

Multi-criteria Optimization of Electromechanical Modules:

Part 1- PROMETHEE method

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Abstract — The paper presents a study conducted to make multi-criteria optimization of electromechanical modules with a goal of providing easier tools for decision-making involving complex decision parameters. The work is motivated upon understanding that the task of both designing electromechanical modules and selecting an appropriate electromechanical drive for concrete application are multivariate. This leads to the need for performing an optimization of the selected electromechanical modules. The work reported in the paper focuses on use of one of the methods in the area, i.e. PROMETHEE method, which has shown good results to find compromise solution of decision-making task under conflicting parameters.

Keywords — optimization; multi-criteria optimization; PROMETHEE; electromechanical modules

I. INTRODUCTION

The electromechanical modules (EMM) represent the constructive unification of the electrical and the mechanical part. In most cases, they are a combination of electric motor and gear reducer, the so-called geared-motor. Figure 1 shows the general structural scheme of an electromechanical module [1].

As illustrated in the figure, the main structural components of an EMM are electric motor, clutch and a gear reducer, though the clutch is not an exclusively necessary component. This is because most companies, that produce EMM, use different types of adaptors as a connecting element between the motor and the gear reducer.

There are many types of electric motors (such as AC or DC, synchronous or asynchronous, servomotors, etc.) and gear units (such as coaxial, with parallel shafts, bevel, worm, etc.) available. In theory, geared-motors can be realized as combination of all available types of structural components. However, in order to achieve an efficient drive, there are several requirements that need to be taken into consideration, which limits the possible combinations of EMM [2], [3]. Still,

depending on the selected type of electric motor, clutch (if needed) and gear reducer, at constant values for the input data (rotational speed of the output shaft in $[\text{min}^{-1}]$ and the torque on the output shaft in $[\text{Nm}]$), a significant number of combinations will be achieved. These possible alternatives need to be analysed by chosen criteria. In order to select the most appropriate variant for a given application, an optimal solution by given target function needs to be found, i.e. an optimization of the alternatives need to be carried out.

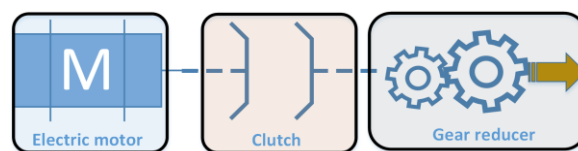


Fig. 1. Structural scheme of EMM

In many real-world decision-making problems, performing optimization is necessary to achieve several objectives including:

- minimizing the cost,
- maximizing the reliability,
- minimizing the risks, etc. [4].

Single-criteria optimization is a useful method for providing an insight to the nature of the problem, but usually cannot provide a number of alternative solutions. Multi-criteria optimization, on the other hand, can be used to identify not only an optimal solution from a selected number of alternatives, but also to set a ranking to these alternatives, so that the decision maker has a better understanding of the problem. Dependence (1) shows a general view of a multi-objective optimization task.

$$\text{Extr}\{k_1(a), k_2(a), \dots, k_h(a), \dots, k_n(a) | a \in K\} \quad (1)$$

In the design of electromechanical systems, optimization is seen as an essential step to achieve a functioning system.

Among others, optimization involves decision-making, in most cases, with conflicting goals. Depending on the complexity of the system and the involved parameters, optimization can be either single objective or multi-objective, also referred to as multi-criteria optimization. For instance, Particle Swarm Optimization (PSO) has been implemented to optimize a coupled electromechanical system (a hybrid electric vehicle) consisting of a combustion engine and electromotor [5]. The optimization is particularly applied on the power train system with a series configuration and the PSO is used to improve the fuel economy. Intended to forecast the state of an electromechanical equipment, a support vector regression based optimization in genetic algorithms has been reported [6]. The proposed optimization technique was implemented for gas turbines and industrial smokes and claimed that it provided satisfactory prediction capability.

Optimization process in general and multi-criteria optimization in particular involves decision-making with several influencing factors or parameters in order to find the best compromise alternative(s). Today, there exist a number of multi-criteria decision aid (MCDA) tools or methods and this paper presents the principle of using PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) method [7, 8] for carrying out a multi-criteria optimization of existing EMM, produced by the German company KEB Antriebstechnik GmbH.

The paper first provides the description of the problem in Section II. Then the approaches used to solve the problem are discussed in Section III. The main part of the article is presented in Section IV where the solution method is demonstrated using a case (an example). Finally, concluding remarks are given in Section V.

II. DESCRIPTION OF THE PROBLEM

The three main structural components of an EMM can be divided into the following groups:

- Group M – electric motors - which includes basic technical characteristics of the motors;
- Group C – clutches (if needed) – each record corresponds to certain type of a clutch;
- Group R – gear reducers – in which the most common gear reducer types and their basic characteristics are given.

In order to facilitate the optimization or the decision-making task, building of a database is needed using the division of the components in the groups. Figure 2 shows the general view of the database structure. As illustrated, each component group corresponds to a separate table in the database. By the means of a search form, the user can find the available information in the database combinations that correspond to the entered values in the search form for the input data (n_{out} and M_{out}).

A basic characteristic of a multi-criteria optimization task is that it is *implicit*, i.e. it does not have just one solution. When solving such optimization task, the first thing to do is to

define the target functions, the requirements and the limitations.

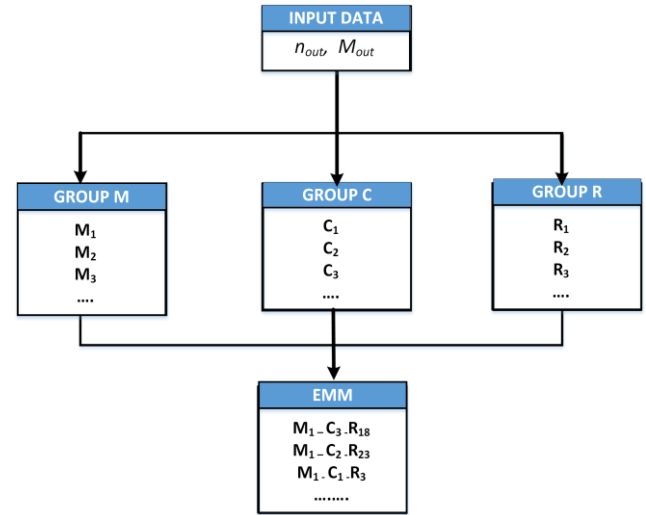


Fig. 2. General view of the database structure

When it comes to conducting optimization of EMM, the optimization criteria can be divided into two groups: (1) static and (2) dynamic. The following are examples for static criteria:

- V_{Σ} - total volume of the EMM, including the volume of the electric motor, of the clutch and of the gear reducer:
- $V_{\Sigma} = V_{mot} + V_{gear} + V_{clutch}$, [mm³];
- $L \times B \times H$ - overall dimensions of the EMM, [mm³];
- η_{total} - total efficiency of the EMM, including the efficiency of the electric motor, of the clutch and of the gear reducer: $\eta_{total} = \eta_{mot} * \eta_{gear} * \eta_{clutch}$, [-];
- m_{total} - total weight of the EMM, [kg];
- w - comparative value assessment, [-];
- a_w - center distance of the gear reducer, [mm], etc.

In addition to the above-mentioned, other criteria, depending on the problem type can also be introduced as optimization parameters.

As a MCDA method, PROMETHEE has been actively used in several research works and it represents family of outranking methods with allocated priorities. Closer review of the literature shows that the basic elements and structure of the PROMETHEE method have been introduced in 1982 by Brans [9]. Since then, diverse versions of the method with different functionalities and applications have been reported in several publications [10-12]. Further details of the methodology, its applications with historical backgrounds and contributions in the decision making research is presented in a comprehensive review conducted by Behzadian, et al. [13]. Review of the literature also shows that the majority of research works refer to PROMETHEE II, which is considered as the basics of application of other methods. As a multi-criteria outranking optimization method, it is also widely used in various industrial fields.

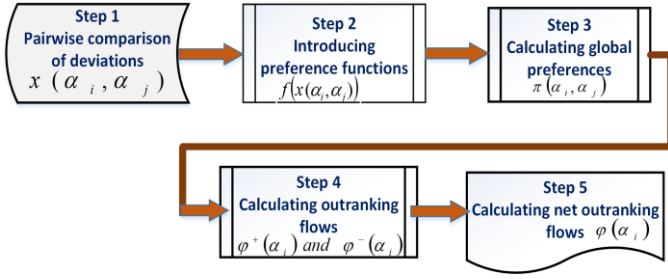


Fig. 3. Stepwise procedures of PROMETHEE method

Furthermore, PROMETHEE II is referred to as method that is found appropriate and successful for MCDA because of the stepwise mathematical procedure and its user friendliness [14]. The methodology ranks feasible alternatives from the best to the worst by pairwise comparison of the available alternatives and evaluating against certain criteria. In so doing, the method uses weights and preference functions that allow proper decision-making particularly in case of too large optimization criteria [15]. The overall steps involved in implementation of PROMETHEE II is depicted in figure 3.

The multi-criteria optimization problem, given in Eq. (1) is considered. In this case, $A = \{a_1, a_2, \dots, a_n\}$ is a set of n alternatives and $k = \{k_1, k_2, \dots, k_s\}$ is a constant set of s criteria.

As illustrated in Fig. 3, the first step in solving the problem, using PROMETHEE method, is to define the differences between the alternatives in pairs (i.e. pairwise comparison), Eq. (2):

$$x = k_s(a_i) - k_s(a_j) \quad a_i, a_j \in A, i \neq j, i, j \in \{1, 2, \dots, m\} \quad (2)$$

At step 2 of the procedure, these differences are valued through especially introduced preference functions. These functions are selected from six basic functions proposed for this purpose [10]:

1. Usual type criterion:

$$P_s^I(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x \geq 0 \end{cases} \quad (3)$$

2. U-shape criterion (Quazi type):

$$P_s^{II}(x) = \begin{cases} 0, & x \leq q \\ 1, & x \geq q \end{cases} \quad (4)$$

3. V-shape criterion with linear preference:

$$P_s^{III}(x) = \begin{cases} x/p, & x \leq p \\ 1, & x \geq p \end{cases} \quad (5)$$

4. Level type criterion:

$$P_s^{IV}(x) = \begin{cases} 0, & x \leq q \\ 1/2, & q \leq x \leq q + p \\ 1, & x > q + p \end{cases} \quad (6)$$

5. V-shape criterion with linear preference and indifference area:

$$P_s^V(x) = \begin{cases} 0, & x \leq q \\ (x - q)/p, & q \leq x \leq q + p \\ 1, & x \geq q + p \end{cases} \quad (7)$$

6. Gaussian type criterion:

$$P_s^{VI}(x) = \begin{cases} 0, & x \leq 0 \\ -x^2 / 2\sigma^2, & x \geq 0 \end{cases} \quad (8)$$

In the above equations, p stands for preference threshold and q is the indifference threshold.

The values for p and q have to be selected by the decision maker, as q represents the largest deviation that is considered as negligible in the comparison of two alternatives and p corresponds (for a given criteria) to the smallest definition that the decision maker considers as definitely important while comparing two alternatives [16].

After a preference function is associated with every criteria, the preference function $f_s(\alpha_i, \alpha_j)$ is defined for each $s = 1, 2, \dots, n$, where n a valued relation between all alternatives can be done. Then, the multi-criteria preference degree of one alternative over the other is calculated in step 3 according to the following dependence function.

$$\pi(a_i, a_j) = \frac{1}{n} \sum_{s=1}^n P_s^{\dots}(a_i, a_j) \quad (9)$$

In order to rank all of the alternatives, an **outgoing flow** ϕ^+ and an **incoming flow** ϕ^- need to be defined using the following relations:

$$\phi^+(a_i) = \sum_{j=1}^m \pi(a_i, a_j) \quad (10)$$

$$\phi^-(a_i) = \sum_{j=1}^m \pi(a_j, a_i) \quad (11)$$

The larger $\phi^+(\alpha_i)$, the more the alternative dominates the other alternatives of k . The smaller $\phi^-(\alpha_i)$, the less the alternative is dominated. Complete ranking without incomparability can be achieved by considering the **net flow** for each alternative in the last step:

$$\varphi(a_i) = \varphi^+(a_i) - \varphi^-(a_i) \quad (12)$$

If $\varphi(a_i) > \varphi(a_j)$, then α_i outranks α_j . If $\varphi(a_i) = \varphi(a_j)$, then α_i is indifferent to α_j .

III. DEMONSTRATIVE EXAMPLE

A multi-criteria optimization of EMM is conducted using method PROMETHEE and at the following input data: $n_{out} = 12 \text{ min}^{-1}$ and $M_{out} = 510 \text{ Nm}$. Based on these values the input power P_{in} and the output power P_{out} are calculated: $P_{in} = 0.67 \text{ kW}$, $P_{out} = 0.64 \text{ kW}$. An electric motor with nominal power $P_{nom} = 0.75 \text{ kW}$ will be able to ensure that the values of the input data can be achieved.

Existing geared motors, produced by the company KEB are used in this example. Their structural components are 0.75 kW asynchronous squirrel cage motor (2-, 4-, 6- and 8-pole motors are available) and a gear reducer (helical, bevel, worm, with parallel shafts and combined gear units). The above-

given values for the input data are achieved with 52 different combinations, which are given in Table I.

For the static criteria, the optimization is conducted using the total volume of the EMM V_{Σ} in $[\text{cm}^3]$, the overall dimensions of the EMM $L \times B \times H$ in $[\text{cm}^3]$, the total efficiency of the module η_{total} and the total weight of the module m in $[\text{kg}]$. The selected preference function is type V, i.e. with linear preference and indifference area. The selected values for the indifference and preference thresholds are listed in Table II.

Following the methodology of PROMETHEE, the differences between all alternatives in pairs are defined and after a preference function is associated with all criteria, a valued relation between all alternatives is created by calculating the multi-criteria preference degree. Based on the calculated values for the incoming and outgoing flows, a ranking of all alternatives by comparing the calculated net flows can be created. All calculations are conducted using Microsoft Office Excel 2013. The results are shown in Table III.

TABLE I. GEARED MOTORS OF THE COMPANY KEB, ACCORDING TO THE INPUT DATA

MR_ID	Gear_ID	MotID	i calc (-)	V_{Σ} (cm ³)	$L \times B \times H$ (cm ³)	η_{total} (-)	m (kg)
MR0001	G33G12	DM80K2	250.00	17670.4	23 242.53	0.73	29.40
MR0002	G43G22	DM80K2	250.00	24872.02	34 582.60	0.73	42.40
MR0003	G53G22	DM80K2	250.00	39339.66	54 428.98	0.73	67.40
MR0004	K43G12	DM80K2	250.00	22803.36	24 478.44	0.73	40.40
MR0005	K53G22	DM80K2	250.00	35247.69	40 788.35	0.73	61.40
MR0006	K63G22	DM80K2	250.00	51933.89	60 350.16	0.73	87.40
MR0007	S32G12	DM80K2	250.00	21494.26	28 197.08	0.67	38.40
MR0008	S42	DM80K2	250.00	24082.4	34 496.55	0.70	51.40
MR0009	S42G22	DM80K2	250.00	34206.64	44 428.89	0.67	57.40
MR0010	F33G12	DM80K2	250.00	20915.55	28 718.26	0.76	33.40
MR0011	F43G12	DM80K2	250.00	29529.66	43 189.86	0.76	46.40
MR0012	F53G22	DM80K2	250.00	47364.12	70 669.55	0.76	72.40
MR0013	F63	DM80K2	250.00	56223.13	78 157.67	0.76	99.40
MR0014	F63G22	DM80K2	250.00	73634.06	104 617.75	0.76	104.40
MR0015	G33	DM80GC4	125.00	13394.31	16 429.14	0.76	26.00
MR0016	G43	DM80GC4	125.00	17650.88	31 491.62	0.76	37.00
MR0017	G53	DM80GC4	125.00	28130.08	42 076.16	0.76	64.00
MR0018	K43	DM80GC4	125.00	17915.40	23 896.30	0.77	38.00
MR0019	K43G12	DM80GC4	125.00	22803.36	29 799.84	0.74	42.00
MR0020	K53	DM80GC4	125.00	26714.91	37 691.84	0.77	56.00
MR0021	K63	DM80GC4	125.00	40088.26	54 667.83	0.77	84.00
MR0022	S22	DM80GC4	125.00	12403.06	15 009.48	0.72	25.00
MR0023	S32	DM80GC4	125.00	16616.93	22 304.10	0.72	36.00
MR0024	S42	DM80GC4	125.00	24082.40	34 496.55	0.71	53.00
MR0025	F33	DM80GC4	125.00	14850.91	21 595.22	0.77	30.00
MR0026	F43	DM80GC4	125.00	20471.00	32 885.06	0.77	43.00
MR0027	F53	DM80GC4	125.00	30866.40	52 212.47	0.77	67.00
MR0028	G33	DM90SC6	83.33	14275.58	18 916.88	0.72	28.90
MR0029	G43	DM90SC6	83.33	18532.15	27 311.65	0.72	39.90
MR0030	G53	DM90SC6	83.33	29011.35	43 228.91	0.72	66.90
MR0031	G63	DM90SC6	83.33	45228.35	65 054.39	0.72	97.90

MR_ID	Gear_ID	MotID	i calc (-)	V_{Σ} (cm ³)	LxHxB (cm ³)	η_{total} (-)	m (kg)
MR0032	K43	DM90SC6	83.33	18796.67	24 437.28	0.73	40.90
MR0033	K53	DM90SC6	83.33	27596.18	38 607.30	0.73	58.90
MR0034	K63	DM90SC6	83.33	40969.53	41 724.76	0.73	86.90
MR0035	K73	DM90SC6	83.33	65196.54	86 413.39	0.73	138.90
MR0036	S32	DM90SC6	83.33	17498.20	23 089.29	0.68	38.90
MR0037	S42	DM90SC6	83.33	24963.67	35 718.25	0.68	55.90
MR0038	F33	DM90SC6	83.33	15732.18	22 741.15	0.73	32.90
MR0039	F43	DM90SC6	83.33	21352.27	34 478.22	0.73	45.90
MR0040	F53	DM90SC6	83.33	31747.67	54 742.79	0.74	69.90
MR0041	G33	DM100L8	62.50	16656.69	21 529.66	0.67	40.00
MR0042	G43	DM100L8	62.50	20913.26	30 740.27	0.67	51.00
MR0043	G53	DM100L8	62.50	31392.46	48 257.12	0.67	78.00
MR0044	K43	DM100L8	62.50	21177.78	27 306.72	0.67	52.00
MR0045	K53	DM100L8	62.50	29977.29	42 542.26	0.67	70.00
MR0046	K63	DM100L8	62.50	43350.64	61 110.02	0.67	98.00
MR0047	S22	DM100L8	62.50	15665.44	18 016.02	0.63	39.00
MR0048	S32	DM100L8	62.50	19879.31	26 306.28	0.63	50.00
MR0049	S42	DM100L8	62.50	27344.78	40 070.08	0.62	67.00
MR0050	F33	DM100L8	62.50	18113.29	26 218.92	0.67	44.00
MR0051	F43	DM100L8	62.50	23733.38	39 483.68	0.67	57.00
MR0052	F53	DM100L8	62.50	34128.78	61 974.96	0.67	81.00

In the above table the following designation are used:

- MR_ID – geared motor identification;
- Gear_ID – gear reducer identification;
- Mot_ID – electric motor identification;
- i cal – calculated value of the ratio;
- G33 – helical gear unit coaxial, size 3, 3-stage;
- K43 – helical bevel gear unit, size 4, 3-stage;
- F33 – helical gear unit with parallel shafts, size 3, 3-stage;
- S22 – helical worm gear unit, size 2, 2-stage;
- DM90SC6 – asynchronous squirrel cage motor series DM, size 90S, 6-pole.

TABLE II. PREFERENCE FUNCTION TYPE AND PREFERENCE AND INDIFFERENCE THRESHOLDS

Criteria	Extremum	Ps - type	q	p
k1	V_{Σ} (cm ³)	min	V	1 500.00
k2	LxHxB (cm ³)	min	V	2 000.00
K3	η_{total}	max	V	0.05
k4	weight (kg)	min	V	20.00

TABLE III. OUTGOING AND INCOMING FLOWS, NET FLOW AND RANKING OF ALL ALTERNATIVES

alternatives	ϕ^+	ϕ^-	$\phi(a_i) = \phi^+ - \phi^-$	rank
a1	18.757	0.640	18.117	5
a2	10.933	6.624	4.310	25
a3	4.051	20.586	-16.534	44
a4	14.863	2.414	12.449	16
a5	6.458	14.725	-8.267	35
a6	2.421	27.134	-24.713	47
a7	13.775	4.658	9.117	20
a8	9.940	6.759	3.181	27
a9	5.684	17.036	-11.352	39

alternatives	ϕ^+	ϕ^-	$\phi(a_i) = \phi^+ - \phi^-$	rank
a10	16.314	2.648	13.665	14
a11	9.141	12.204	-3.063	33
a12	3.713	24.758	-21.045	41
a13	2.839	31.902	-29.063	50
a14	1.991	34.059	-32.068	51
a15	25.299	0.000	25.299	1
a16	16.446	2.703	13.742	15
a17	8.507	12.096	-3.589	32
a18	19.056	0.718	18.337	7
a19	13.473	3.777	9.696	19
a20	10.341	9.167	1.174	28
a21	5.305	23.818	-18.513	45
a22	24.975	0.000	24.975	2
a23	18.363	0.406	17.957	8
a24	9.919	6.460	3.459	26
a25	22.367	0.232	22.134	3
a26	14.539	3.909	10.629	18
a27	7.104	16.868	-9.763	37
a28	21.841	0.059	21.781	4

alternatives	φ^+	φ^-	$\varphi(a_i) = \varphi^+ - \varphi^-$	rank
a29	15.505	1.528	13.977	13
a30	6.522	13.311	-6.789	34
a31	2.146	29.110	-26.964	48
a32	16.467	0.954	15.513	10
a33	8.144	10.185	-2.041	31
a34	5.041	21.066	-16.025	43
a35	1.098	36.893	-35.795	52
a36	17.089	1.825	15.265	11
a37	8.940	8.647	0.293	29
a38	19.414	0.371	19.043	6
a39	11.817	4.831	6.986	24
a40	5.386	18.225	-12.838	40
a41	17.884	2.296	15.589	9
a42	12.002	5.356	6.645	23
a43	4.958	19.909	-14.951	42
a44	12.934	4.425	8.509	21
a45	5.999	16.005	-10.006	36
a46	2.078	30.318	-28.240	49
a47	19.838	5.559	14.279	12
a48	13.894	7.150	6.744	22
a49	6.935	17.870	-10.935	38
a50	15.285	3.110	12.175	17
a51	8.508	10.042	-1.534	30
a52	3.191	24.146	-20.955	46

The results show that alternative **a15** (MR0015 - G33 DM80GC4 – helical geared motor with 4-pole asynchronous squirrel cage motor) is the optimal solution among all alternatives, according to the predefined criteria. Since the multi-criteria optimization does not give just one single solution, the decision maker has the opportunity to shift out large number of alternatives and to determine which of them will be most suitable for a given application. It is possible to conduct further optimization of the chosen alternative by the decision maker in order to find the most suitable alternatives, which will contribute to even higher precision at applying method PROMETHEE. For example, the alternatives ranked from 1st up to 10th place have small differences between the values for the incoming, outgoing and net flows. Therefore, if the decision maker is uncertain of which alternative will be best suited for given application, another optimization of these ten selected alternatives can be carried out, at which new criteria can be introduced as well.

IV. CONCLUSION

The main conclusions drawn from the study reported in this article is that method PROMETHEE offers an exact ranking of multiple alternatives at multiple criteria and

contributes to an optimized decision-making. The main advantage is that this method gives the opportunity for fine setting of the preferences by the decision maker.

As a disadvantage can be pointed out the need of preliminary knowledge of the method, in order for the decision maker to be able to select suitable preference function and appropriate values for the indifference and preference thresholds.

This method can be easily applied at solving multi-criteria optimization tasks and permits automation of the optimization process using a different software (such as Matlab, Visual Promethee, Diviz, etc.).

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