

Optimization Research on Multi-objects Aircraft Maintenance Shop Scheduling Problem

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Abstract—To make the maintenance adapt for the modern aircraft, the modeling and scheduling algorithm of aircraft maintenance shop scheduling problem are discussed. At the base of maintenance shop scheduling modeling referring to the formalization of flexible job shop scheduling problem, genetic algorithm is adopted as framework, where the execution steps are designed, and the coupling operator is introduced to adapt the operation sequence part of chromosome to avoid the situation that the chromosome might violate the coupling constraint and could not decode. Subsequently, applicability and quality of multi-objects scheduling algorithm are verified through maintenance shop example.

Keywords—military aircraft; maintenance shop; coupling constraint; multi-objects

I. INTRODUCTION

As the reference of aircraft maintenance, maintenance shop scheduling should meet the demands to optimize and integrate task element and resource element. However, the contradiction among scheduling objects and resource elements made maintenance use to mainly rely on the artificial scheduling and coordination, which is inefficient, inflexible and difficult to spread. As a result, discuss the automatic generation of scheduling scheme to made maintenance shop adapt for modern aircraft, will have important theoretical value and practical significance. In the optimization and integration of task element and resource element, process constraint is always the basis of element relationship, so flexible job shop scheduling optimization research could provide help for the aircraft maintenance shop scheduling based on process constraint.

Based on the fitness assignment strategy of Pareto dominance, NSGA^[1], NPGA adopted fitness sharing mechanism and niche technology to keep the population diversity and to avoid converging to a single Pareto curve, SPEA^[2], SPEA2^[3], PAES^[4], NSGA-II^[5], NPGA2^[6] introduced external population to hold non-dominant vector and derived out the interactive mode and maintenance technology of external population and internal population.

At the same time of achievements for reference, we also need to notice that the process constraint of flexible job shop scheduling problem adopts chain constraint, which is only suit to describe the relationship among the operations in single-component systems. Nevertheless, as the representative of multi-component systems, aircraft maintenance is not only involve the chain constraint of operations in the same task, but

also refer to the coupling constraint of operations in the different task. Therefore, we take advantage of scheduling optimization algorithm achievement, expend the existence form of process constraint, and composite the chain constraint and coupling constraint to explore the aircraft maintenance shop scheduling optimization.

II. MULTI-OBJECTS AIRCRAFT MAINTENANCE SHOP SCHEDULING MODEL CONSTRUCTION

The paper chooses formalization of flexible job shop scheduling problem to create model. Let s_v be the start time, p_{v,μ_v} be the process time, e_v be the finish time of operation v .

A. Task Model

Given a set $J = \{J_i | i = 1, 2, \dots, n\}$ for tasks, which is composed of the operation set $O = \{O_{ij} | i = 1, 2, \dots, n, j = 1, 2, \dots, n_i\}$, $n_i = |J_i|$ will be the number of operations belonging to J_i . Task model describes the constraints from the perspective of process constraint:

1. Task attribution constraints: For an arbitrary operation $v \in O$, there exists unique $\rho_v = J_i \in J$, which denotes the operation v is the component of task J_i .

2. Chain constraint: For arbitrary operations v, u that belong to the same task, where exists process order. Set v is priori to u , $s_u \geq s_v + p_{v,\mu_v}$ should be meet, and let $JP[v]$ and $JS[v]$ be the chain adjacent predecessor and chain adjacent successor of operation v .

3. Coupling constraint: For arbitrary operations v, u that belong to the different task, where exists process order. Set v is priori to u , and let $OP[v]$ and $OS[v]$ be the coupling adjacent predecessor and coupling adjacent successor of operation v .

B. Scheduling Model

Given a set $M = \{M_k | k = 1, 2, \dots, m\}$ for units, which is composed of the allocation and sequence set $\prod_{k=1}^m Q_k = \{v | \mu_v = M_k\}$, $m_k = |Q_k|$ will be the number of operations belonging to M_k . Scheduling model describes the constraints from the perspective of unit constraint:

1. Unit attribution constraints: For an arbitrary operation $v \in O$, there exists unique $\mu_v = M_k \in M$, which denotes the operation v will be processed on unit M_k , and M_k could be arbitrarily selected in units set C_v .

2. Unit constraint: For arbitrary operations v, u that belong to the same unit, where exists process order. Set v is priori to u , $s_u \geq s_v + p_{v, \mu_v}$ should be meet, and let $MP[v]$ and $MS[v]$ be the unit adjacent predecessor and unit adjacent successor of operation v .

C. Object Model

According to practical demands of maintenance, the paper sets objects in common use as follows:

1. Complete time setting: Let C_{J_i} be the complete time of J_i , and set

$$f_1 = \min \max_{J_i \in J} C_{J_i} \tag{1}$$

2. Load time setting: Let $L_k = \sum_{v \in O} p_{v, M_k}$ be the load time of M_k , and set

$$f_2 = \min \max_{M_k \in M} L_k \tag{2}$$

$$f_3 = \min \sum_{M_k \in M} L_k \tag{3}$$

III. MULTI-OBJECTS SCHEDULING OPTIMIZATION ALGORITHM DESIGN

The paper adopts genetic algorithm as framework and introduces the adaptive mechanism of coupling constraint to design the scheduling algorithm around the step.

A. Chromosome Encoding Mechanism Design

Maintenance shop scheduling needs to consider the operation allocation and operation sequence at the same time, chromosome encoding mechanism could be designed segmentally.

1. The operation allocation part of chromosome is composed of $\sum_{i=1}^n |J_i|$ genes, which is sequenced by the priority of O_{ij} belonging to J_i . For an arbitrary gene v , the operation that v denotes is invariant, whose assignment scope is units set C_v .

2. The operation sequence part of chromosome is composed of $\sum_{i=1}^n |J_i|$ genes as well. For an arbitrary gene $v = i$, if the occurrence number of i equals to j by the end of v , that means v denotes operation O_{ij} .

B. Scheduling Scheme Initialization

Let JA be the task allocation array, $JA(i)$ denotes the number of allocated operations belonging to J_i , $JA(i) = 0$; let PM be the unit allocation array, $PM(k)$ denotes the cumulative time of allocated operations belonging to M_k , $PM(k) = 0$; let AL denotes the list that operations to be allocated, $AL = \emptyset$.

1. Test the first operation O_{ij} to be allocated in J_i in proper order, and let O_{st} be the coupling adjacent predecessor of O_{ij} . If $JobA(s) \geq t$, it means that O_{ij} could be allocated, and adds i to the tail of AL ; otherwise, it means that O_{ij} is not meet the allocation condition.

2. Randomly select an operation in AL , and let O_{ij} be the selection, $JA(i) + 1$, and wipe AL .

3. Randomly select a unit in unit set of O_{ij} , and let M_k be the selection, $PM(k) + p_{O_{ij}, M_k}$.

4. According to chromosome encoding mechanism, set the selection O_{ij} and M_k into corresponding genes, and loop execute the step 1 to 4.

C. Crossover Operator Design

Crossover operator simulates the inheritance patterns of biological intergeneration, which inherits the excellent characteristics from parent chromosomes, and needs to be designed into the operation allocation part and the operation sequence part.

1. In operation allocation part, the operation that arbitrary gene v denotes is invariant, the paper adopts uniform cross to design crossover operator.

2. In operation sequence part, the operation that arbitrary gene v denotes is variant, the paper adopts POX cross to design crossover operator.

D. Mutation Operator Design

Mutation operator simulates the mutation patterns of biological intergeneration, which improves population diversity and escapes local minima with a chance, and needs to be designed into the operation allocation part and the operation sequence part.

1. In operation allocation part, the operation that arbitrary gene ν denotes is invariant, so that mutation operator could be designed based on the same genes.

(1) Randomly select N genes in operation allocation part of chromosome, $N \in [0, \sum_{i=1}^n |J_i|]$.

(2) Mutate the selected N genes in proper order, and let gene ν denotes operation O_{ij} , make ν randomly assign in the scope of C_ν .

2. In operation sequence part, the operation that arbitrary gene ν denotes is variant, so that mutation operator could be designed based on the different genes.

(1) Randomly select N genes in operation sequence part of chromosome, $N \in [0, \sum_{i=1}^n |J_i|]$.

(2) Mutate the selected N genes in proper order, and let gene ν select mutation pattern from insert pattern and exchange pattern, where the position is also random.

E. Coupling Operator Design

The operation sequence part of encoding mechanism that paper designed could only satisfy the demand of chain constraint, so that the chromosome might violate the coupling constraint and could not decode after crossover and mutation. Therefore, the paper introduces the adaptive mechanism of coupling constraint what would be described as coupling operator.

Let ST be the task sequence array, $ST(i)$ denotes the number of operations adapted, $ST(i) = 0$; let SL be the list that operations to be adapted, $SL = \emptyset$.

1. Test the first gene ν to be adapted in parent chromosome $P(T)$, and let O_{st} be the coupling adjacent predecessor of O_{ij} what ν denotes. If $ST(s) \geq t$, it means that O_{ij} could be adapted, and adds ν to the tail of child chromosome $C(T)$, $ST(i) + 1$; otherwise, adds ν to the tail of list SL that operations to be sequenced.

2. Test the genes in the list SL in proper order, if operation O_{ij} what ν denotes satisfy the adaptive condition, move ν from SL to the tail of child chromosome $C(T)$, $ST(i) + 1$.

3. Loop execute the step 1 to 2, until all the genes in $P(T)$ are adapted and SL is empty.

Coupling operator reflect the constraint against chromosome sequence, which could be the connection between chain constraint and coupling constraint.

F. Chromosome Decoding Mechanism Design

Chromosome decoding mechanism is the inverse operation of encoding mechanism, which should be designed to exclude the semi-activity schedule.

Let AM be the allocation unit array, where $AM(i, j)$ denotes the unit that O_{ij} selected; let AP be the allocation time array, where $AP(i, j)$ denotes the process time of O_{ij} in the unit that $AM(i, j)$ decides. The key to generate the scheduling scheme is the decoding of operation sequence part.

1. Identify the operation O_{ij} that the first gene ν denotes in the operation sequence part of chromosome, remove ν from the chromosome and test that weather O_{ij} is the first operation of the task or the unit that belongs, and coupling adjacent predecessor of O_{ij} is not existed.

2. If the condition is satisfied, $s_{O_{ij}} = 0$, compute the end time of O_{ij} ; otherwise, compute $e_{JP[O_{ij}]}$, $e_{OP[O_{ij}]}$ and the earliest embeddable interval $[ps_i, pe_i]$, $s_{O_{ij}} = \max\{e_{JP[O_{ij}]}, e_{OP[O_{ij}]}, ps_i\}$, compute the end time of O_{ij} .

3. Loop execute the step 1 to 2, until the operation in operation sequence part of chromosome is empty.

G. Selection Operator Design

Selection operator simulates natural selection and the performance of chromosome is not a single, which needs to adopt multi-objects technology to design.

1) Fitness Assignment Strategy

The choice of the scheduling schemes depends on assessment of chromosomes, and Pareto dominate strategy divides chromosomes into different level to sequence the schemes. The paper chooses the NSGA-II method to divide the chromosomes into levels with dominated relationship, and let Z_f be the population, $Z_f(k)$ be the swarm in non-dominated level k .

1. Selects all the non-dominated chromosomes in population Z_f to form $Z_f(k)$, $k = 0$.

2. Selects all the non- dominated chromosomes in swarm $Z_f - \bigcup_{i=1}^k Z_f(k)$ to form $Z_f(k)$.

3. Loop execute the step 1 to 2 until $Z_f - \bigcup_{i=1}^k Z_f(k) = \emptyset$.

2) *Diversity Retaining Strategy*

At the basis of non-dominated levels, it still needs to differentiate for the chromosomes in the same level to avoid to converge to a single scheduling scheme and to make the non-dominated chromosomes distributed uniformly in the non-dominated surface, the paper chooses Hamming distance to measure the distances between chromosomes.

The measure of Hamming distance depends on the sum of the same gene with different attribution in chromosomes. Let x^p, x^q be arbitrary chromosomes in feasible space, $x^{p,allocation}, x^{q,allocation}$ denotes operation allocation part, $x^{p,sequence}, x^{q,sequence}$ denotes operation sequence part, and the Hamming distance between x^p and x^q could be described as follow:

$$H(x^p, x^q) = \sum_{j=1}^n \sum_{i=1}^{|J_i|} h(x_j^{p,allocation}, x_j^{q,allocation}) + \sum_{j=1}^n \sum_{i=1}^{|J_i|} h(x_j^{p,sequence}, x_j^{q,sequence}) \quad (6)$$

The similarity of x^p in non-dominated level $Z_f(k)$ is:

$$S(x^p) = \sum H(x^p, x^q), x^q \in Z_f(k) \quad (7)$$

The similarity $S(x^p)$ describes the distance between x^p and the other chromosomes in swarm $Z_f(k)$, which denotes the search degree of direction that x^p represented. In order to ensure the uniform distribution of search directions, the paper chooses $S(x^p)$ as the evolution probability of x^p .

3) *Elite Retaining Strategy*

In multi-objects optimization, the contradiction among scheduling objects leads to form numerous non-dominated chromosomes, which could guide search and be benefited to diversity strategy. Therefore, the paper introduces the memory to store the non-dominated chromosomes formed in the search process.

To promote the rate of convergence, memory needs to take part in the selection of chromosomes. Mix $P(T), C(T)$ and memory into mixture population, which would divide into non-dominated swarm $Z_f(k)$ based on NSGA-II method.

To avoid to early convergence, $P(T+1)$ needs to be selected respectively in all the swarms. The number of selection in $Z_f(k)$ depends on $[(n-k+1)/\sum_{k=1}^n k] \times |Z_f|$, where $[\]$ denotes the integer part, and the selection probability of chromosome x^p in swarm $Z_f(k)$ is $S(x^p)$.

After population selection, update the memory according to non-dominated swarm $Z_f(1)$.

H. *Terminal Condition Design*

The contradiction among scheduling objects makes the terminal condition design could only refer to the swarm search information obtained in search process, the paper chooses number of Iterations hits the set point as the terminal condition.

Comprehensive the design of algorithm execution step, the multi-objects scheduling optimization algorithm flow chart is shown in figure 1.

IV. CASE ANALYSIS

A. *Aircraft Maintenance Shop Scheduling Case Analysis*

To examine the applicability of multi-objects scheduling optimization algorithms, the paper takes aircraft maintenance shop flow for reference to create the formalization model, which is shown in figure 2, the process time of unit and operation is shown as table 1.

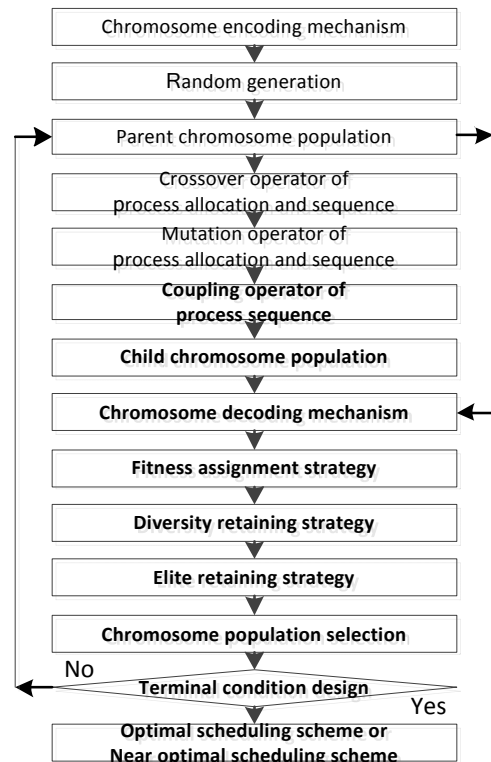


FIGURE I. ALGORITHM FLOWCHART OF MULTI-OBJECTS SCHEDULING OPTIMIZATION ALGORITHM

The paper designs program in java, and adopts the computer with Intel(R) Core(TM)i3-2310M 2.10GHz CPU and 2.00G memory to run the case.

Let the scale of population be $5 \times (n + m)$, crossover probability be 90%, mutation probability be 20%, iteration

point be 500, and f_1, f_2, f_3 are adapted as scheduling objects. The experimental result is shown in table 2, figure 3.

Compositing the experimental results, scheduling schemes obtained makes clear that multi-objects scheduling optimization algorithm the paper designed is suit to solve aircraft maintenance shop scheduling problem. Compositing the 3-D perspective of distribution of non-dominated chromosomes in state space, the optimal surface of coupling constraints is not present as a smooth surface, which indicates that coupling constraints increases the warp degree of feasible space. In the meantime, the strategy of generating only active schedules in chromosome decoding mechanism will promote efficiency, but will also lead to the situation that different chromosomes representative the same scheduling scheme.

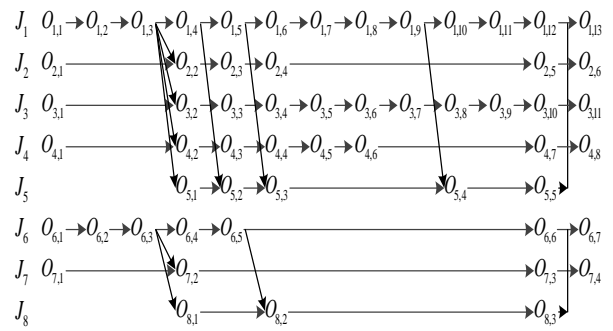


FIGURE II. TASK SET, OPERATION SET AND CHAIN CONSTRAINT, COUPLING CONSTRAINT

TABLE I. PROCESS TIME OF UNIT AND OPERATION

	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆	M ₇	M ₈	M ₉	M ₁₀	M ₁₁	M ₁₂	M ₁₃	M ₁₄	M ₁₅					
O _{1,1}	30	30	-	-	-	-	O _{2,1}	30	-	O _{3,1}	30	-	-	O _{4,1}	30	-	O _{5,1}	25	28	
O _{1,2}	23	29	14	30	20	22	O _{2,2}	40	39	O _{3,2}	14	20	13	O _{4,2}	14	13	O _{5,2}	32	27	
O _{1,3}	60	70	49	62	50	70	O _{2,3}	43	49	O _{3,3}	-	11	14	O _{4,3}	12	15	O _{5,3}	25	31	
O _{1,4}	35	46	37	25	-	28	O _{2,4}	16	19	O _{3,4}	20	15	18	O _{4,4}	36	27	O _{5,4}	24	21	
O _{1,5}	46	37	42	35	50	25	O _{2,5}	26	29	O _{3,5}	14	-	21	O _{4,5}	24	21	O _{5,5}	12	15	
O _{1,6}	55	62	42	-	57	56	O _{2,6}	60	-	O _{3,6}	12	14	11	O _{4,6}	15	20	O _{5,6}	24	25	
O _{1,7}	40	-	26	37	31	29	O _{2,7}	-	22	25	O _{3,7}	-	22	O _{4,7}	23	17	O _{5,7}	36	39	
O _{1,8}	-	35	17	42	32	41	O _{2,8}	12	11	-	O _{3,8}	12	14	15	O _{4,8}	60	-	O _{5,8}	12	14
O _{1,9}	30	21	34	26	-	35	O _{2,9}	-	-	-	O _{3,9}	12	14	15	-	-	-	-	-	
O _{1,10}	52	43	38	48	50	52	O _{2,10}	-	-	-	O _{3,10}	37	26	40	-	-	-	-	-	
O _{1,11}	93	-	85	80	94	83	O _{2,11}	-	-	-	O _{3,11}	60	-	-	-	-	-	-	-	
O _{1,12}	31	26	22	30	25	27	O _{2,12}	24	21	32	O _{3,12}	24	21	32	-	-	-	-	-	
O _{1,13}	60	60	-	-	-	-	O _{2,13}	14	13	15	O _{3,13}	14	13	15	-	-	-	-	-	
O _{6,1}	24	35	31	24	19	21	O _{7,1}	32	28	25	O _{8,1}	32	28	25	-	-	-	-	-	
O _{6,2}	14	13	16	10	12	11	O _{7,2}	35	21	26	O _{8,2}	35	21	26	-	-	-	-	-	
O _{6,3}	21	23	25	20	24	22	-	-	-	-	-	-	-	-	-	-	-	-	-	
O _{6,4}	14	21	15	23	13	17	-	-	-	-	-	-	-	-	-	-	-	-	-	
O _{6,5}	25	21	26	21	24	22	-	-	-	-	-	-	-	-	-	-	-	-	-	
O _{6,6}	31	25	36	32	27	25	-	-	-	-	-	-	-	-	-	-	-	-	-	
O _{6,7}	37	31	32	36	31	38	-	-	-	-	-	-	-	-	-	-	-	-	-	

TABLE II. SCHEDULING SCHEME PERFORMANCE OF COUPLING CONSTRAINT MODEL

f_1	f_2	f_3	f_1	f_2	f_3
449	198	1494	459	168	1491
480	134	1524	473	136	1511
480	135	1523	461	158	1499
466	133	1529	477	142	1499
480	133	1526	479	138	1496
466	134	1527	474	146	1497
466	156	1525	465	162	1498
466	135	1526	457	168	1486
466	135	1526	459	158	1498
449	198	1501	457	166	1491
475	162	1497	467	152	1501
$\Delta = 0.62748$					
$\Delta_v = 0.04387$					

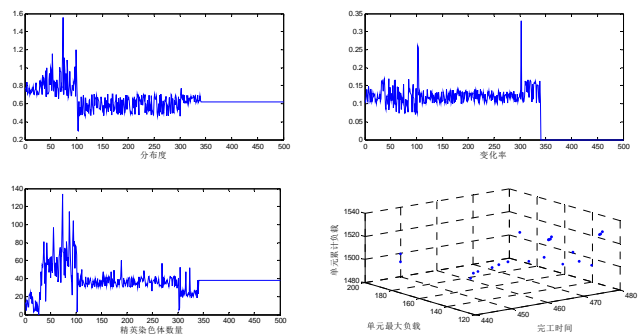


FIGURE III. TEST PROCESS OF COUPLING CONSTRAINT SCHEDULING

V. SUMMARY

To make the maintenance adapt for modern aircraft, the modeling and scheduling algorithm of aircraft maintenance shop scheduling problem are discussed. First of all, at the base of maintenance shop scheduling modeling referring to the formalization of flexible job shop scheduling problem, the connection and the distinction between chain constraint and

coupling constraint are discussed. In the next place, genetic algorithm is adopted as framework, where chromosome encoding and decoding mechanism, scheduling scheme initialization, crossover operator, mutation operator and selection operator are all designed, and the coupling operator is introduced to adapt the operation sequence part of chromosome to avoid the situation that the chromosome might violate the coupling constraint and could not decode. At the end, the maintenance shop scheduling case is established to examine the applicability and quality of multi-objects scheduling optimization algorithms.

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