Parameter Setting Method of Fractional Order $PI^{\lambda}D^{\mu}$ Controller Based on Bode's Ideal Transfer Function and its Application in Buck Converter

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Abstract. The Bode's ideal transfer function is applied to the parameter setting of fractional order $PI^{\lambda}D^{\mu}$ controller in this paper. The closed loop system composed of Bode's ideal transfer is used to be a reference model in this optimized system, where the controller parameters are searched by genetic algorithm and the simulation software. The optimized system is similar to the reference model as much as possible and possesses its excellent properties. The validity of the method is proved by simulation results.

In addition, the above method is applied to the BUCK converter with constant power load. The simulation results show that the optimal controller is superior to the effect of the integer order PID controller.

Introduction

The order of calculus is promoted to fraction even to complex field by fractional calculus. But because of its computation complexity and lacking of definite physical meaning, the theoretical research has just been applied to mathematics.

With the rapid development of computer science, the application of fractional calculus has become possible[1,2]. The theory of fractional order control is the summary and supplement of the traditional integer order control theory, which would be more simulate and robust. At present, several important fractional order controllers, such as $PI^{\lambda}D^{\mu}$ controller[3], CRONE controller[4] and TID controller[5] have been promoted.

Among them, the $PI^{\lambda}D^{\mu}$ controller has greatest influence and has been widely used. Because the fractional order $PI^{\lambda}D^{\mu}$ controller has two adjustable parameters more than the integer order PID controller, it could adjust the five parameters more flexibly and reasonably to achieve better results. But as a result of the λ, u, Kp, KI, KD parameters need to design, it makes the design harder. Therefore, parameter setting and optimization of fractional $PI^{\lambda}D^{\mu}$ controller is one of the focus on the research topic.

The Bode's ideal transfer function is applied to the parameter setting of fractional order $PI^{\lambda}D^{\mu}$ controller in this paper. It can not only satisfy the system requirements of the cut-off frequency and phase margin, but also make the phase frequency characteristic curve of Bode diagram of compensated system has a level area near the cut-off frequency. That is to say, the closed-loop system is robust to the change of gain. In addition, this method is applied to control of the Buck converter and the good control effect has been achieved.

Fractional Order Calculus and Fractional Order $PI^{\lambda}D^{\mu}$ Controller

Fractional Order Calculus. Generally, the $d^n y / dx^n$ is n order differentiation of y for x. But what is the meaning if n is equal to the 1/2 ? This problem has been put forward by the French mathematician Guillaume more than 300 years ago. Although the fractional order calculus theory had been raised more than 300 years ago, it mainly focused on theoretical research in the early days. However, with the development of the computer in recent years, the application field of fractional order calculus is more and more wide. For example, fractional order control has emerged in the field of automatic control theory. In terms of modeling, some systems are difficult to accurately describe by using an integer order differential equation. Introducing fractional differential operator, the fractional order differential equation can be used to describe the systems, and it would be more close to the nature of the system characteristic.

In addition, because of the characteristics of fractional order calculus operator, fractional order controller also has a lot of superiority that can't be realized by integer order controller [6].

In the process of the development of fractional order calculus, there have been many definitions. The Riemann Liouville calculus is most widely used among them. The definition is as follows:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t-jh) \quad . \tag{1} $$

According to the definition, it has an association with all the value from the initial to that point, which is different from the limit of integer order differential. As a result, it has memory that makes the information of the past has an influence on the present and the future[7].

Fractional Order PI$^{\lambda}$D$^{\mu}$ Controller. With the development of fractional order calculus, TID, CRONE and PI$^{\lambda}$D$^{\mu}$ controller appeared. Fractional order differential equation of PI$^{\lambda}$D$^{\mu}$ controller is as follows:

$$ u(t) = K_P e(t) + K_I D^{-\lambda} e(t) + K_D D^{\mu} e(t) \quad . \tag{2} $$

The transfer function is:

$$ G_C(s) = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \qquad \lambda, \mu > 0 \, . \tag{3} $$

The appropriate values u and λ of PI$^{\lambda}$D$^{\mu}$ controller can be selected according to the order, characteristics and design requirements of the control system. The dynamic and static characteristics of system would be improved to get better control effect.

Parameter Setting of Fractional Order PI$^{\lambda}$D$^{\mu}$ Controller Based on Bode's Ideal Transfer Function

Bode's Ideal Transfer Function. An ideal form of a kind of open loop transfer function was put forward by Bode in the design of feedback amplifier:

$$ G(s) = \left(\frac{\omega_c}{s}\right)^{\alpha}, \quad \alpha \in R \, . \tag{4} $$

Where ωc is the cut-off frequency of the system, that is to say $\left|G\left(j\omega_c\right)\right| = 1$. Alpha can be fraction or integer. Substantially, G (s) is a fractional order differentiator when alpha is less than zero and a fractional integrator when alpha is greater than zero.

The Bode diagram of Bode' s ideal transfer function is very simple when 1 <α< 2. The amplitude-frequency characteristic curve is a straight line that the slope is -20αdB/dec, and the phase frequency characteristic curve is a level straight line with the value of -απ/2 [7]. Figure 1 is the Bode diagram of ideal transfer function when ωc=1,α=1.2.

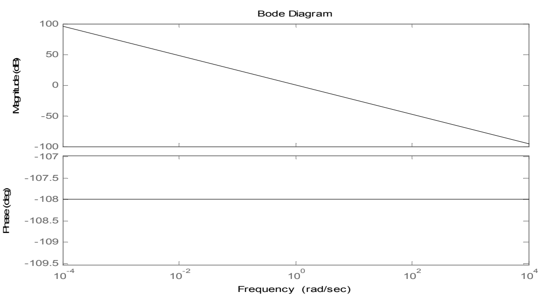


Fig.1 Bode plot of Bode's ideal transfer function

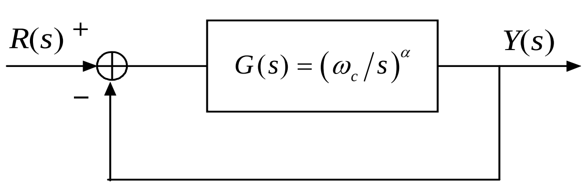


Fig.2Fractional-order control system with ideal transfer function

Figure 2 is the block diagram of unit feedback system including the ideal transfer function. The ideal transfer function makes the closed-loop system has the expected characteristic that is not sensitive to gain change. The cut-off frequency ωc will change when the gain changes, while the phase margin remains the same (φm=π-απ/2).This means that the system has strong robustness to gain change. The overshoot of the system is only related with the alpha and has nothing to do with gain. The excessive overshoot caused by gain changes can be avoided. Because of the sensor or environment change, the variation of the gain is difficult to be avoided in the process of actual control. Therefore, the robust feature is one of expected features of control system.

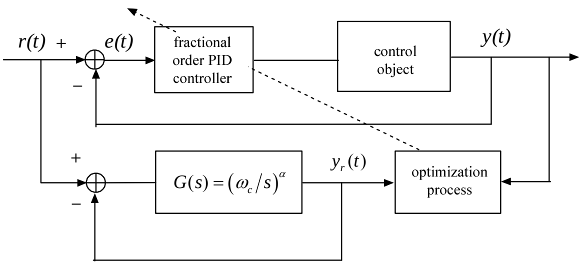
Given the ideal transfer function has excellent properties, it can be considered as a goal to design the controller. The controller is designed to make the open loop transfer function of the compensated system same as the ideal transfer function. As the ideal transfer function is fractional order, this vision can't be achieved using an integer order controller. The development of the theory of fractional order calculus and fractional order control make it possible to this idea.

Parameter Setting of Fractional Order $PI^{\lambda}D^{\mu}$ Controller Based on Bode's Ideal Transfer Function. In recent years, Bode's ideal transfer function has gotten wide attention of the researchers. The ideal transfer function performed as a reference model for integer order control in literature [8]. It is used for parameter setting of integer order controller in literature [7].

The fractional order PID controller has two degrees of freedom , order of integral and differential item, more than the conventional ones. As a result it has a greater adjustment range, which can make the control system a better approximation of fractional order reference mode. Therefore, the fractional order controller is optimized using closed loop system constituted by Bode's ideal transfer function as a reference model in this article. The response of optimized system is made close to reference model as much as possible and possesses its excellent properties. The phase frequency response curve of Bode diagram of the control system is not exactly the same as that of the ideal transfer function, but you can make it there is a "flat" area near the cut-off frequency. The closed-loop system is robust to the change of gain.

The principle of parameter setting of fractional order controller based on the ideal transfer function is shown in figure 3. The control object can be either integer order or fractional order.

Setting steps are as follows: first of all, Bode's ideal transfer function is calculated according to the system requirements in the time domain or frequency domain. Secondly, the closed-loop transfer function of the ideal system is used as a reference model. The ITAE or ISE of y (t) - y_r (t), that is to say, the error of the system output and the reference model output, is used as optimization indexes. Finally, the most optimal fractional order controller is searched by the genetic algorithm based on Simulink block diagram.



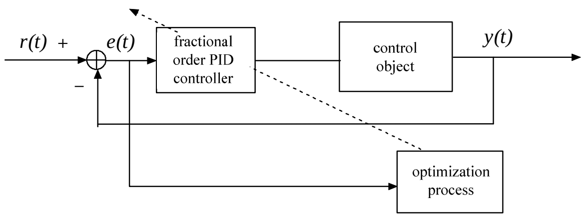


Fig.3 System structure for $PI^{\lambda}D^{\mu}$ controller tuning Fig.4 System structure for traditional $PI^{\lambda}D^{\mu}$ controller tuning

The optimal design method [9, 10] based on ITAE of system error for fractional order PID controller is an optimization design method promoted from common integer order PID controller. To illustrate the difference and contact of the ideal transfer function method and the optimal design method based on ITAE of system error and to facilitate simulation and compare, the brief introduction is given as follows. The principle block diagram is shown in figure 4.

Except that its optimization indexes are different from the ideal transfer function method, the optimization algorithm, the algorithm of fractional order module and simulation environment are all same as ideal transfer function method.

A third-order object is used as an example to testify the above parameters setting methods as follow. The transfer function of the object is:

$$G_P(s)=\frac{1}{(s+1)^3}\,. \tag{5}$$

Selecting phase margin φm=π/4 and cutoff frequency ωc=1, the corresponding ideal transfer function s - 3/2 is get. The fractional order PID controller adjusted by the ideal transfer function method is:

$$G_{c1}(s)=3.68+1.93s^{-1.17}+1.92s^{1.13}\,. \tag{6}$$

The integer order PID controller adjusted by the ideal transfer function method is(order of calculus λ and μ are fixed of 1 in the tuning process):

$$G_{c2}(s)=2.68+2.24s^{-1}+2.21s\,. \tag{7}$$

The controller applying to equation (5) obtained according to the method illustrated in figure 4 is:

$$G_{c3}(s)=6.86+2.24s^{-1}+4.68s^{1.19}\,. \tag{8}$$

The Bode diagrams of the systems that the object of equation (5) are controlled by GC1, GC2 and GC3 controller respectively are compared in Figure 5 . It can be seen from the diagram, there is a "flat" area near the cut-off frequency in the phase frequency characteristic curves of GC1 and GC2 control system adjusted using the ideal transfer function. This means that the system has robustness to the gain change. This area of fractional order controller is broader than that of integer order, so the robustness will be stronger.

Although the phase margin of GC3 control system is greater than GC1 and GC2 control system (the design value of phase margin of GC1, GC2 system is π/4, but the phase margin of GC3 system is obtained from another optimization method), it does not have the "flat" area.

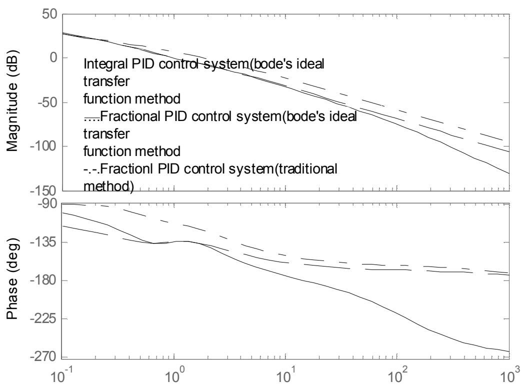


Fig5.Bode plot of G_{C1},G_{C2} and G_{C3} control system

In order to verify the robustness of GC1 control system for gain change, the gain of the object is set to 0.5,1 and 1.5. The step response curves are compared in Figure 6. It can be seen from the diagram, the response speed is affected when the gain is changed, while the change of overshoot is very small. Figure 7 shows the step response of GC3 control system for gain change. It is obvious that the system is not robust to the change of gain.

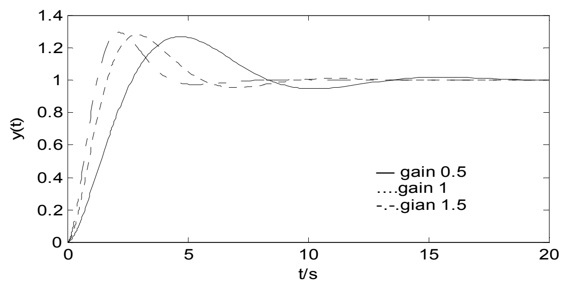


Fig6.Unit step response of G_{C1} control system for deferent gain

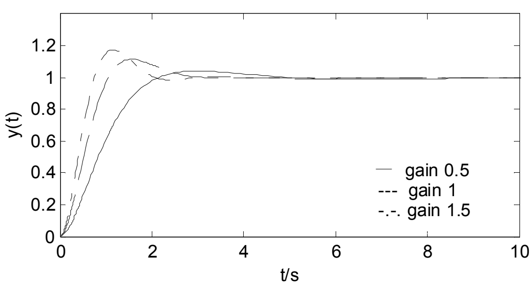


Fig7.Unit step response of G_{C3} control system for deferent gain

Application in Buck Converter of the Setting Method

In order to verify the control effect of obtained fractional $PI^{\lambda}D^{\mu}$ controller to drive the BUCK converter driving constant power load, the simulation has been carried on by MATLAB software.

The principle diagram of BUCK converter driving constant power lord is shown in figure 8. Parameters of main circuit are R = 5 Ω, L = 5mH, C = 50μF and vin = 20V. The output voltage v_0 and power P are 10V and 10W respectively.

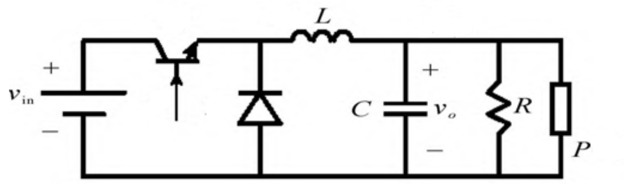


Fig8. Principle diagram of BUCK converter

Using the above method and genetic algorithm to optimize the parameters of fractional order controller, the obtained $PI^{\lambda}D^{\mu}$ controller is

$$G_{cf}(s) = 1.1 + \frac{277}{s^{0.8}} + 0.0001 s^{1.1} . \quad (9)$$

The traditional PID controller obtained by Z - N setting method is [11]

$$G_c(s) = 0.145 + \frac{247}{s} + 2.12 \times 10^{-5} s \ . \quad (10)$$

The curves of step response of the fractional order and the traditional integer order control are shown in figure 9. Obviously, fractional $PI^{\lambda}D^{\mu}$ controller is significantly better than the integer order PID controller in response speed, adjust time and overshoot, etc.

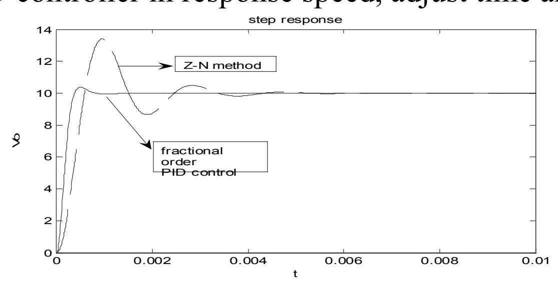


Fig9. Step response

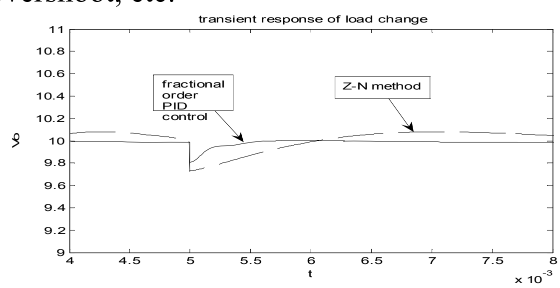


Fig10. Response curve while load change

Figure 10 is the response curves of fractional order $PI^{\lambda}D^{\mu}$ and conventional PID control strategy for load resistance jump from 5Ω to 2.5 Ω. It is shown that fractional $PI^{\lambda}D^{\mu}$ controller has stronger robustness when the system load is changed.

Conclusion

The fractional order controller has less sensitivity and more robustness for the change of parameters of the controlled system. The static and dynamic characteristics of the control system would be dramatically improved.

The Bode's ideal transfer function is applied to parameter setting of fractional order PID controller in this article. The examples prove the method can not only obtain satisfactory dynamic and static characteristic, also can improve the robustness of system to gain change.

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