New Models for Capacitated Air-cargo Scheduling Problems

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***Abstract*—This paper examines hub location and scheduling of planes for an express company. Two Mixed Integer Programming (MIP) models are constructed for this problem, restricted model (RM) and extensible model (EM). The comparison between the two models shows that the first one performs better than the existing models in terms of solution time, while the second one with different variable definition may further reduce the total cost by allowing more visiting hubs for serving an O-D pair. Due to the NP-hardness of the problem, a two-stage hybrid algorithm is developed to solve the large-scale instances. Numerical experiments are conducted to test the performance of the algorithm.**

Keywords-air-cargo; vehicle; hub-and-spoke; airline; hub location.

I. INTRODUCTION

With the increasing aerial freight volume, outsourcing cannot fully satisfy the requirement of the express company. In order to deliver the goods timely and take advantages of the economies of scale, more and more express companies begin to combine the self-owned planes and outsourcing. Thus, coordinating the self-owned planes with outsourcing to meet all service demands in a short time is a challenge.

The aerial freight scheduling is often turned into a problem associated with the Hub Location Problem (HLP) which was first put forward by Hakimi [1]. Later, Toh and Higgins [2] applied the hub location problem to the aerial transport industry in which the high no-load rate of trips was discussed and an alternative solution by introducing hubs and using smaller-capacitated planes was suggested in order to decrease the operation cost. The first mathematical formulation and solution method based on the HLP were given by O'Kelly [3,4, 5]. Since O'Kelly's formulation is quadratic, Campbell [6] developed a linear mathematical formulation with p-hub median location problem (p-HMLP). Later, Ernst and Krishnamoorthy [7] proposed a 0-1 formulation to the p-HMLP. In the above work, each demand service with an original node and a destination node (called an O−D pair for short) is required to go through a path with exactly two hubs. Later, lots of work has been done based on the p-HMLP model. In this paper, the situation that the transportation volume of an O−D pair can be assigned to several paths has not been considered, and the capacity of hub nodes is pre-specified as given, which is different from the reality however.

This paper attempts to integrate the hub location and the scheduling of air cargo planes, and solve them in a single model. Sender and Clausen [8] is one of the few papers that constructed an MIP model to handle this integrated problem. The model, however, takes a long time to get the optimal solutions without an efficient algorithm. In this paper, a more efficient model, which requires less variables and constraints than Sender and Clausen, is provided. Moreover a competitive algorithm is also developed for large-scale instances. The remainder of this paper is organized as follows: two formulations are constructed in section 2; a hybrid algorithm for the formulation is given in section 3; numerical experiments are conducted in Section 4; and the conclusions are drawn in Section 5.

II. PROBLEM FORMULATION

This problem is to select a set of hubs from dispersed candidate nodes, assign O−D pairs to hubs in order to route the demand of each O−D pair through the network, and at the same time deploy or outsource the airplanes to engine the network, with the objective of minimizing the total cost including the hub setup cost and transportation cost. The constraints mainly include: the flow conservation constraint, the transportation capacity constraint, and the restriction on the number of self-owned planes. The related assumptions of this problem are presented as follows:

1) Only the nodes that provide transfer service can be seen as a hub;

2) Self-owned plane is required to go back to its original node at the end of the planning horizon, while the outsourcing planes do not have this kind of requirement;

3) The demand volume of each O−D pair can be assigned to multiple paths which connect O and D but via different hubs.

A. Model 1: The Model with Restriction on the Number of Visiting Hubs (RM)

In the following, we will present an MIP model in which the number of hubs for an O–D pair is restricted to no more than two for each path connecting O and D. The related notations are presented as follows:

Input parameters:

L the set of plane types indexed by l. Index l_0 represents the outsourcing plane.

n^l the number of self-owned l-type planes.

p^l the capacity of l-type plane.

N the set of nodes indexed by i, j, O and D.

f_i the operational cost for node i if it is set up as a hub.

h_{ij} 1, if there are no commercial direct flights between nodes i and j; 0, otherwise.

d_{ij} the average flight time between nodes i and j.

d^{OD} the demand from node O to node D.

c_{ij}^{l} the unit-time transportation cost of using a l-type plane from node i to node j

M a sufficiently large positive value.

Decision variables:

Y_i 1, if node i is set up as a hub; 0, otherwise.

X_{ij}^{OD} the flow amount of the O−D pair assigned to link $i \to j$ whose immediately previous node is O and whose immediately following node is D.

R_{ij}^{l} the number of l-type planes assigned to link $i \to j$. $R_{ij}^{l_0}$ interprets that of outsourcing.

The formulation is given as follows.

$$\min \sum_i f_i Y_i + \sum_l \sum_i \sum_j c_{ij}^l d_{ij} R_{ij}^l \tag{1}$$

$$X_{ij}^{OD} \le MY_i, \forall O, D, i, j \in N, i \ne O \tag{2}$$

$$X_{ij}^{OD} \le MY_j, \forall O, D, i, j \in N, j \ne D \tag{3}$$

$$\sum_i \sum_j X_{ij}^{OD} = d^{OD}, \forall O, D \in N \tag{4}$$

$$\sum_O \sum_D X_{ij}^{OD} + \sum_k \sum_m \left(X_{jk}^{im} + X_{mi}^{kj} \right) \le \sum_l p^l R_{ij}^l, \forall i, j \in N \tag{5}$$

$$R_{ij}^{l_0} = 0, \forall i, j \in N, where, h_{ij} = 0 \tag{6}$$

$$R_{ij}^{l} = R_{ji}^{l}, \forall i, j \in N, l \in L, where, l \ne l_0 \tag{7}$$

$$\sum_i \sum_j R_{ij}^l \le n^l, \forall l \in L, j < i, where, l \ne l_0 \tag{8}$$

$$X_{iO}^{OD} = 0, \forall O, D, i \in N \tag{9}$$

$$\sum_j X_{Oj}^{OD} = X_{OD}^{OD}, \forall O, D \in N \tag{10}$$

$$X_{Dj}^{OD} = 0, \forall O, D, j \in N \tag{11}$$

$$\sum_i X_{iD}^{OD} = X_{OD}^{OD}, \forall O, D \in N \tag{12}$$

In the formulation, the object function (1) is to minimize the total cost including the operation cost of employed hubs and the transportation cost of self-owned and outsourcing planes. Constraints (2)-(3) indicate that any transfer node within a path should be a hub, which corresponds to Assumption 1. Constraints (4) guarantee each O−D pair demand should be satisfied. Constraints (5) make sure that the total capacity of employed planes should not be violated. Constraints (6) mean that no outsourcing can be applied for the link without commercial direct flight. Constraints (7) correspond to Assumption 2. Constraints (8) restrict the number of self-owned planes that can be allocated. Constraints (9)-(12) confine some variables without physical meaning equal to zero. The constraints for defining variable range are omitted in this paper.

This model will encounter application difficulty when more number of transfer times is needed. Therefore, a more general model, which can accommodate more hubs for each O-D pair, will be proposed in the next section.

B. *Model 2: The Model with the Number of Visiting Hubs Controllable (EM)*

All notations presented in Section A will be used in this section except for X_{ij}^{OD}, which will be replaced by W_{ij}^{OD}. The specific explanations are given in the following:

Additional input parameters:

the maximum number of visiting hubs in each path for serving the O−D pair.

Additional decision variables:

W_{ij}^{OD} the flow amount of the O−D pair assigned to link $i \to j$. As opposed to X_{ij}^{OD}, it does not require that the immediately previous node of link $i \to j$ be O and the immediately following node be D. It only means the flow amount of the O−D pair that passes through link $i \to j$.

Z_{ij}^{OD} 1, if the link $i \to j$ is used by O−D pair; 0, otherwise.

S_i^{OD} the sequence number of node i visited by O−D pair.

The formulation is presented as follows.

$$\min \sum_i f_i Y_i + \sum_l \sum_i \sum_j c_{ij}^l d_{ij} R_{ij}^l \tag{13}$$

$$W_{ij}^{OD} \le MY_i, \forall O, D, i, j \in N, i \ne O \tag{14}$$

$$W_{ij}^{OD} \le MY_j, \forall O, D, i, j \in N, j \ne D \tag{15}$$

$$\sum_i W_{ik}^{OD} = \sum_j W_{kj}^{OD}, \forall O, D, k \in N, where, k \ne O, k \ne D \tag{16}$$

$$\sum_j W_{Oj}^{OD} = d^{OD}, \forall O, D \in N \tag{17}$$

$$\sum_i W_{iD}^{OD} = d^{OD}, \forall O, D \in N \tag{18}$$

$$\sum_O \sum_D W_{ij}^{OD} = \sum_l p^l R_{ij}^l, \forall i, j \in N \tag{19}$$

$$R_{ij}^{l_0} = 0, \forall i, j \in N, where, h_{ij} = 0 \tag{20}$$

$$R_{ij}^{l} = R_{ji}^{l}, \forall i, j \in N, l \in L, where, l \ne l_0 \tag{21}$$

$$\sum_i \sum_j R_{ij}^l \le n^l, \forall l \in L, j < i, where, l \ne l_0 \tag{22}$$

$$W_{ij}^{OD} \le MZ_{ij}^{OD}, \forall O, D, i, j \in N \tag{23}$$

$$S_i^{OD} + 1 - S_j^{OD} - M\left(1 - Z_{ij}^{OD}\right) \le 0, \forall O, D, i, j \in N \tag{24}$$

$$S_O^{OD} = 0, \forall O, D \in N \tag{25}$$

$$S_D^{OD} \le t + 1, \forall O, D \in N \tag{26}$$

$$W_{iO}^{OD} = 0, \forall O, D, i \in N \tag{27}$$

$$W_{Dj}^{OD} = 0, \forall O, D, j \in N \quad (28)$$

$$W_{kk}^{OD} = 0, \forall O, D, k \in N \quad (29)$$

In the formulation, the objective function (15) is the same as the objective function (1). Constraints (16)-(17) indicate that any transfer node within a path should be a hub, which corresponds to Assumption 1. Constrains (18) are the flow conservation constraint. Constraints (19)-(20) guarantee that the outflow volume from the source is equal to the inflow volume to the sink, both of which are equal to the demand volume of the corresponding O−D pair. Constraints (21) make sure that the total capacity of employed planes should not be violated. Constraints (22) mean that no outsourcing can be applied for the link without commercial direct flight. Constraints (23) correspond to Assumption 2. Constraints (24) restrict the number of self-owned planes that can be allocated. Constraints (25)-(26) show that if $W_{ij}^{OD} > 0$, the inequality $S_j^{OD} - S_i^{OD} \geq 1$ holds. Constraints (27)-(28) ensure that any path could not visit more than t hubs. Similar to the previous, constraints (29)-(31) confine some variables without physical meaning equal to zero. Similar to the previous, the constraints for defining variable range are omitted in this paper.

III. GREEDY DEMAND PAIR HYBRID ALGORITHM

Due to the NP-hardness of p-HMLP [9], in the following, we will develop a hybrid algorithm called greedy demand pair hybrid algorithm (GDPHA), which includes two stages. The basic idea of this algorithm is to assign the self-owned planes in the first stage and then solve the modified model with fixed assignment of self-owned planes in the second stage by commercial solver package such as CPLEX. The specific procedure is presented as follows:

Step 1: Set a search parameter F and threshold value V;

Step 2: Sequence the plane list in an increasing order of the plane's unit cost;

Step 3: For every g in interval $[0, f_{\max} / F]$

Step 3.1: Calculate d_{ij}' for each link $i \to j$, where d_{ij}' the allocating demand for a set of is links which are close enough to link $i \to j$;

Step 3.2: $a_{ij} = 1$, if $d'_{ij} > p^l$; else, $a_{ij} = 0$;

Step 3.3: Select the link with the longest flight time and satisfying $a_{ij} + a_{ji} = 2$, and then go to Step 3.4; otherwise, select the next g and go back to Step 3.1;

Step 3.4: If $V > (CB_{ij} + CB_{ji}) / (CA_{ij} + CA_{ji})$ (where CB represents cost before allocating and CA represents cost after allocating), link (i, j) is selected as a candidate link, and then select next g and go back to Step 3.1;

Step 4: Assign the plane

Step 4.1: Select the link with the maximum value of $\left[(CB_{ij} + CB_{ji}) - (CA_{ij} + CA_{ji})\right]$ from candidate links passed from Step 3;

Step 4.2: The plane is assigned to the link selected in Step 4.1 and both ends of the link are set as the hub automatically. In addition, the consolidated O-D pairs included in the link will be assigned to the link, in an increasing order of the flight time, until the capacity of the plane is completely depleted. Update the remaining demand for each involved O-D pair.

Step 5: Fix the plane assignment and the hub setting information and solve the modified model by CPLEX to generate a complete solution result.

IV. NUMERICAL EXPERIMENTS

In this section, two sets of experiments are conducted. The first one is to validate the advantage of the proposed models by comparing them with the Sender's formulation; another one is to test the performance of GDPHA. All experiments are coded by JAVA and run on a computer with a 16GB RAM and a 2.4 GHz CPU.

The coordinates of all cities will be generated randomly within a 200 by 200 grid. It is assumed that the longest flight time in this grid is 300 minutes. The flight time between two nodes is proportional to their distance. Plane information is directly obtained from the express company. Whether there exists an airline depends on the type of cities and the flight time between them. Demand of link depends on the type of cities in both ends.

A. Model Validation

TABLE I shows the comparison results of three models. The variable number and constraint number increase as the city number increases, while the (RM) has the smallest number of variables and constraints among the three models although their optimal values are the same. It can also be seen that no more than two hubs are needed for each O-D pair. From the table, we also see that the runtimes of the Sender's formulation are much larger than those of RM. It seems that RM outperforms others in terms of efficiency. Although extendable model (EM) does not show any advantage from TABLE I, it may show its value when more than 2 hubs are needed for serving an O-D pair. It can be seen from TABLE II which shows the computational results of another chain instance that EM has a significant improvement on cost when parameter t is more than 2.

TABLE I. COMPARISON RESULTS FOR RM, EM AND SENDER'S FORMULATION

	Variable Number			Constraint Number			Objective Value			Runtime[s]		
CN*	**Sender**	**RM**	**EM**	**Sender**	**RM**	**EM**	**Sender**	**RM**	**EM**	**Sender**	**RM**	**EM**
14	4494	350	350	7072	2684	23934	1213	1213	1213	91	22	162
15	5224	409	409	8402	2804	29451	944	944	944	91	67	176
16	6434	446	446	10306	3635	39822	1340	1340	1340	179	82	951
17	7687	513	513	12361	4397	50357	1301	1301	1301	141	99	1035
18	9312	600	600	14852	5588	67214	1757	1757	1757	30	46	376
19	10720	669	669	17254	6224	84879	1826	1826	1826	205	128	1068
20	12472	732	732	20122	7301	98814	1966	1966	1966	157	188	715
21	14218	799	799	23102	8144	117489	2145	2145	2145	205	135	1201
22	16300	878	878	26534	9361	138840	2151	2151	2151	1448	262	1201
23	18583	965	965	30293	10699	160510	2497	2497	2497	577	249	1205

* CN is city number. Each instance includes three big cities.

B. *Algorithm Validation*

In this section, we will test the performance of the GDPHA on large-scale instances for which the number of cities increase to 28, and more planes for each type are deployed. Parameters $F = 2$, $V = 1.25$ are set for the subsequent experiments. TABLE I II shows the statistics result of 20 large-scale instances solved by CPLEX and GDPHA within 1200 seconds. It can be seen from the table that GDPHA can in most instances provide a better solution than CPLEX, which means the heuristic algorithm performs well. TABLE I V shows the statistics run time of another 50 instances. We can find GDPHA have a better time efficiency. These two dimensions show that GDPHA has advantages over CPLEX in terms of both solving quality and time efficiency.

TABLE II. RESULT OF CHAIN INSTANCE

Model	Parameter	Optimal Value	GAP
RM	-	139.94	13.58%
EM	t=3	123.2	0%
EM	t=6	123.2	-

TABLE III. COMPARISON RESULTS FOR CPLEX AND GDPHA

	Average Optimal Value	Better Ratio
CPLEX	2384.96	20%
GDPHA	2293.84	80%

TABLE IV. STATISTICS OF 50 INSTANCES' RUNTIME

	(0, 600]	(600, 1200]	(1200, 1800]	(1800, 2400]	(2400, infinite)	Average Runtime[s]
GDPHA	0	38	12	0	0	1011.023
CPLEX	0	14	20	12	4	1628.961

V. CONCLUSION

This paper focus on air express companies air-cargo problem. An enhanced model of p-HMLP, RM, can provide a satisfied solution in shorter time, while an extensible model is developed to balance both time efficiency and cost efficiency. Related results have been proved by some instances. Meanwhile, this paper gives an effective hybrid algorithm to solve large-scale instance. Compared with the result of ILOG CPLEX, GDPHA could provide a better solution with higher time efficiency in some instances. Future work along this line of research will be focused on changing assumptions. Moreover, new algorithm will be approached to solve extensible algorithm.

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