p(Q) - c(K_i, \phi_i S))q_i}{r} - K \right] \tag{3-8}$$

Assuming the same companies involved in Cournot competition, get

$$B_i = \alpha_i \left[ \frac{p(Q)HHIq_i}{-\varepsilon(Q)r} - K \right] = \alpha_i \left[ \frac{p(Q)QHHI^2}{-\varepsilon(Q)r} - K \right] \tag{3-9}$$

**4. Social welfare analysis**

In this section, we analyze the changes of social welfare caused by the external variables in the above model By definition, can be obtained $U(Q) = \int_0^Q p(x)dx$ Symmetric Cournot equilibrium with fixed marginal cost of generation can be obtained by solving the following equation:

$$\underset{Q}{Max} W(Q) = \frac{1}{N}\left[ p(Q) - c \right] Q + \frac{N-1}{N}\left[ U(Q) - cQ \right] \tag{4-1}$$

In order to show the welfare changes due to the change of variables, this paper established inverse demand function. Suppose the demand for wireless services is the function of price (P), income level (Y) and their substitute (F). We use the following demand function in this paper:

$$Q = \lambda Y^{\delta} F^{\rho} p^{\varepsilon} \tag{4-2}$$

$Q^*$ is the optimal solution of (4-1) in this paper, combined with (4-2) can be obtain:

$$W(Q^*) = \frac{1}{N}\left[ p(Q^*) - c \right] Q^* + \frac{N-1}{N}\left[ U(Q^*) - cQ^* \right] \tag{4-3}$$

Among them,

$$Q^* = \frac{\lambda Y^{\delta} F^{\rho}}{c^{-\varepsilon}} \left[ 1 + \frac{HHI}{\varepsilon} \right]^{-\varepsilon} \tag{4-4}$$

$$p(Q^*) = \frac{c}{\left[ 1 + \frac{HHI}{\varepsilon} \right]} \tag{4-5}$$

We note that when HHI= 1, corresponding to a monopoly situation siege. When HHI= 0, corresponding to the case of perfect competition.

For a given $Q_M < Q^* < Q_c$, we can know that social welfare is increasing in $Q^*$. By (4-3) can be found, social welfare increasing with income, reducing with marginal cost and market focus.

Social welfare can be obtained under the control of Cournot competition:

$$SW(Q^*) = U(Q^*) - cQ^* - NK \quad (4\text{-}6)$$

In addition, the above derivation can be found to increase the number of enterprises can increase competition. However, it is possible to bring two negative effects, namely the number of frequencies assigned to each license reduced, but increased the marginal cost; the second is to increase the number of networks, improve the cost of inputs. The optimal number solved by the model balanced these points.

## 5. Conclusions

Marketing allocation of frequency resource is today's trend, the authorities is necessary to analysis the social welfare of frequency allocation Policy.

According to the HHI index and the social welfare model of frequency allocation policy built in t he paper, through rigorous mathematical derivation and analysis, we can get that we should make de cision about the optimal number of frequency license before allocating a certain frequency to achiev e maximum social welfare, so that we can find balance in terms of cost, price, etc.

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