

Maneuverable Warhead Tracking Based on CKF

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Abstract. The performance of contemporary weapon systems has been enhanced by the development of military high-tech, which makes the maneuverable warhead as a new challenge of ballistic midcourse tracking. Aiming to solve the maneuvering warhead tracking, nonlinear filter algorithm, which is specially designed to track target with time-changing state and measurement models, is proposed. Recently the cubature Kalman filter (CKF) has been proposed to estimate the nonlinear system, providing more accurate state estimation. In this paper, the midcourse maneuvering warhead movement and measurement models are established. Then the cubature Kalman filter is presented to track the maneuvering warhead. The simulation results demonstrate the improved performance of CKF over traditional Kalman filter, which shows the shorter execution time of the former algorithm than the latter and the faster rate of convergence.

Keywords: Maneuverable Warhead; Cubature Kalman Filter; Ballistic Tracking.

1. Introduction

Ballistic missile has been the world most violent and destructive weapons in modern battle field. Early detection and pre-warning can improve the possibility of precision interception, which has always aroused widely attention of major military powers. The flight of ballistic missile can be divided into three phases: the boost phase, the midcourse ballistic phase and the reentry phase. As the main interception stage, the midcourse ballistic phase shows certain characteristics like long-time interception and relatively stable ballistic trajectory. However, the presence of maneuverable warhead has become the highlight in the target tracking field. The movement of Ballistic target is obvious nonlinear because of the effects from gravity, which makes the target motion state and radar measurements are non-linear changes state. Several of maneuvering targets tracking algorithms are developed, such as extended Kalman filter (EKF), unscented Kalman filter (UKF), or particle filter (PF) [1-3]. The extended Kalman filter is developed to solve the nonlinear filtering problems, based on applying traditional Kalman filter to the linearized models which has been linearized by Taylor expansion [4]. However, it suffers from widespread drawback that may cause serious filter divergence. Then the unscented Kalman filter has been introduced as a modified alternative to EKF for nonlinear state estimation [3]. The UKF has prominent performance in dealing with nonlinear problems, whose updating operation is realized by designing a few sigma points and calculating the propagation of these sigma points via non-linear functions. It simplifies the process of calculation and lessens the filter divergence [3]. The idea of the particle filter is to compute the conditional probability distribution function consists of a finite set of random particles and corresponding weights, which solves the integrals appearing in the filtering problem by means of stochastic Monte Carlo integration [5]. However, it has the inherent drawbacks like samples less of diversity [6]. Lately, Arasaratnam and Haykin invented the cubature Kalman filter (CKF) [7, 8]. The CKF is to convert nonlinear filter into the calculation problem of the product between the nonlinear function and the Gaussian probability density function, based on a third degree spherical-radial cubature rule in the nonlinear Bayesian filter. The CKF performs better than the EKF and UKF [9].

The paper is organized as follows. The maneuvering midcourse ballistic warhead motion and measurement models are described first, and then cubature Kalman filter is used for ballistic missile tracking. The comparison of the CKF with the traditional tracking algorithms has been done by simulations and analysis in this paper shows that the new algorithm provides better estimation accuracy with minimal computational effort.

2. Mathematic Model of Multi-rotor

2.1 Movement Model

Sometime after the launch of ballistic missile warhead flight separation from projectile, the target enters the middle phase of flight. Because the long distance between the radar and the warheads, and the Earth is actually elliptical shape, added second-order harmonic coefficients of the cue ball factor in the acceleration of gravity model based on the standard ellipsoid Earth gravity model, can achieve precise description of warhead motion model [10].

In order to use ellipsoid model for the Earth's gravity midcourse warhead trajectory modeling, the Earth-centered coordinate system $O_E - X_F Y_F Z_F$ is established, which has its origin at the Earth center O_E , its axes Z_F , and fundamental plane $O_E X_F Z_F$ coincident with the equatorial plane. Its axe $O_E X_F$ and $O_E Y_F$, however, rotate with the Earth around the Earth's spin axis $O_E Z_F$ as $O_E X_F$ points to the prime meridian. L is the radar earth longitude, B is the radar earth latitude; if T is the set target warhead.

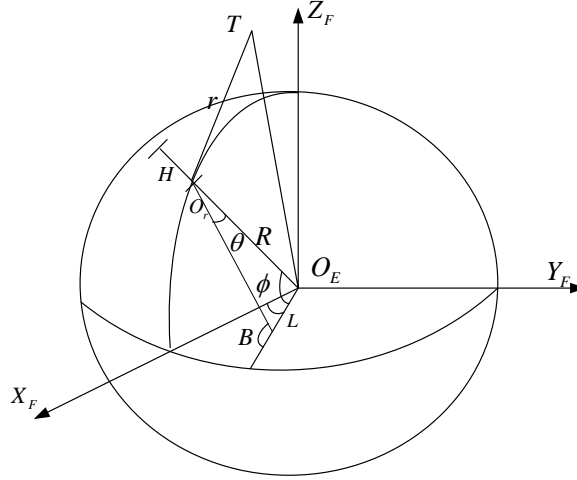


Fig 1. Ballistic Trajectory comparison

The forces act on the warhead T can be expressed as

$$a_E = a_G + a_T \quad (1)$$

Where a_G and a_T denote the accelerations induced by gravity and drag, respectively. Throughout this paper, let the target position and velocity vectors be $p = [x, y, z]^T$ $v = \dot{p} = [\dot{x}, \dot{y}, \dot{z}]^T$ respectively, with $x = [p', v']^T$, of the state-space models of a ballistic target has the form

$$\dot{x} = [v \ a]^T \quad (2)$$

Thus, the target acceleration model under EC coordinates can be expressed as [11] :

$$a_G = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -\frac{\mu}{p^3} \begin{bmatrix} \left(1 + (c_e / p^3) \left(1 - 5(z/p^3)^2\right)\right) x \\ \left(1 + (c_e / p^3) \left(3 - 5(z/p^3)^2\right)\right) y \\ \left(1 + (c_e / p^3) \left(1 - 5(z/p^3)^2\right)\right) z \end{bmatrix} \quad (3)$$

$$p = \sqrt{x^2 + y^2 + z^2} \quad (4)$$

$$c_e = 3J_2 R^2 / 2 \quad (5)$$

$\mu = 3.986005 \times 10^{14}$ is the Earth gravitational constant, $J_2 = 0.108267989 \times 10^{-2}$ is the shape of the Earth dynamics factor, $R = 6378137m$ is the Earth radius.

Maneuvering of midcourse ballistic missile is produced by the thrust made by the rocket rooster built-in within a short time, which changes the acceleration of the warhead and the trajectory. The acceleration of the thrust is decided by the thrust-vector engines T , the vertical direction u_r and the missile mass ratio m , as

$$a_T = Tu_r / m \quad (6)$$

And the missile mass ratio reduces as the fuel combustion:

$$T = (dm / dt) g I_{sp} \quad (7)$$

(dm/dt) Is the missile fuel consumption per second, $g = \mu / R^2$, I_{sp} is the specific impulse.

We add a small increment to the azimuth angle and elevation angle of the velocity to make the thrust vector, the azimuth angle and elevation angle are:

$$\alpha_v = \arctan(\dot{y} / \dot{x}) \quad (8)$$

$$\lambda_v = \arctan\left(\dot{z} / \left(\sqrt{\dot{x}^2 + \dot{y}^2}\right)\right) \quad (9)$$

If the small increment of the azimuth angle and elevation angle are $\Delta\alpha$ and $\Delta\lambda$, so the azimuth angle and elevation angle of the thrust vector are:

$$\alpha_T = \arctan(\dot{y} / \dot{x}) + \Delta\alpha \quad (10)$$

$$\lambda_T = \arctan\left(\dot{z} / \left(\sqrt{\dot{x}^2 + \dot{y}^2}\right)\right) + \Delta\lambda \quad (11)$$

So the thrust movement model can be expressed as:

$$u_T = \frac{dm}{dt} \frac{g I_{sp}}{m} \begin{bmatrix} \cos(\lambda_T) \cos(\arctan(\dot{y} / \dot{x}) + \Delta\alpha) \\ \cos(\lambda_T) \sin(\arctan(\dot{y} / \dot{x}) + \Delta\alpha) \\ \sin(\lambda_T) \end{bmatrix} \quad (12)$$

Furthermore, the midcourse maneuvering warhead movement model can be expressed as [12]:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{dm}{dt} \frac{g I_{sp}}{m} \begin{bmatrix} \cos(\lambda_T) \cos(\arctan(\dot{y} / \dot{x}) + \Delta\alpha) \\ \cos(\lambda_T) \sin(\arctan(\dot{y} / \dot{x}) + \Delta\alpha) \\ \sin(\lambda_T) \end{bmatrix} - \frac{\mu}{p^3} \begin{bmatrix} \left(1 + (c_e / p^3)\right) \left(1 - 5(z / p^3)^2\right) x \\ \left(1 + (c_e / p^3)\right) \left(3 - 5(z / p^3)^2\right) y \\ \left(1 + (c_e / p^3)\right) \left(1 - 5(z / p^3)^2\right) z \end{bmatrix} \quad (13)$$

2.2 Measurement Model

In the formula (13) is provided on the basis of the operating state variables middle warhead $X = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$, the missile in the ENU coordinate system state equation can be expressed as [13]:

$$\frac{dX}{dt} = [\dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}]^T \quad (14)$$

Discretization of the formula to obtain:

$$X_{k+1|k} = X_k + f(X_{k|k}) \Delta t + F(X_{k|k}) f(X_{k|k}) \Delta t^2 / 2 \quad (15)$$

Where Δt the sampling interval determined by the tracking data rate, $F(X_{k|k})$ is the Jacobi matrix as $f(X_{k|k}, k)$ relative to $X_{k|k}$:

$$F(X_{k|k}) = \left. \frac{df}{dX} \right|_{X=X_{k|k}} = \begin{bmatrix} O_{3 \times 3} & I_{3 \times 3} \\ \frac{\partial \ddot{r}}{\partial r} & -2\omega\phi \end{bmatrix}_{X_{k|k}} \quad (16)$$

The nonlinear measurement equation form[13] is then

$$Z(k) = h(X_k) + \omega(k) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2 + z_k^2} \\ \tan^{-1}(y_k / x_k) \\ \tan^{-1}(z_k / (\sqrt{x_k^2 + y_k^2})) \end{bmatrix} + \begin{bmatrix} n_k^R \\ n_k^\phi \\ n_k^\theta \end{bmatrix} \quad (17)$$

Where $\omega(k) = [n_k^R, n_k^\phi, n_k^\theta]^T$ is the measurement noise vector, n_k^R , n_k^ϕ and n_k^θ are independent process and measurement Gaussian noise with zero means and variances σ_R^2 , σ_ϕ^2 , σ_θ^2 . Therefore, the measurement noise covariance matrix can be expressed as:

$$R_k = \begin{bmatrix} \sigma_R^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_\theta^2 \end{bmatrix} \quad (18)$$

The Jacobi array $H(X_{k|k})$ can be expressed as:

$$H(X_{k|k}) = \frac{dh}{dX} \Big|_{X=X_{k|k}} = \begin{bmatrix} \frac{x}{r} & \frac{z}{r} & 0 & 0 \\ \frac{y}{x^2+y^2} & 0 & 0 & 0 \\ -\frac{xz}{r^2\sqrt{x^2+y^2}} & \frac{\sqrt{x^2+y^2}}{r^2} & 0 & 0 \end{bmatrix} \Big|_{X=X_{k|k}} \quad (19)$$

The nonlinear function $f(\cdot)$, $h(\cdot)$ and the expression of their Jacobian matrix F and H have been determined; can use filtering technology to CKF mixed terminal ballistic target coordinates to track.

3. A Target tracking based on CKF algorithm

3.1 Spherical-radial rule

Middle warhead tracking is a typical nonlinear filtering problem in the process of target tracking. The gut of the CKF is to find a set of points and weights to compute the mean and covariance of the first two moments of the state in numeral. The CKF find the points and weights based on the third-degree spherical-radical rule.

The target motion equation of state can be expressed as:

$$X_{k+1} = f_k(X_k) + v_k \quad (20)$$

$$Z_{k+1} = h_{k+1}(X_{k+1}) + w_{k+1} \quad (21)$$

Where X_k is the state of the dynamic system, $f_k(\cdot)$, $h_{k+1}(\cdot)$ are some known functions; v_k and w_{k+1} are independent process and measurement Gaussian noise with zeros means and covariances Q_k and R_k , respectively.

Assume at time k that the3posterior density of the state vector $p(X_k|Z^k)$ is known, the Bayesian filter in accordance with the following two steps:

(1)Time update

$$p(X_k|Z^k) = \int p(X_{k+1}|X_k)p(X_k|Z^k)dX_k \quad (22)$$

(2)Measurement update

$$p(X_{k+1}|Z^{k+1}) = p(Z_{k+1}|X_{k+1})p(X_{k+1}|Z^k)/(p(Z_{k+1}|Z^k)) \quad (23)$$

Among them $p(Z_{k+1}|Z^k)$ normalizing factor as:

$$p(Z_{k+1}|Z^k) = \int p(Z_{k+1}|X_{k+1})p(X_{k+1}|Z^k)dX_{k+1} \quad (24)$$

3.2 Cubature Kalman Filter

According to the Bayesian filter theory, the integrands of nonlinear filter are all of the form nonlinear plus Gaussian density. Aim to any arbitrary function $f(x)$; consider a multi-dimensional weighted integral of form,

$$I(f) = \int_{R^{n_x}} f(x)N(x; \mu, P)dx \approx \sum_{i=1}^{2n_x} \omega_i f(\mu + S\chi_i) \quad (25)$$

R^{n_x} is the region of the integration, N is the Gaussian distribution, n_x is the state vector dimension. χ_i is the volume of points, $\omega_i = 1/2n_x$ is the volume point weights and $P = SS^T$ is covariance matrix for the Cholesky decomposition factors, namely, the volume of which point to set:

$$\sqrt{n} \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -1 \end{bmatrix} \right\} \quad (26)$$

CKF can be summarized as follows:

Step1: CKF time update, factorize the covariance $P_{k|k}$ and evaluate the cubature points:

$$\begin{cases} P_{k|k} = S_k S_k^T \\ \xi_{l,k+1} = S_k \cdot \chi_l + \hat{X}_k, l=1, \dots, 2n_x \end{cases} \quad (27)$$

Evaluate the propagated cubature points according to the state transition function $f_{k+1}(\cdot)$

$$\xi_{l,k+1|k} = f(\xi_{l,k+1}) = f(S_k \cdot \chi_l + \hat{X}_k), l=1, \dots, 2n_x \quad (28)$$

Estimate the predicted state and the predicted error covariance

$$\hat{X}_{k+1|k} = \sum_{i=1}^{2n_x} \omega_i \xi_{i,k+1|k} \quad (29)$$

$$P_{k+1|k} = Q_{k+1} + \sum_{i=1}^{2n_x} \omega_i \xi_{i,k+1|k} \xi_{i,k+1|k}^T - \hat{X}_{k+1|k} \hat{X}_{k+1|k}^T \quad (30)$$

Step 2: CKF measurement update, factorize the covariance $P_{k|k}$ and evaluate the cubature points:

$$\begin{cases} P_{k+1|k} = S_{k+1|k} S_{k+1|k}^T \\ \xi_{i,k+1|k} = S_{k+1|k} \cdot \chi_i + \hat{X}_{k+1|k}, i=1, \dots, 2n_x \end{cases} \quad (31)$$

Evaluate the propagated cubature points according to the state transition function $h_{k+1}(\cdot)$

$$\psi_{i,k+1|k} = h(\xi_{i,k+1|k}) = h(S_{k+1|k} \cdot \chi_i + \hat{X}_{k+1|k}), i=1, \dots, 2n_x \quad (32)$$

Estimate the predicted measurement and the innovation error covariance matrix

$$\hat{Z}_{k+1|k} = \sum_{i=1}^{2n_x} \omega_i \psi_{i,k+1|k} \quad (33)$$

$$P_{Z,Z} = R_{k+1} + \sum_{i=1}^{2n_x} \omega_i (\psi_{i,k+1|k} - \hat{Z}_{k+1|k})(\psi_{i,k+1|k} - \hat{Z}_{k+1|k})^T \quad (34)$$

Estimate the CKF gain K_{k+1}

$$K_{k+1} = P_{X,Z} P_{Z,Z}^{-1} \quad (35)$$

Estimate cross- covariance matrix

$$P_{X,Z} = \sum_{i=1}^{2n_x} \omega_i (\xi_{i,k+1|k} - \hat{X}_{k+1|k})(\psi_{i,k+1|k} - \hat{Z}_{k+1|k})^T \quad (36)$$

Estimate the update state and the corresponding error covariance

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1} (Z_{k+1} - \hat{Z}_{k+1|k}) \quad (37)$$

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1} P_{Z,Z} K_{k+1}^T \quad (38)$$

4. Simulation and Analysis

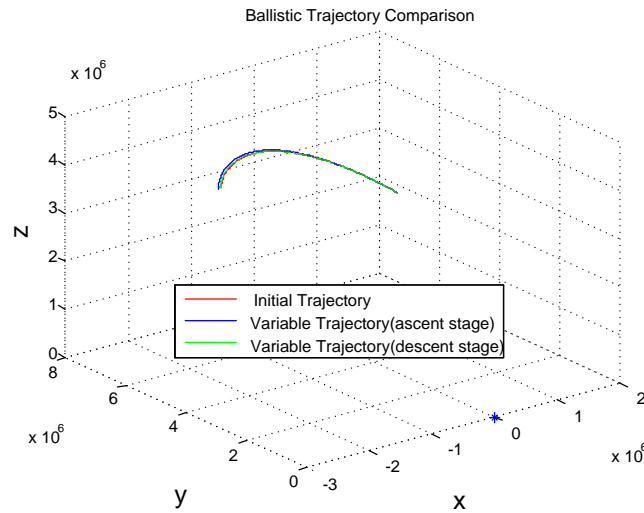


Fig 2. Ballistic Trajectory comparison

The trajectory for a maneuvering midcourse warhead is shown in Fig.2. The position, velocity and ballistic coefficient of a typical target trajectory are shown. The parameter of this typical target are: initial longitude 130.4, initial altitude 43.5, initial height 166 km, initial velocity 6.8km/s, slant angle 43.3, azimuth angle 248.7, the missile mass ratio 2300kg. the missile fuel consumption per second 50kg/s, the specific impulse 50s, length of maneuvering time 10s radar's data rate is 10Hz, $\Delta\alpha = 0.02\text{rad}$, $\Delta\lambda = 0.5\text{rad}$, 200-time Monte Carlo approach is adopted for simulations. The simulation of the maneuvering midcourse warhead model is shown in Fig 3, we can tell that the

influence in the ascent state of the missile is larger than that in the descent state. The maneuvering of the missile warhead influent the land position and the midcourse flight time will increase as well. The velocity of both the maneuvering missile warhead in the ascent state and the descent state enhanced when the missile midcourse flight ends, which may also let other problems like tracking loss or estimate errors in the boost phase tracking.

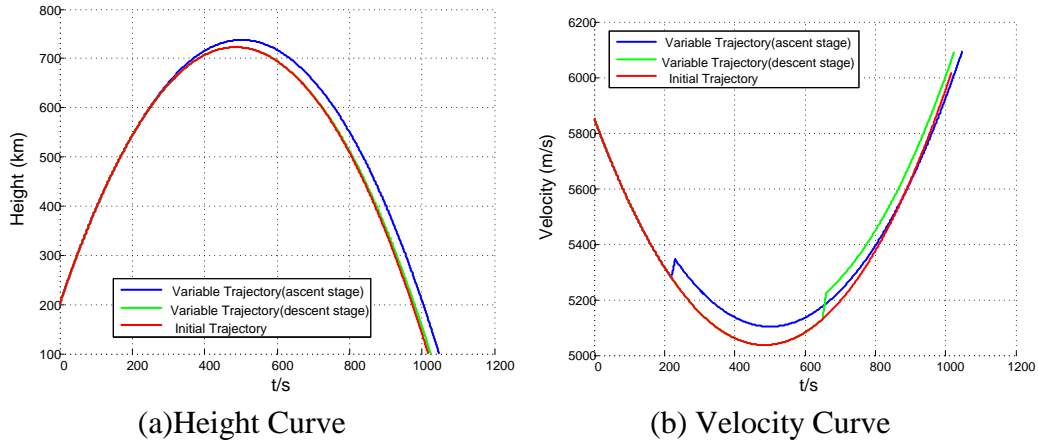


Fig 3. Ballistic Trajectory characters comparison

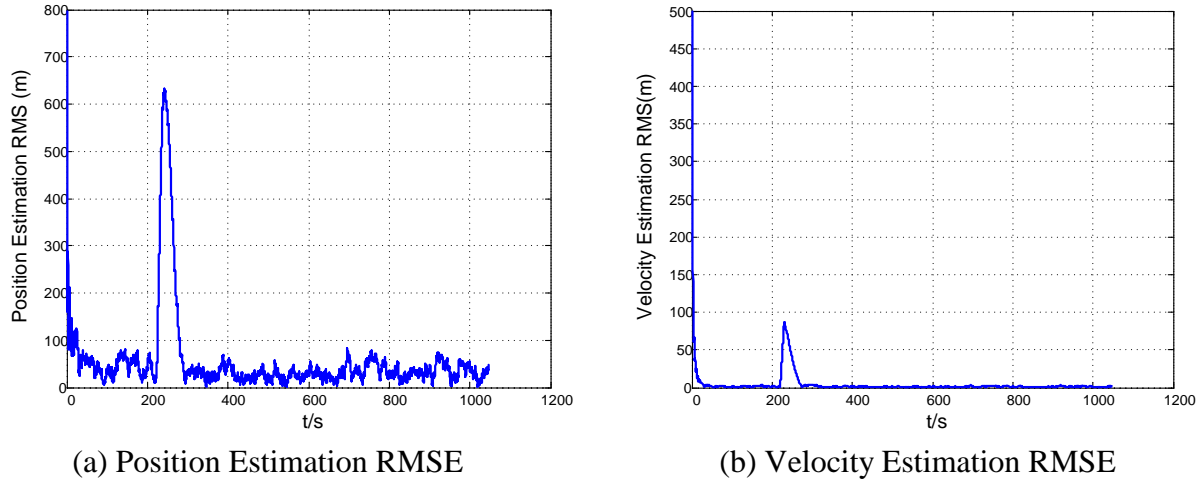


Fig 4.Characters of the maneuvering model in ascent stage

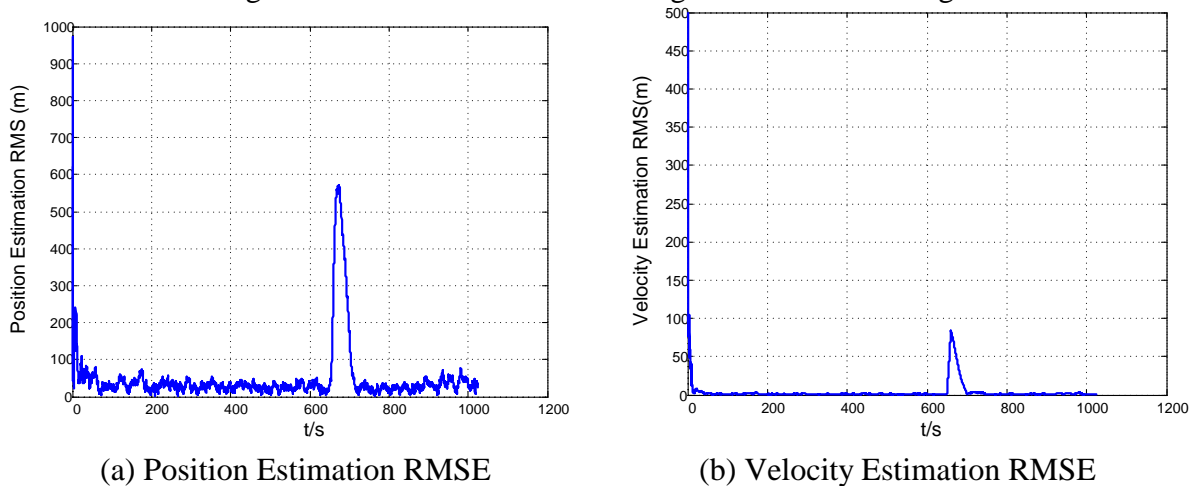
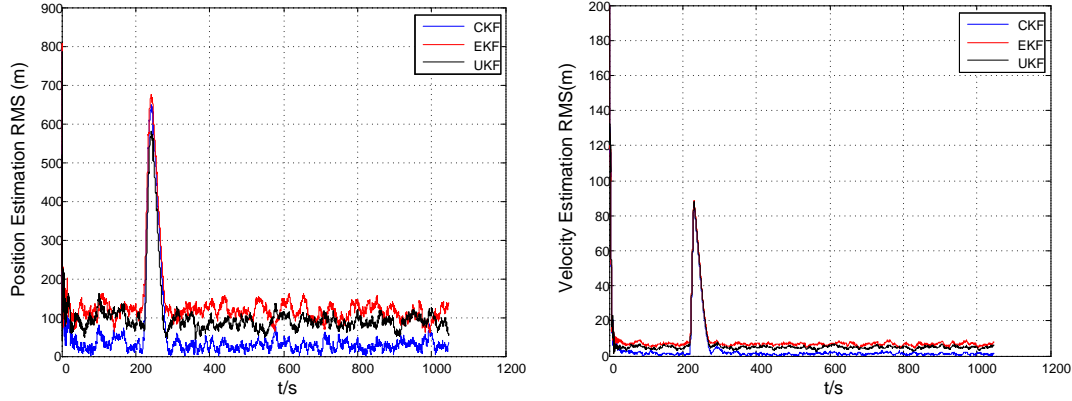


Fig 5.Characters of the maneuvering model in descent stage

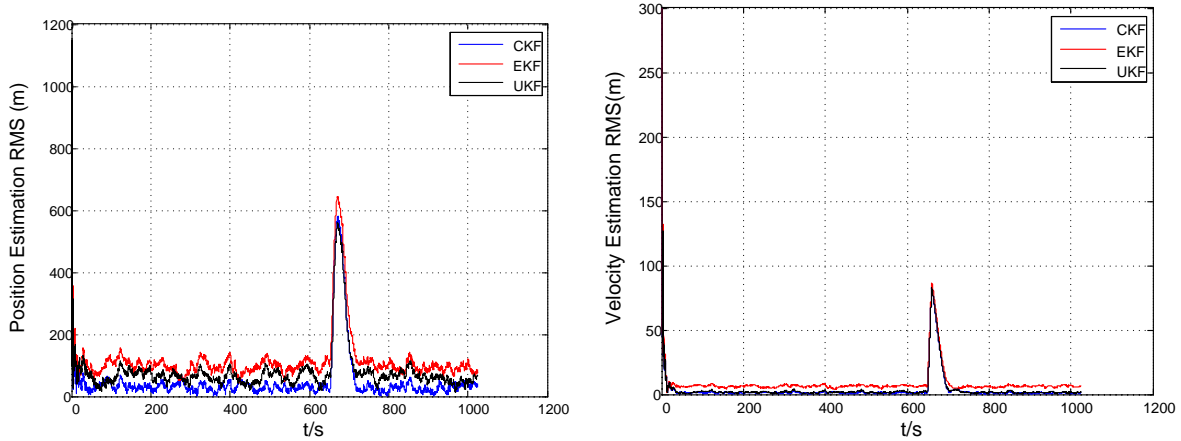
In order to test the performance of the CKF, the EKF and UKF are used to compare with it. First the midcourse maneuvering model of ballistic missile is established. There are two types of midcourse maneuvering model, one's trajectory changes in the ascent stage and the other one change in the descent stage. The two kinds of midcourse maneuvering model are shown in Fig.3, which are calculated by the Runge-Kutta and compared with the non-maneuver cause. It can be seen that the performance of the CKF filter was demonstrated based on the Root Mean Square Error (RMSE) in

target position and velocity. The position error curves were estimated by averaging 200 Monte-Carlo runs in Fig 4 and Fig 5. Fig 4 indicates that when the acceleration change of the ballistic missile. In Fig.6 and Fig.7, we have plotted the filtered estimates produced by the three filters, which indicates that when the acceleration change of the ballistic missile is caused by the thrust from the maneuvering warhead, CKF outperforms EKF and UKF. It can be seen that the estimates produced by CKF and UKF are close to each other and much closer to the true trajectory than that of EKF.



(a) Position Estimation RMSE (b) Velocity Estimation RMSE

Fig 6. RMSEs of CKF, EKF and UKF in ascent stage



(a) Position Estimation RMSE (b) Velocity Estimation RMSE

Fig.7 RMSEs of CKF, EKF and UKF in descent stage

The comparison is incomplete without mentioning the execution time of the three filters. The average central processing unit time is used to show as the execution time, which is displayed in table .1: The simulations show that CKF is more time-consuming than UKF. Considering 200-time Monte Carlo simulations, the time consumed by CKF is about 2.01 times as that by UKF. If the measurement equation is strong-nonlinear, this number will increase.

Table. 1 Average execution time of CKF, EKF, UKF

Filters	CKF	EKF	UKF
Average time(s)	0.974	0.452	1.365

5. Conclusion

With the development of the ballistic missile defense penetration technology, the midcourse ballistic missile can make maneuverable trajectory-change and there are abrupt changes in acceleration of target. In this paper, maneuvering midcourse ballistic warhead motion and measurement models are studied, and the characteristics of cubature Kalman filter are analyzed respectively. The simulation results show that provides better estimation accuracy with minimal computational effort. Three filters such as EKF, UKF and CKF are compared for their performance and computational complexity. Simulation results show that the CKF performs the best in reducing the estimate errors, while the EKF is efficient approach for maneuvering target tracking with the

worst performance. If the target is maneuverable and its acceleration has abrupt changes, CKF can lower the filtering error evidently to achieve an ideal performance with good precision and in less time.

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