The Simplified Quasi-Optimal Estimates of the Time and Power Parameters of a Low-Frequency Random Pulse with Arbitrary Modulating Function

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***Abstract*—We introduce a simpler approach for obtaining of usable processing algorithms of fast fluctuating Gaussian pulses with arbitrary modulation function in conditions of parametric prior uncertainty. We carry out the synthesis and analysis of the quasi-optimal measurer of a low-frequency random pulse signal with unknown appearance time, mathematical expectation and dispersion. We find the asymptotically exact expressions for the conditional biases and variances of the resulting estimates. By methods of statistical computer modeling the usefulness and efficiency of the considered technique is corroborated, the working capacity of offered measurer is established, and applicability borders of asymptotically exact formulas for its characteristics are also defined.**

Keywords-fast fluctuating random signal; maximum likelihood method; decision statistics; unknown parameters; Cramer-Rao bound; quasi-optimal estimation; local Markov approximation method; statistical modeling

I. INTRODUCTION

We are going to use a low-frequency random pulse signal with arbitrary modulating function to mean the multiplicative combination of the form of [1-3, et al.]

$$s(t) = \xi(t)\, f[\gamma(t-\lambda_0)]\, I\left(\frac{t-\lambda_0}{\tau}\right), \quad I(x) = \begin{cases} 1, & |x| \le 1/2, \\ 0, & |x| > 1/2. \end{cases} \qquad (1)$$

here λ_0 is the appearance time, τ is the duration of a signal, $f(t)$ is the modulating function (envelope), which is describing the pulse form and is normalized so that $\max f(t) = 1$, γ is a scale coefficient, and $\xi(t)$ is the realization of the stationary centered Gaussian random process with mathematical expectation (ME) $\langle \xi(t) \rangle = a_0$ and spectral density

$$G(\omega) = (2\pi D_0/\Omega)\, I(\omega/\Omega) \qquad (2)$$

In Eq. (2) the designations are the following: Ω is the bandwidth, and D_0 is the dispersion of the process $\xi(t)$.

We presuppose that fluctuations $\xi(t)$ are "fast", that is the pulse duration τ and the characteristic changing time Δt of the function $f(t)$ essentially exceed the correlation time of the process $\xi(t)$, so the following conditions are satisfied

$$\tau >> 4\pi/\Omega \quad (\mu = \tau\Omega/4\pi >> 1), \quad \Delta t >> 4\pi/\Omega \qquad (3)$$

In [1] the estimation problem of the appearance time $\lambda_0 \in [\Lambda_1, \Lambda_2]$ of a signal (1) observed against Gaussian white noise with one-sided spectral density N_0 is considered provided that all other pulse parameters are a priori known. However, in a number of practical tasks ME a_0 and dispersion D_0 of the process $\xi(t)$ can also be unknown. In this connection, it is of interest to find the structure and characteristics of the feasible measurer of time and power parameters of a signal (1).

II. THE ESTIMATION ALGORITHM SYNTHESIS

When synthesizing the estimation algorithm we use a maximum likelihood method [3-5]. Under this method, it is necessary to form the decision statistics represented by the logarithm of the functional of likelihood ratio (FLR) as the function $L(\lambda, a, D)$ of current values λ, a, D of the unknown parameters λ_0, a_0, D_0. If the conditions (3) are fulfilled, then according to [1, 2] we have

$$\begin{aligned} L(\lambda, a, D) = {} & \frac{q}{N_0} \int_{\lambda-\tau/2}^{\lambda+\tau/2} \frac{f^2[\gamma(t-\lambda)]\, y^2(t)}{1+qf^2[\gamma(t-\lambda)]} dt + \\ & + \frac{2a}{N_0} \int_{\lambda-\tau/2}^{\lambda+\tau/2} \frac{f[\gamma(t-\lambda)]\, x(t)}{1+qf^2[\gamma(t-\lambda)]} dt - \\ & - \frac{a^2}{N_0} \int_{-\tau/2}^{\tau/2} \frac{f^2(\gamma t)}{1+qf^2(\gamma t)} dt + \frac{\Omega}{4\pi} \int_{-\tau/2}^{\tau/2} \ln\left[1+qf^2(\gamma t)\right] dt, \end{aligned} \qquad (4)$$

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where $q = D/E_N$, $E_N = N_0\Omega/4\pi$ is the average power of noise $n(t)$ within bandwidth of the process $\xi(t)$, and $y(t) = \int_{-\infty}^{\infty} x(t')\,h(t-t')\,dt'$ is output signal (response) of the filter with transfer function $H(\omega)$, which satisfies to a condition $|H(\omega)|^2 = I(\omega/\Omega)$, on the observable data realization $x(t) = s(t) + n(t)$.

Then maximum likelihood estimates (MLEs) λ_m, a_m and D_m of the appearance time λ_0 , ME a_0 and dispersion D_0 of a random pulse (1) are determined as the position of the greatest maximum of decision statistics $L(\lambda, a, D)$:

$$\lambda_m = \underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup} L(\lambda, a_m, D_m),$$
$$(a_m, D_m) = \underset{a\in(-\infty,\infty), D\geq 0}{\arg\sup} L(\lambda_m, a, D). \quad (5)$$

It is easy to see that the measurer (5) has multichannel structure and the infinitely many channels are required for its exact implementation that it is hardly probably in practice. Thereupon, it might be useful to find single-channel quasi-optimal estimation algorithms of time and power parameters of a signal (1) close to the optimal algorithm (5) by their accuracy characteristics.

Similarly to [5] it can be shown that MLE $\lambda_m \to \lambda_0$ in mean square, if $\mu \to \infty$. Then, according to [6], the characteristics of MLEs a_m and D_m (5) coincide asymptotically (with increasing μ) with characteristics of the estimates

$$(a_{m0}, D_{m0}) = \underset{a\in(-\infty,\infty),\, D\geq 0}{\arg\sup} L(\lambda_0, a, D) \quad (6)$$

In general, the measurer (6) has also multichannel organization, while the minimum variances $V_{a_{\min}}$ and $V_{D_{\min}}$ of the estimates a_{m0} and D_{m0} (6) determined by the Cramer-Rao formula [4] are equal to

$$V_{a_{\min}} = -1\Big/\left\langle d^2 L(\lambda_0, a, D_0)\big/ da^2 \right\rangle\Big|_{a=a_0} = E_N / 2\mu G_{21} ,$$
$$V_{D_{\min}} = -1\Big/\left\langle d^2 L(\lambda_0, a_0, D)\big/ dD^2 \right\rangle\Big|_{D=D_0} =$$
$$= E_N^2 \Big/ \left[\mu G_{42} + \left(G_{42}^2 - 2G_{21}G_{63}\right)\big/ G_{21}^2 \right]. \quad (7)$$

here $G_{mn} = \int_{-1/2}^{1/2} f^m(\tilde{\gamma}\tilde{t})\,d\tilde{t} \Big/ \left[1 + q_0 f^2(\tilde{\gamma}\tilde{t})\right]^n$, $q_0 = D_0/E_N$, $\tilde{\gamma} = \tau\gamma$ and $\tilde{t} = t/\tau$ is the normalized time. As it is noted in [4], the variances of MLEs (6) coincide asymptotically (with increasing an output signal-to-noise ratio (SNR)) with Eqs. (7).

We introduce some simpler quasi-optimal estimates (QOEs) a_{q0} and D_{q0} of the pulse (1) ME a_0 and dispersion D_0 instead of MLEs a_{m0} and D_{m0} (6). In the synthesis of QOEs a_{q0} and D_{q0} we use the condition for closeness of their variances to the minimum variances (7) provided that QOEs can be technically implemented by single-channel units. Besides, in some limiting cases QOEs a_{q0} and D_{q0} should move to MLEs a_{m0} and D_{m0} (6). As a result, we come to the estimates in terms of

$$a_{q0} = L_2(\lambda_0)/\tau G_{10} ,$$
$$D_{q0} = \max\left[0, \left(L_1(\lambda_0)/\tau - E_N\right)/G_{20} - \left(L_2(\lambda_0)/\tau G_{10}\right)^2\right], \quad (8)$$

where $L_1(\lambda) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} y^2(t)\,dt$, $L_2(\lambda) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} x(t)\,dt$, and E_N , $y(t)$ are determined the same as in Eq. (4).

Carrying out in Eqs. (8) the averaging through all possible realizations $x(t)$ (with fixed values of all unknown parameters), for conditional biases $b(a_{q0}|a_0)$, $b(D_{q0}|D_0)$ and variances $V(a_{q0}|a_0)$, $V(D_{q0}|D_0)$ of QOEs a_{q0} , D_{q0} we get

$$b(a_{q0}|a_0) = 0 , \quad V(a_{q0}|a_0) = E_N(1 + q_0 G_{20})/2\mu G_{10}^2 ,$$
$$b(D_{q0}|D_0) = -E_N(1 + q_0 G_{20})/2\mu G_{10}^2 , \quad (9)$$
$$V(D_{q0}|D_0) = \frac{E_N^2}{\mu}\left\{ \frac{1}{G_{10}^2}\int_{-1/2}^{1/2}\left[1 + q_0 f^2(\tilde{\gamma}\tilde{t})\right]^2 d\tilde{t} - \right.$$
$$- \frac{1}{\mu G_{20}}\left[\frac{1}{G_{10}}\int_{-1/2}^{1/2}\left[1 + q_0 f^2(\tilde{\gamma}\tilde{t})\right]^2 d\tilde{t} - \frac{3(1 + q_0 G_{20})^2}{4G_{20}}\right] +$$
$$\left. + \eta^2\left[\frac{1}{G_{20}} - \frac{1}{G_{10}} + q_0\int_{-1/2}^{1/2} f^2(\tilde{\gamma}\tilde{t})\left(\frac{1}{\sqrt{G_{20}}} - \frac{f(\tilde{\gamma}\tilde{t})}{G_{10}}\right)^2 d\tilde{t}\right]\right\}.$$

here $\eta^2 = 2a_0^2/E_N$ and q_0 is determined the same as in Eq. (7).

As follows from Eqs. (7), (9), with fulfillment of conditions (3) the variances of the estimates a_{q0} , D_{q0} (8) differs from ultimate variances by no more than 5 % for a large class of modulating functions $f(t)$. If $f(t) \equiv 1$, then variances (7) and (9) coincide, i.e. QOEs (8) converges to MLEs (6) with approach of the form of the modulating function $f(t)$ to rectangular. It allows us to recommend the single-channel algorithm (8) instead of more complex multichannel (6) to measure the pulse signal (1) ME and dispersion in practical applications without significant loss in accuracy of a passed estimate.

If the parameter λ_0 is unknown, then from Eq. (8) we obtain the estimates of the kind of

$$a_q = L_2(\lambda_q)/\tau G_{10},$$
$$D_q = \max\left[0, \left(L_1(\lambda_q)/\tau - E_N\right)/G_{20} - \left(L_2(\lambda_q)/\tau G_{10}\right)^2\right], \quad (10)$$

where $\lambda_q = \underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup}\, L(\lambda, a_q, D_q)$ is the estimate of the appearance time of a pulse (1). Substituting Eq. (10) in Eq. (4) and, following [7], carrying out optimization of the estimation algorithm we move to the estimate of the appearance time

$$\lambda_q = \underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup}\, L_q(\lambda), \quad (11)$$
$$L_q(\lambda) = \mu\left\{L_1(\lambda)/\tau E_N - \ln\left[\left(L_1(\lambda) - f^2(\gamma\tau/2)L_2^2(\lambda)/\tau\right)/\tau E_N\right] - 1\right\}$$

instead of the estimate λ_m (5). We also name the estimates (10), (11) as QOEs. Indeed, if $f(t) \equiv 1$, then QOEs (10), (11) go over the corresponding MLEs of the ME, dispersion and appearance time of a low-frequency random pulse with rectangular modulating function [8].

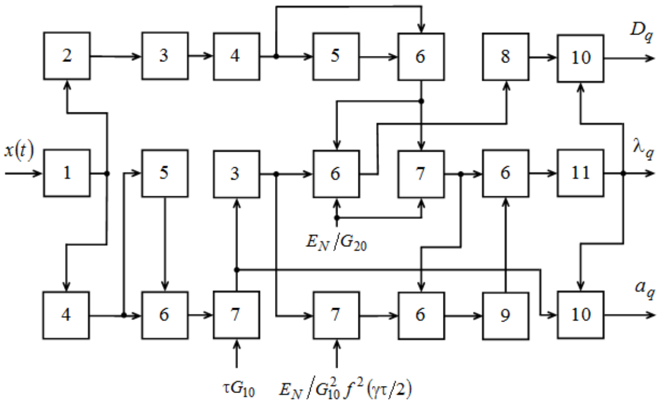

FIGURE I. THE QUASI-OPTIMAL MEASURER OF THE APPEARANCE TIME, MATHEMATICAL EXPECTATION AND DISPERSION OF A LOW-FREQUENCY RANDOM PULSE WITH ARBITRARY MODULATING FUNCTION

The measurer (10), (11) of the time and power parameters of a pulse signal (1) can be implemented on the basis of the block diagram presented in Figure 1. Here are the designations: 1 is the switch that is open for time $[\Lambda_1 - \tau/2, \Lambda_2 + \tau/2]$, 2 is a filter with transfer function $H(\omega)/\sqrt{\tau G_{20}}$ (4), 3 is the squarer, 4 is an integrator, 5 is a delay line for the period τ, 6 is the substracter, 7 is the divider, 8 is the nonlinear element with characteristic $\max(0, x)$, 9 is the logarithmic amplifier, 10 is the sampling device forming to its output the input signal sample at the instant time $t = \lambda_q + \tau/2$, 11 is the retriever of the location of the input signal greatest maximum (extremator).

III. THE ESTIMATION ALGORITHM ANALYSIS

Let us find the characteristics of the estimates (10), (11). During the analysis process, we find it expedient to divide all QOEs (11) of the pulse appearance time into the two classes: reliable and anomalous [4, 5]. The estimate λ_q is reliable, if it is within the interval limits $\Gamma_S \equiv [\lambda_0 - \tau, \lambda_0 + \tau]$. If QOE λ_q is out of an interval Γ_S, i.e. $\lambda_q \in \Gamma_N = [\Lambda_1, \lambda_0 - \tau) \cup (\lambda_0 + \tau, \Lambda_2]$, then the estimate and the corresponding estimate error are designated as anomalous [1, 2, 5]. It is necessary to consider the anomalous errors, if the length of the prior interval of the possible values of appearance time λ_0 is much greater than the length of the interval Γ_S of the reliable estimate, i.e. the following condition holds

$$m = (\Lambda_2 - \Lambda_1)/\tau >> 1 \quad (12)$$

According to [4, 5], while executing ratio (16), the conditional bias $b(\lambda_q|\lambda_0) = \langle \lambda_q - \lambda_0 \rangle$ and variance $V(\lambda_q|\lambda_0) = \langle (\lambda_q - \lambda_0)^2 \rangle$ of the estimate λ_q, with the allowance for the anomalous errors, can be written down as follows:

$$b(\lambda_q|\lambda_0) = P_0 b_0(\lambda_q|\lambda_0) + (1 - P_0)[(\Lambda_1 + \Lambda_2)/2 - \lambda_0],$$
$$V(\lambda_q|\lambda_0) = P_0 V_0(\lambda_q|\lambda_0) + \quad (13)$$
$$+ (1 - P_0)\left[\left(\Lambda_1^2 + \Lambda_1\Lambda_2 + \Lambda_2^2\right)/3 - \lambda_0(\Lambda_1 + \Lambda_2) + \lambda_0^2\right].$$

here $b_0(\lambda_q|\lambda_0)$, $V_0(\lambda_q|\lambda_0)$, $P_0 = P\left[\,|\lambda_q - \lambda_0| \le \tau\right]$ are, correspondingly, conditional bias, conditional variance and probability of a reliable estimate λ_q (11).

While determining $b_0(\lambda_q|\lambda_0)$, $V_0(\lambda_q|\lambda_0)$ and P_0, we will be limited to a condition of a high posterior accuracy, when the output power SNR of the algorithm (10), (11) is sufficiently great. We also consider that $f(\gamma t)$ is even function (for simplicity of overall expressions) and does not vanish in points $t = \pm\gamma\tau/2$ (the useful signal (1) is discontinuous [5]). Then, based on the results of works [2, 8, 9] we obtain

$$P_0 \approx \frac{2\psi z}{\sigma\sqrt{\mu}} \exp\left(\frac{\psi^2 z^2}{2} + \psi z^2\right) \int_H^\infty \exp\left[-m\,\phi\left(\sqrt{\frac{u}{2}}, \frac{1}{f(\tilde{\gamma}/2)}\right)\right] \times$$
$$\times \left\{ \exp\left(-\frac{\psi z u}{\sigma\sqrt{\mu}}\right) \Phi\left[\frac{u}{\sigma\sqrt{\mu}} - z(\psi + 1)\right] - \right. \quad (14)$$
$$\left. - \exp\left[\frac{3\psi^2 z^2}{2} + \psi z\left(z - \frac{2u}{\sigma\sqrt{\mu}}\right)\right] \Phi\left[\frac{u}{\sigma\sqrt{\mu}} - z(2\psi + 1)\right] \right\} du,$$

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$$b_0\left(\lambda_q|\lambda_0\right)=0, \quad (15)$$

$$V_0\left(\lambda_q|\lambda_0\right)=\frac{13\tau^2}{8}\frac{\left\{\left[1+q_0 f^2(\tilde{\gamma}/2)\left(1+\left(q_0+\eta^2\right)f^2(\tilde{\gamma}/2)\right)\right]\times}{\mu^2 f^4(\tilde{\gamma}/2)\times}\to$$

$$\to\frac{\times\left[q_0G_{20}+\left(\eta^2/2\right)\left(G_{20}-G_{10}^2 f^2(\tilde{\gamma}/2)\right)\right]^2+}{\times\left\{\eta^2G_{10}f(\tilde{\gamma}/2)+\left(q_0+\eta^2/2\right)\times\right.}\to$$

$$\to\frac{+\eta^2G_{10}f^3(\tilde{\gamma}/2)\left[G_{10}f(\tilde{\gamma}/2)+\left(1+q_0 f^2(\tilde{\gamma}/2)\right)\times\right.}{\times\left[q_0G_{20}+\left(\eta^2/2\right)\times\right.}\to$$

$$\to\frac{\left.\left.\times\left(G_{10}f(\tilde{\gamma}/2)+2q_0G_{20}+\eta^2\left(G_{20}-G_{10}^2 f^2(\tilde{\gamma}/2)\right)\right)\right]\right\}^2}{\left.\left.\times\left(G_{20}-G_{10}^2 f^2(\tilde{\gamma}/2)\right)\right]\right\}^4}.$$

here

$$H=\frac{2f(\tilde{\gamma}/2)}{\sqrt{1+f^2(\tilde{\gamma}/2)}},\quad z^2=\frac{\mu}{\sigma^2}\left[G_{20}\left(q_0+\eta^2/2\right)-\ln(1+Q)\right]^2,$$

$$\psi=\frac{2\sigma A_S\sqrt{\mu}}{z\left(\sigma_1^2+\sigma_2^2\right)},\quad A_S=\frac{f^2(\tilde{\gamma}/2)}{1+Q}\left[Q\left(q_0+\eta^2/2\right)+\eta^2G_{10}f(\tilde{\gamma}/2)\right],$$

$$\sigma^2=\left\{Q^2\left[1+q_0G_{20}+\left(G_{20}+q_0G_{40}\right)\left(q_0+\eta^2\right)\right]+2\eta^2QG_{10}\times\right.$$
$$\left.\times f^2(\tilde{\gamma}/2)\left(G_{10}+q_0G_{30}\right)+\eta^2G_{10}^2f^4(\tilde{\gamma}/2)\left(1+q_0G_{20}\right)\right\}/(1+Q)^2,$$

$$\sigma_1^2=\left[1+q_0f^2(\tilde{\gamma}/2)\right]\left\{Q^2\left[1+\left(q_0+\eta^2\right)f^2(\tilde{\gamma}/2)\right]+\right.$$
$$\left.+\eta^2f^3(\tilde{\gamma}/2)G_{10}\left[G_{10}f(\tilde{\gamma}/2)+2Q\right]\right\}/(1+Q)^2,$$

$$\sigma_2^2=\frac{Q^2+\eta^2G_{10}^2f^4(\tilde{\gamma}/2)}{(1+Q)^2},\quad Q=q_0G_{20}+\frac{\eta^2}{2}\left[G_{20}-G_{10}^2f^2(\tilde{\gamma}/2)\right],$$

$$\phi(h,\gamma)=\left\{4h^2\left[1+\gamma^2-\left(\gamma^2-1\right)I_1\left(h^2\left(\gamma^2-1\right)\right)/I_0\left(h^2\left(\gamma^2-1\right)\right)\right]-1\right\}\times$$
$$\times\gamma I_0\left[h^2\left(\gamma^2-1\right)\right]\exp\left[-h^2\left(\gamma^2+1\right)\right],$$

and $I_0(x)$, $I_1(x)$ are modified zero-order and first-order Bessel functions, respectively. The accuracy of the formulas (13), (14) increases with μ, z, m, as well as the accuracy of the formulas (15) increases with μ and z. If $f(t)\equiv 1$, then from Eqs. (14), (15) we get the expressions for probability and conditional bias and variance of the reliable MLE of the appearance time of a rectangular random pulse (1) with unknown ME and dispersion [8].

From [1] and Eq. (15) follows that for the large class of modulating functions $f(t)$ the variance of the reliable QOE (11) does not differ from the variance of MLE of the appearance time of a signal (1) with a priori known other parameters by more than 5-15 %, if output SNR is smaller than 2…3 (in times), and by more than 5 %, if output SNR is greater than 3. Thus, the offered measurer (11) ensures the estimation accuracy which is comparable with optimal measurer, without having the information about ME and dispersion of a pulse (1), as well as form of its modulating function $f(t)$.

Now we consider the characteristics of QOEs a_q and D_q (10). It follows from formulas (9) that when $\lambda_q=\lambda_0$ and the conditions (3) are fulfilled, the variances of the estimates (10) are not decreasing faster than μ^{-1}. At the same time, the variance (15) of the estimate λ_q (11) is proportional μ^{-2}. Therefore, similarly to [6] it can be shown that with $\mu>>1$, $z>>1$ (SNR $>>1$), when $P_0\approx 1$ and the estimate λ_q is reliable, the characteristics of the estimates a_q and D_q (10) coincide with the characteristics (9) found with the known value of the parameter λ_0. The accuracy of formulas (9) increases with μ and z.

Assuming in Eqs. (9) $f(t)\equiv 1$, we obtain the expressions for conditional biases and variances of MLEs of the ME and dispersion of a rectangular random pulse (1) with unknown appearance time [8].

IV. THE RESULTS OF STATISTICAL MODELING

For the purpose of an experimental functional test of the introduced quasi-optimal simultaneous estimation algorithm of the appearance time, ME and dispersion of a random pulse (1) and an establishment of the applicability borders of the asymptotically exact formulas (9), (13)-(15) for its characteristics, we applied the statistical computer modeling of the measurer (10), (11). The modeling technique of processing algorithms of random pulse signals is described in detail in [9].

Some results of statistical modeling for $\lambda_0=(\Lambda_2+\Lambda_1)/2$, $\Lambda_1=\tau/2$, $\Lambda_2=(m+1/2)\tau$, $f(t)=\exp\left(-\gamma^2t^2\right)$, $\gamma=1/\tau$ and $z_0^2=2a_0^2\tau/N_0=10$ are shown in Figures 2-4. Each experimental value in Figures 2-4 was obtained as a result of the processing of no less than 10^4 realizations of the observable data $x(t)$. So, with probability of 0.9 confidence intervals boundaries deviate from experimental values no more than for 10...15 %.

In Figure 2 solid lines represent dependences (13) of the normalized conditional variance $\tilde{V}_\lambda(q_0)=12V\left(\lambda_q|\lambda_0\right)/\left(\Lambda_2-\Lambda_1\right)^2$ of QOE λ_q (11) from parameter q_0 (7) taking into account anomalous errors, if $m=20$ (12). Here analogous dependences (15) of the normalized variance $\tilde{V}_{0\lambda}(q_0)=12V_0\left(\lambda_q|\lambda_0\right)/\left(\Lambda_2-\Lambda_1\right)^2$ of the reliable QOE λ_q (11) are also drawn by dashed lines. Curves 1 are calculated with $\mu=50$, 2 – 100, 3 – 200. The experimental values for $\mu=50$, 100 and 200 are denoted by rectangles, crosses, and rhombuses (for variance $\tilde{V}_\lambda$ of the estimate λ_q with the allowance for anomalous errors), respectively, as well as by circles, triangles and asterisks (for variance $\tilde{V}_{0\lambda}$ of the reliable estimate λ_q).

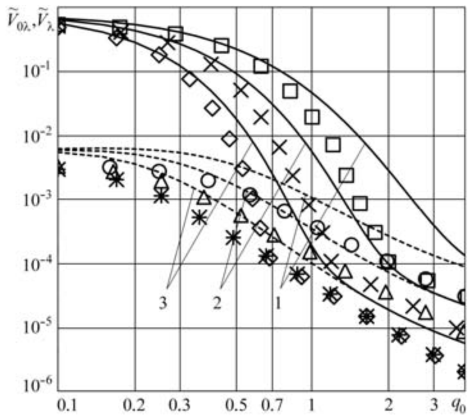

FIGURE II. THE NORMALIZED VARIANCE OF THE ESTIMATE OF THE PULSE APPEARANCE TIME.

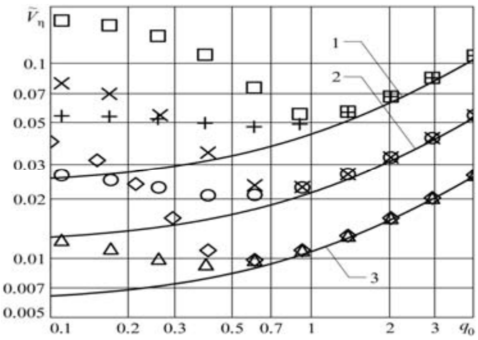

FIGURE III. THE NORMALIZED VARIANCE OF THE ESTIMATE OF THE PULSE MATHEMATICAL EXPECTATION.

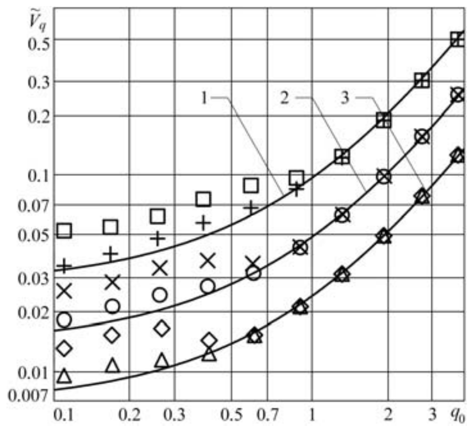

FIGURE IV. THE NORMALIZED VARIANCE OF THE ESTIMATE OF THE PULSE DISPERSION.

In Figures 3, 4 the theoretical dependences (9) of the normalized conditional variances $\widetilde{V}_{\eta}(q_0)=2V(a_q|a_0)/E_N$, $\widetilde{V}_q=V(D_q|D_0)/E_N^2$ of the estimates a_q, D_q (10) are traced by solid lines. Curves 1 correspond to $\mu=50$, 2 – 100, 3 – 200. Experimental values of variances $\widetilde{V}_{\eta}$ $\widetilde{V}_q$ in Figures 3, 4 are shown by squares, crosses, rhombuses, when $m=2$ and the estimate of the appearance time l_q is reliable, and by pluses, circles, triangles, when $m=20$ and anomalous errors are possible measuring the appearance time.

As is evident from Figures 2-4, the theoretical dependences (9), (13)-(15) for characteristics of the measurer (10), (11) of time and power parameters of a random pulse (1) approximate the experimental results in a satisfactory manner, at least, for $m \geq 20$, $\mu \geq 50$, $z \geq 1.5 \ldots 2$, $(f_{\max}-f_{\min})/\mu < 4\cdot 10^{-3}$. Here $f_{\min}=\min f(t)$ and $f_{\max}=1$ are minimum and maximum values of the function $f(t)$.

So, the obtained results make it possible to perform a valid choice between offered and other estimation algorithms of the appearance time, ME and dispersion of a low-frequency random pulse signal with arbitrary modulation function depending on the available prior information and the required accuracy and simplicity of hardware measurer implementation.

V. CONCLUSION

Optimal measurers of several unknown parameters of random pulse signals designed with help of the traditional (maximum likelihood, Bayesian) approaches have sufficiently complex multichannel structure. In order to obtain the effective single-channel estimation algorithms the technique based on closeness of accuracy of formed estimates of continuous signal parameters to the potential accuracy (Cramer-Rao bound) can be used. Application of the specified technique allows us to synthesize the quasi-optimal estimation algorithm of the appearance time of a Gaussian pulse which is invariant to the mathematical expectation and dispersion of a useful signal, as well as the form of its modulating function. At the same time, the characteristics of quality rating of the introduced algorithm are comparable to the corresponding characteristics of the technically more complex maximum likelihood estimation algorithm.

To implement the quasi-optimal measurer of the power parameters of a random pulse with unknown appearance time we need to know only the first and second moments of modulating function. Accuracy of the quasi-optimal estimates of mathematical expectation and dispersion has almost no differs from accuracy of the corresponding maximum likelihood estimates for a wide class of modulating functions. Conclusions and recommendations are valid, if the output signal-to-noise ratio is greater than 2…3.

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